Numerical studies on the early universe by large-scale numerical computations in the Lorentzian IIB matrix model

Yuta Ito (SOKENDAI)
In collaboration with
Jun Nishimura (KEK,SOKENDAI), Asato Tsuchiya (Shizuoka U.)

1. Introduction

Lorentzian version of the IIB matrix model

- ✓ A non-perturbative formulation of superstring theory
- ✓ Eigenvalues of A_0 represent the "real time" coordinates.

$$S_b = -\frac{1}{4g^2}\mathrm{tr}\,[A_\mu, A_\nu]^2$$

$$S_f = -\frac{1}{2g^2}\mathrm{tr}\,\Psi \mathcal{C}\Gamma^\mu [A_\mu, \Psi]$$

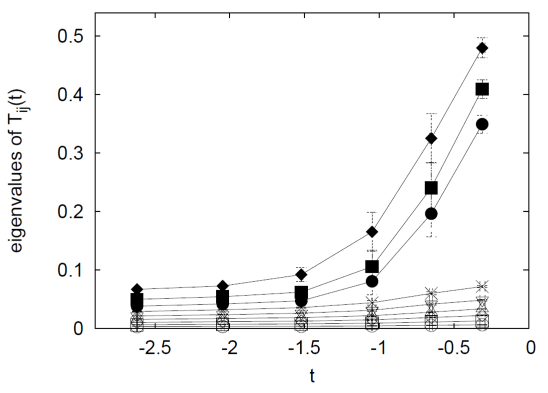
$$\eta_{\mu\nu} = \mathrm{diag}\,(-1, 1, \cdots, 1)$$

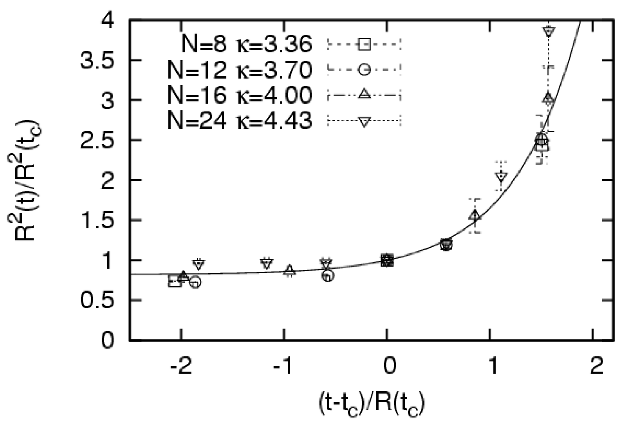
$A_\mu, \Psi : N \times N$ Hermitian matrices

SO(9,1) symmetry

[Ishibashi, Kawai, Kitazawa, Tsuchiya, Nucl. Phys. B **498** (1997) 467]

In previous works





- ✓ SSB of SO(9,1)
- ✓ Exponentially expanding

[Kim, Nishimura, Tsuchiya, Phys.Rev.Lett. 108 (2012) 011601]

2. Effects of the fermion action

Fermion action

$$S_f = \mathrm{tr}\,\Psi_\alpha\,(\Gamma^\mu)_{\alpha\beta}\,[A_\mu, \Psi_\beta]$$
$$= \mathrm{tr}\,\Psi_\alpha\,(\Gamma^0)_{\alpha\beta}\,[A_0, \Psi_\beta] + \mathrm{tr}\,\Psi_\alpha\,(\Gamma^i)_{\alpha\beta}\,[A_i, \Psi_\beta]$$

dominant at early times $A_0 \gg A_i$

Pfaffian

$$\mathrm{Pf}\,\mathcal{M}(A) = \Delta(\alpha)^{2(d-1)} = \prod_{I<J}(\alpha_I - \alpha_J)^{2(d-1)}$$

repulsive force between A_0 eigenvalues

- Time extends to infinity.
- Exponential expansion

dominant at late times $A_0 \ll A_i$

As a toy model,

$$\mathrm{Pf}\,\mathcal{M}(A) = 1$$

➡ Bosonic model

3. Bosonic IIB matrix model

$$Z_b = \int \mathcal{D}A\, e^{iS_b}$$

Constraint : $\frac{1}{N}\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} = 0$

IR Cut off : $\frac{1}{N}\mathrm{tr}\,A_i^2 = 1$

Gauge fixing : $A_0 = \mathrm{diag}\,(\alpha_1, \alpha_2, \cdots, \alpha_N)$, $\alpha_1 < \alpha_2 < \cdots < \alpha_N$

$\Delta(\alpha)^2 = \prod_{I<J}(\alpha_I - \alpha_J)^2$: FP determinant

✓ Note: No need for any temporal cutoffs.

