Anti-evaporation in massive/bi-gravity

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1. Introduction

It is well known that horizon radius of the black hole usually degreases by the Hawking radiation.

⟶ Black hole evaporation [Hawking (1974)]

However, the black hole radius can increases by the quantum correction for the Nariai black hole in GR.

⟶ Black hole anti-evaporation [Bousso and Hawking (1997)]

- ☐ The anti-evaporation occurs in modified gravity (i.e. F(R) gravity). [Nojiri and Odintsov (2013)]
- ☐ Does the anti-evaporation occur in massive gravity & bigravity? [de Rham, Gabadadze and Tolley (2011)] [Hassan and Rosen (2012)]

2. Anti-evaporation

We consider the Schwarzschild-de Sitter (SdS) solution.

$$ds^2 = -V(r)dt^2 + V(r)^{-1} + r^2 d\Omega^2\,, \quad V(r) = 1 - \frac{2\mu}{r} - \frac{\Lambda}{3}r^2$$

- ☐ Λ is positive cosmological constant and μ is mass parameter.
- ☐ The limit $\mu \to 0$ corresponds to the de Sitter solution and $\Lambda \to 0$ corresponds to the Schwarzschild solution.
- ☐ The topology of the spacelike sections is $S^1 \times S^2$.

For $0 < \mu < \frac{1}{3}\Lambda^{-1/2}$, V has two positive roots r_c and r_b, corresponding to the cosmological and black hole horizons.

☐ In the limit $\mu \to \frac{1}{3}\Lambda^{-1/2}$, the size of the black hole horizon approaches that of cosmological horizon. → Nariai solution

☐ We consider the perturbations for the Nariai spacetime in GR.

Spherically symmetric ansatz.

$$ds^2 = e^{2\rho(t,x)}\left(-dt^2 + dx^2\right) + e^{-2\phi(t,x)} d\Omega^2$$

We consider N massless scalar fields as radiation.
Then, the effective action with quantum effects is given by

$$S = \frac{1}{16\pi}\int d^2x\sqrt{-g}\Big[\Big(e^{-2\phi} + \frac{\kappa}{2}(Z + \omega\phi)\Big)R \\ -\frac{\kappa}{4}(\nabla Z)^2 + 2 + 2e^{-2\phi}(\nabla\phi)^2 - 2e^{-2\phi}\Lambda\Big]$$

where $\kappa \equiv \frac{2N}{3}$, Z is auxiliary fields and ω is arbitrary constant.

Now, we consider the metric perturbation. $e^{2\phi} = \Lambda_2[1 + 2\epsilon\sigma(t)\cos x]$

In classical case, $\kappa = 0$, no evaporation takes place and horizon size remains.
In quantum case, $\kappa > 0$, there are two types of perturbations, one type is stable at least initially and the BH size increases.

3. Massive gravity & Bigravity

We consider the anti-evaporation in massive gravity and bigravity as modified gravity theory.

☐ Massive gravity describes interacting a massive graviton.

- Dynamical metric $g_{\mu\nu}$ and fixed (reference) metric $f_{\mu\nu}$
- Background dependence because of $f_{\mu\nu}$

$$S = M_g^2\int d^4x\sqrt{-g}R(g) - 2m_0^2 M_g^2\int d^4x\sqrt{-g}\sum_{n=0}^{3}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

[de Rham, Gabadadze and Tolley (2011)]

☐ Bigravity describes interacting massive and massless gravitons.

- Two dynamical metrics $g_{\mu\nu}$ and $f_{\mu\nu}$
- Background independence (general coordinate trans. inv.)

$$S = M_g^2\int d^4x\sqrt{-g}R(g) + M_f^2\int d^4x\sqrt{-f}R(f) \\ -2m_0^2 M_{\rm eff}^2\int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

[Hassan and Rosen (2012)]

Planck mass scales: $M_g, M_f, \frac{1}{M_{\rm eff}^2} = \frac{1}{M_g^2} + \frac{1}{M_f^2}$ $\quad \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\rho}\left(\sqrt{g^{-1}f}\right)^{\rho}{}_{\nu} = g^{\mu\rho}f_{\rho\nu}$

Free parameters: β_n, Mass of massive spin-2 field (massive graviton): m_0

$$e_0(\mathbf{X}) = 1\,, \quad e_1(\mathbf{X}) = [\mathbf{X}]\,, \quad e_2(\mathbf{X}) = \frac{1}{2}\left([\mathbf{X}]^2 - [\mathbf{X}^2]\right)\,, \quad e_3(\mathbf{X}) = \frac{1}{6}\left([\mathbf{X}]^3 - 3[\mathbf{X}][\mathbf{X}^2] + 2[\mathbf{X}^3]\right)$$

$$e_4(\mathbf{X}) = \frac{1}{24}\left([\mathbf{X}]^4 - 6[\mathbf{X}]^2[\mathbf{X}^2] + 3[\mathbf{X}^2]^2 + 8[\mathbf{X}][\mathbf{X}^3] - 6[\mathbf{X}^4]\right) = \det(\mathbf{X})\,, \quad [\mathbf{X}] = X^{\mu}{}_{\mu}$$

4. SdS solutions in Bigravity

In order to discuss the anti-evaporation, we need to confirm that asymptotically de-Sitter solutions are realized.
At first, we discuss the existence of the SdS solution in bigravity.

