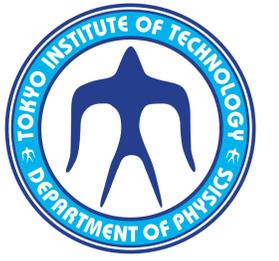


# Supersymmetric backgrounds from 5d $\mathcal{N} = 1$ supergravity

Hiroki Matsuno (Tokyo Institute of Technology)

Based on JHEP **1407** (2014) 055 [arXiv:1404.0210] with Yosuke Imamura



## Introduction

The construction of rigid SUSY on Euclidean curved backgrounds

↓ Computation by localization  
 $S^4$  [Pestun '07],  $S^3$  [Kapustin-Willet-Yaakov '09], ...

Nonperturbative properties of SUSY field theory and dualities

The **systematic** construction of SUSY on curved backgrounds

[Festuccia-Seiberg '11]

1. Start from off-shell SUGRA, and set the gravity multiplet as background
2. SUSY condition:  $\delta_Q \psi_\mu = 0$ 
  - ▶ By solving  $\delta_Q \psi_\mu = 0$ , we obtain backgrounds which admit rigid SUSY
    - ▷ 4d: Hermitian manifold or (squashed)  $S^4$  [Klare-Tomasiello-Zaffaroni, Dumitrescu-Festuccia-Seiberg, Dumitrescu-Festuccia '12]
    - ▷ 3d: Almost contact metric structure with a certain integrability condition [Closset-Dumitrescu-Festuccia-Komargodski '12]
  - ▶ Using this formulation, we can know whether the deformation of the backgrounds corresponds the Q-exact deformation of the action or not
    - Parameter dependence of 4d/3d partition function [Closset-Dumitrescu-Festuccia-Komargodski '13]

How about in 5d?

Motivation to 5d: the close relation to 6d (2, 0) theory

- ▶ KK mode in 6d is conjectured as instanton in 5d [Douglas, Lambert '10]
- ▶  $\mathcal{N}^3$  behavior of  $S^5$  partition function [Kallen-Minahan-Nedelin-Zabzine '12]
- ▶ and also for  $S^3 \times \Sigma$ ,  $S^2 \times M_3$ , ...

## 5d $\mathcal{N} = 1$ Weyl multiplet [Kugo-Ohashi '00]

|          | fields                    | dof | $Sp(1)_R$ |                                        |
|----------|---------------------------|-----|-----------|----------------------------------------|
| bosons   | vielbein                  | 10  | <b>1</b>  | $e_\mu^\nu$                            |
|          | $U(1)_Z$ gauge field      | 4   | <b>1</b>  | $a_\mu$ (field strength $f_{\mu\nu}$ ) |
|          | anti-sym. tensor          | 10  | <b>1</b>  | $v^{\mu\nu}$                           |
|          | $Sp(1)_R$ triplet scalars | 3   | <b>3</b>  | $t_a$                                  |
|          | $Sp(1)_R$ gauge field     | 12  | <b>3</b>  | $V_\mu^a$                              |
|          | scalar                    | 1   | <b>1</b>  | $C$                                    |
| fermions | gravitino                 | 32  | <b>2</b>  | $\psi_{I\mu\alpha}$                    |
|          | fermion                   | 8   | <b>2</b>  | $\eta_\alpha$                          |

$$\delta_Q(\xi)\psi_\mu = D_\mu\xi - f_{\mu\nu}\gamma^\nu\xi + \frac{1}{4}\gamma_{\mu\rho\sigma}v^{\rho\sigma}\xi - t\gamma_\mu\xi$$

$$\delta_Q(\xi)\eta = -2\gamma_\nu\xi D_\mu v^{\mu\nu} + \xi C + 4(\not{D}t)\xi + 8(\not{f} - \not{y})t\xi + \gamma^{\mu\nu\rho\sigma}\xi f_{\mu\nu}f_{\rho\sigma}$$