Partition function in simulations

$$Z_b = \int \mathcal{D}A_i \prod_{I=1}^{N} d\alpha_I\, \Delta(\alpha)^2\, \delta\left(\frac{1}{N}\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}\right)\delta\left(\frac{1}{N}\mathrm{tr}\,A_i^2 - 1\right)$$

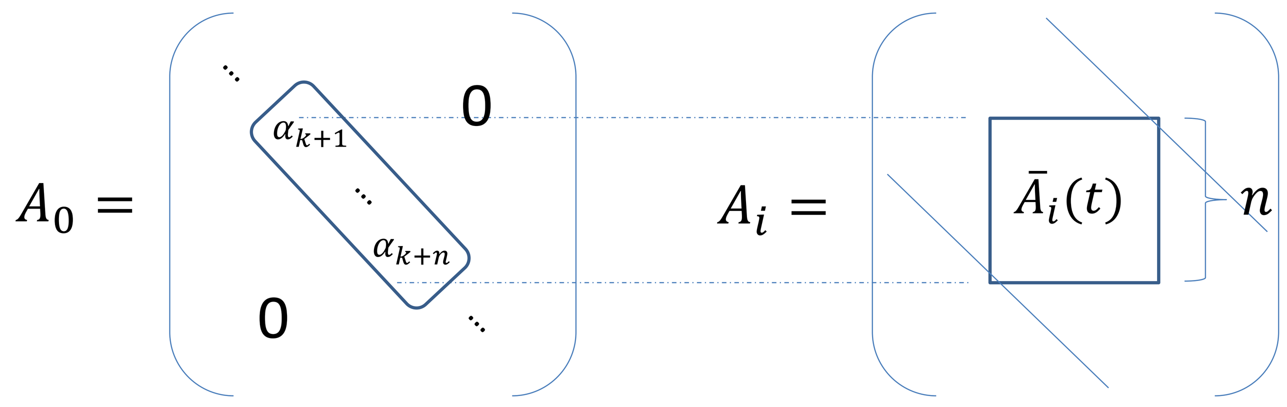
Bosonic 1-loop effect

$$\Delta(\alpha)^{-2d} = \prod_{I<J}(\alpha_I - \alpha_J)^{-2d}$$

attractive force ⇨ Time extent becomes finite.

4. Evaluating the time evolution

Structure of matrices



$t = \frac{1}{n}\sum_{i=1}^{n}\alpha_{k+i}$: time

$\bar{A}_i(t)$: state of the universe at t

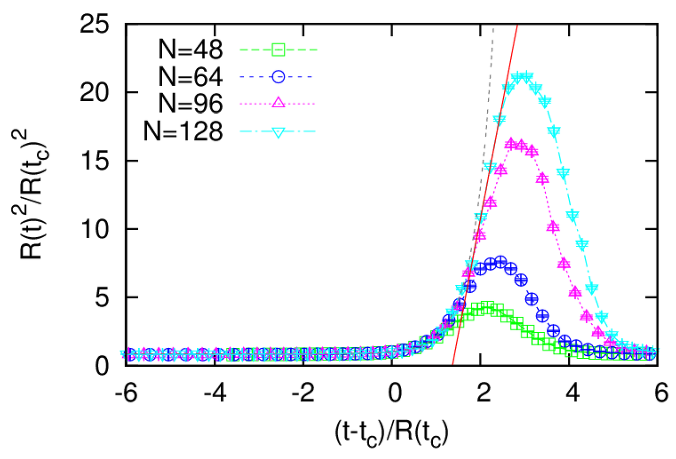
The extent of spacetime

$$R^2(t) = \frac{1}{n}\sum_i \mathrm{tr}\,\bar{A}_i^2(t)$$

The moment of inertia tensor

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\,(\bar{A}_i(t)\,\bar{A}_j(t))$$

6d bosonic IKKT model



Eigenvelues of $T_{ij}(t)$ represent the extent in each spatial direction.

5. Numerical result

SSB from SO(d) to SO(3)

There exists a critical value N_c such that the SSB occurs for $N > N_c$.