We consider a particular class of solutions: $f_{\mu\nu} = C^2 g_{\mu\nu}$
In this assumption, the equations of motion are given by

$$0 = R_{\mu\nu}(g) - \frac{1}{2}R(g)g_{\mu\nu} + \left(\frac{m_0 M_{\rm eff}}{M_g}\right)^2\left[\beta_0 + 3|C|\beta_1 + 3C^2\beta_2 + C^2|C|\beta_3\right]g_{\mu\nu}$$

$$0 = R_{\mu\nu}(f) - \frac{1}{2}R(f)f_{\mu\nu} + \left(\frac{m_0 M_{\rm eff}}{M_f}\right)^2\frac{1}{C^2|C|}\left[\beta_1 + 3|C|\beta_2 + 3C^2\beta_3 + C^2|C|\beta_4\right]f_{\mu\nu}$$

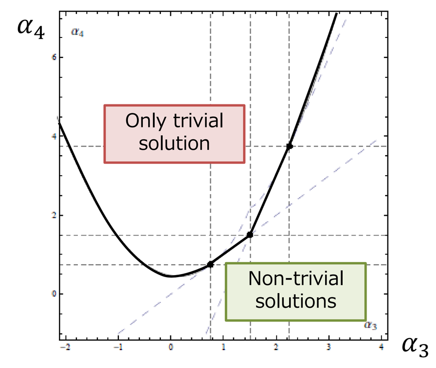
cosmological constants

And we consider 2-para. family of bigravity

$$\beta_0 = 6 - 4\alpha_3 + \alpha_4, \quad \beta_1 = -3 + 3\alpha_3 - \alpha_4$$
$$\beta_2 = 1 - 2\alpha_3 + \alpha_4, \quad \beta_3 = \alpha_3 - \alpha_4, \quad \beta_4 = \alpha_4, \quad M_f = M_g$$

We find that above models of bigravity has solution $C = 1$, when the cosmological constants vanish for arbitrary (α_3, α_4).

Now, we classify two parameters when we obtain non-trivial solution with $C \neq 1$ and non-vanishing cosmological constants.
We obtain following results.



[TK (2013)]

Regarding cosmological constant:

- The sign depends on α_3, α_4 and corresponding C.
- For instance, the model with $(\alpha_3, \alpha_4) = (1, -1), (-1, 1), (-1, -1)$ have asymptotically de Sitter solution.

→ Nariai solutions exist.

5. Stability of Nariai spacetime

Next, we evaluate the stability of the Nariai spacetime in Biravity.

At first, we investigate the classical stability because the anti-evaporation occurs without quantum effects in F(R) gravity. [Nojiri and Odintsov (2013)]

In the same way as the case of GR, we consider the spherically symmetric ansatz for $g_{\mu\nu}$ and $f_{\mu\nu}$.

$$g_{\mu\nu}dx^\mu dx^\nu = e^{2\rho_1(t,x)}\left(-dt^2 + dx^2\right) + e^{-2\varphi_1(t,x)}d\Omega^2$$
$$f_{\mu\nu}dx^\mu dx^\nu = e^{2\rho_2(t,x)}\left(-dt^2 + dx^2\right) + e^{-2\varphi_2(t,x)}d\Omega^2$$

We also assume $f_{\mu\nu} = C^2 g_{\mu\nu}$ and $M_f = M_g$ because we can obtain the Nariai solution with these conditions.

☐ When we consider perturbations for the Nariai spacetime in bigravity, two sets of the perturbations for $g_{\mu\nu}$ and $f_{\mu\nu}$ are required because two metric tensors are independent.

Finally, we obtain the equations for the perturbations in the following forms.

$$\delta G_{\mu\nu}(g) + \delta I^{\lambda}{}_{\nu}(\sqrt{g^{-1}f})g_{\mu\lambda} + \bar{I}^{\lambda}{}_{\nu}(\sqrt{g^{-1}f})\delta g_{\mu\lambda} = 0$$
$$\delta G_{\mu\nu}(f) + \delta I^{\lambda}{}_{\nu}(\sqrt{f^{-1}g})f_{\mu\lambda} + \bar{I}^{\lambda}{}_{\nu}(\sqrt{f^{-1}g})\delta f_{\mu\lambda} = 0$$

where $G_{\mu\nu}$s are the Einstein tensors, $\bar{I}^{\lambda}{}_{\nu}$s and $\delta I^{\lambda}{}_{\nu}$s are the interaction terms and their perturbations.

☐ Moreover, the perturbations should satisfy the Bianchi identities.

→ I'm trying to calculate… (in progress)

6. Discussion and Future directions

- ☐ We need to evaluate the stability with/without quantum corrections.
- ☐ If we find that the anti-evaporation doesn't occur classically, we need to take the quantum effects into account.

- ■ Can we analyze the stability without specifying the parameters in massive/bi-gravity? We can classify the parameter regions where the perturbations are stable/unstable.
- ■ Can we evaluate the BH entropy?
- ■ Can we estimate the number of primordial BH? This estimation also allows us to limit the parameter regions.

Thank you for your attention!