$D_\mu$ : covariant derivative for Lorentz  $Sp(2)_L = SO(5)_L$ ,  
R-sym.  $Sp(1)_R = SU(2)_R$  and central charge sym.  $U(1)_Z$

- ▶ We must solve not only  $\delta_Q \psi_\mu = 0$  but also  $\delta_Q \eta = 0$
- ▶ Partial analysis, which focused on  $\delta_Q \psi_\mu = 0$  was done in [Pan '13]

## SUSY backgrounds

We assume the existence of one symplectic Majorana spinor  $\xi_{I\alpha}$  which is a SUSY transformation parameter

Bilinears of  $\xi$ :

$$S = (\xi\xi), \quad R^\mu = (\xi\gamma^\mu\xi), \quad J_{\mu\nu}^a = \frac{1}{S}(\xi\tau^a\gamma_{\mu\nu}\xi)$$

Define the 5th direction:  $R^\mu\partial_\mu = \partial_5$ ,  $m, n, \dots = 1, \dots, 4$

We can locally take

$$e^{\hat{m}} = e_n^{\hat{m}} dx^n, \quad e^{\hat{5}} = S(dx^5 + \mathcal{V}_m dx^m)$$

$J_{\mu\nu}^a$  satisfies

$$J_{\hat{m}\hat{5}}^a = 0, \quad -\frac{1}{2}\epsilon_{\hat{m}\hat{n}\hat{p}\hat{q}}^{(4)} J_{\hat{p}\hat{q}}^a = J_{\hat{m}\hat{n}}^a, \quad J_{\hat{m}\hat{p}}^a J_{\hat{p}\hat{n}}^b = \delta_{ab}\delta_{\hat{m}\hat{n}} + i\epsilon_{abc}J_{\hat{m}\hat{n}}^c$$

By  $\delta_Q \psi_\mu = \delta_Q \eta = 0$ , all fields (including  $e^{\hat{m}}$ ,  $S$  and  $\mathcal{V}_m$ ) are  $x^5$ -independent and, up to gauge transformation,

$$a_5 = \frac{1}{2}S,$$

$$v_{\hat{p}\hat{q}} = \epsilon_{\hat{p}\hat{q}\hat{m}\hat{n}}^{(4)} \left( \frac{S}{4} \mathcal{W}_{\hat{m}\hat{n}} - f_{\hat{m}\hat{n}} + t_a J_{\hat{m}\hat{n}}^a \right),$$

$$V_{\hat{m}}^a = \frac{1}{4} \omega_{\hat{m}\hat{p}\hat{q}}^{(4)} J_{\hat{p}\hat{q}}^a + \frac{1}{2} J_{\hat{m}\hat{p}}^a v^{\hat{p}\hat{5}},$$

$$V_{\hat{5}}^a = \frac{1}{2} J_{\hat{m}\hat{n}}^a \left( f_{\hat{m}\hat{n}} - \frac{S}{2} \mathcal{W}_{\hat{m}\hat{n}} \right) + t_a,$$

$$C = 2D_{\hat{m}}^{(4)} v^{\hat{m}\hat{5}} + 4t_a J_{\hat{m}\hat{n}}^a f_{\hat{m}\hat{n}} + 32t_a t_a - \epsilon_{\hat{m}\hat{n}\hat{p}\hat{q}}^{(4)} \left( f_{\hat{m}\hat{n}} - \frac{S}{2} \mathcal{W}_{\hat{m}\hat{n}} \right) \left( f^{\hat{p}\hat{q}} - \frac{S}{2} \mathcal{W}^{\hat{p}\hat{q}} \right)$$

( $\mathcal{W}$ : field strength of  $\mathcal{V}$ )

Independent fields are  $e_n^{\hat{m}}$ ,  $S$ ,  $\mathcal{V}_m$ ,  $a_{\hat{m}}$ ,  $v^{\hat{m}\hat{5}}$  and  $t^a$

(Similar analysis can be done for background vector multiplets)