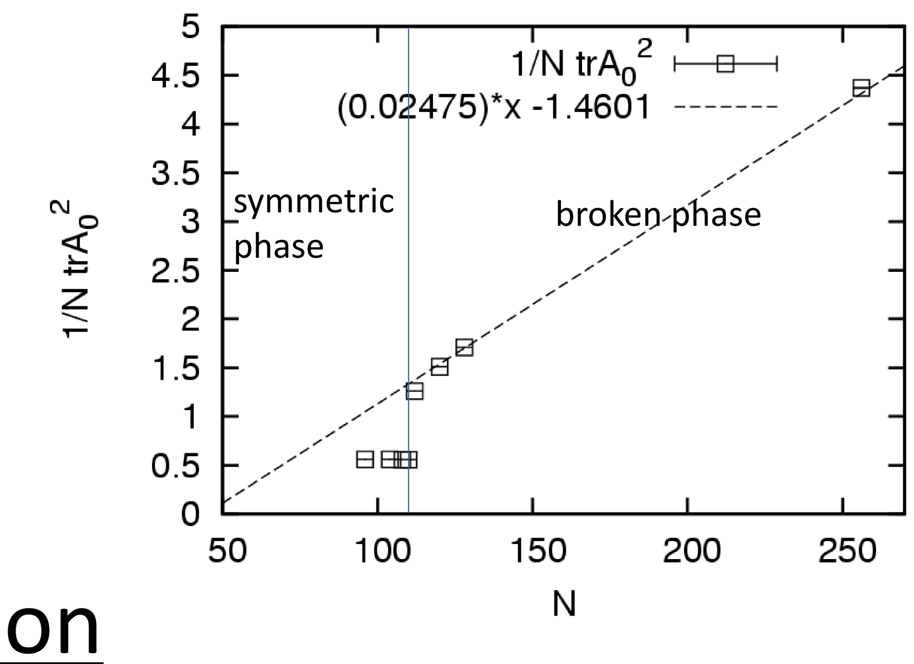
6d case $N_c \cong 34$

10d case $N_c \cong 112$

Time extent

$$\frac{1}{N}\mathrm{tr}\,A_0^2$$

10d bosonic IKKT model



Power-law expansion

- Scaling of R(t)
- Expanding behavior of 3d space

✓ at early times

$$R(t) \sim e^{\Lambda t}$$

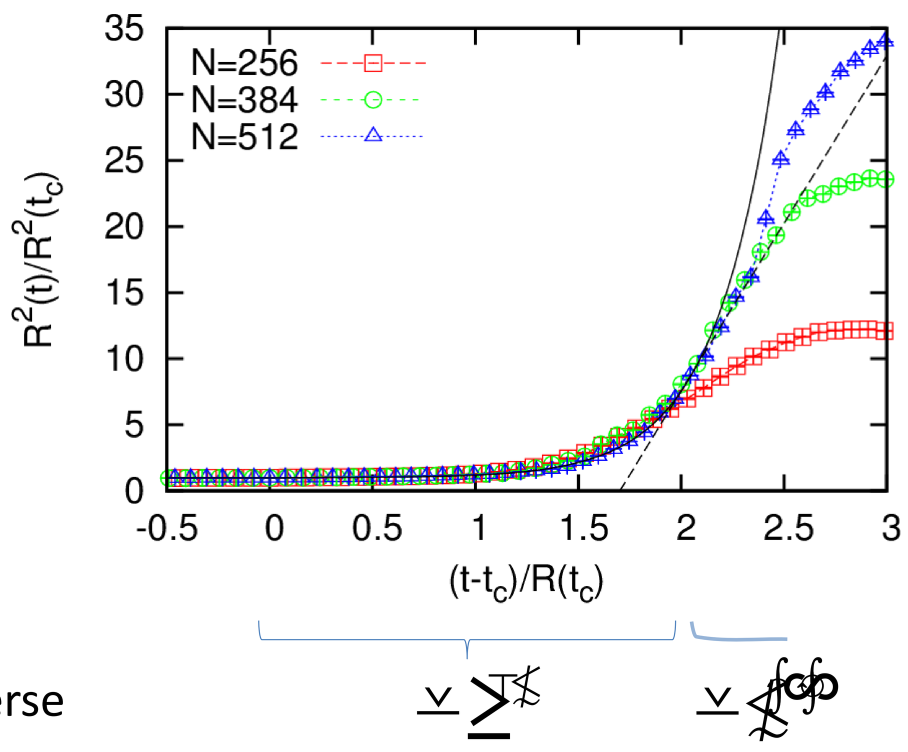
transition

✓ at late times

$$R(t) \sim t^{1/2}$$

cf) radiation dominated FRW universe

10d bosonic IKKT model



6. Summary

- It turns out that the SSB from SO(d) to SO(3) occurs even in the bosonic IIB matrix model.
  The time has a finite extent without any cutoffs.
  There exists a critical matrix size N_c such that the SSB occurs for $N > N_c$.
- Scaling of R(t) is confirmed at late times, where the expanding behavior changes from exponential expansion into a power-law ($t^{1/2}$).

→ It may be interpreted as the transition from inflation to radiation dominated FRW universe.