## Q-exact deformations

A small deformation of the background gives the change of the action

$$S_1 = \int d^5x \sqrt{g} \left[ -\delta e_\mu^\nu T_\nu^\mu + \delta V_\mu^a R_a^\mu + (\delta\psi_\mu S^\mu) - \delta a_\mu J^\mu \right. \\ \left. + \delta v^{\mu\nu} M_{\mu\nu} + \delta C \Phi + (\delta\eta\chi) + \delta t_a X_a \right]$$

( $T_\nu^\mu, R_a^\mu, \dots$ : components of the supercurrent multiplet)

On the other hand, a Q-exact deformation which can be regarded as a change of background fields generally takes the form

$$S_Q(\xi; H_\mu, K) = \delta_Q(\xi) \int \sqrt{g} d^5x [H_\mu S^\mu + K\chi]$$

For example, by using the transformation law for  $\chi$ ,

$$S_Q(\xi; 0, \frac{4}{S} k^a \xi \tau_a) = \int d^5x \sqrt{g} \left( k^a R_a^{\hat{5}} - 2k^a J_{\hat{m}\hat{n}}^a M^{\hat{m}\hat{n}} \right. \\ \left. + (4k^a J_{\hat{m}\hat{n}}^a f^{\hat{m}\hat{n}} + 64k^a t_a) \Phi + k^a X_a \right)$$

This can be realized as the deformation  $\delta t_a = k^a$  and associated deformations of dependent fields

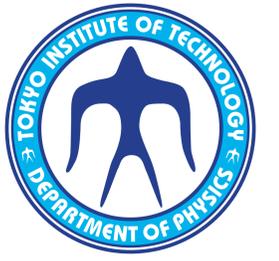
Similarly, we obtain the result that **all SUSY-preserving local deformations give Q-exact deformations and do not change the partition function**

(We obtain similar results for background vector multiplets)

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## Vector multiplet and Lagrangian [Kugo-Ohashi '00]

|          | fields           | dof | $Sp(1)_R$ |                                        |
|----------|------------------|-----|-----------|----------------------------------------|
| bosons   | gauge field      | 4   | <b>1</b>  | $A_\mu$ (field strength $F_{\mu\nu}$ ) |
|          | scalar           | 1   | <b>1</b>  | $\phi$                                 |
|          | auxiliary fields | 3   | <b>3</b>  | $D_a$                                  |
| fermions | gaugino          | 8   | <b>2</b>  | $\lambda_{I\alpha}$                    |
|          | "prepotential"   |     |           | $\mathcal{F}(\phi)$                    |

The Lagrangian for vector multiplets in 5d  $\mathcal{N} = 1$  SUGRA is

$$e^{-1}\mathcal{L}_{\text{SUGRA}}^{(V)} = e^{-1}\mathcal{L}_0^{(V)} + e^{-1}\mathcal{L}_1^{(V)},$$

$$e^{-1}\mathcal{L}_0^{(V)} = -\frac{1}{2}\mathcal{F}_i[\lambda, \lambda]^i$$

$$+ \mathcal{F}_{ij} \left( \frac{1}{4}F_{\mu\nu}^i F^{\mu\nu j} + \frac{1}{2}D_\mu \phi^i D^\mu \phi^j - \frac{1}{2}D_a^i D_a^j - \frac{1}{2}\lambda^i \not{D} \lambda^j \right)$$

$$+ \mathcal{F}_{ijk} \left( \frac{i}{6}[\text{CS}]_5^{ijk} + \frac{1}{4}\lambda^i (i\not{F}^j + D^j) \lambda^k \right),$$

$$e^{-1}\mathcal{L}_1^{(V)} = \mathcal{F} (C - 20t_a t_a - 4f_{\mu\nu} v^{\mu\nu} - 6f_{\mu\nu} f^{\mu\nu})$$

$$- i\mathcal{F}_i F_{\mu\nu}^i (v^{\mu\nu} + 2f^{\mu\nu}) + \frac{1}{4}\mathcal{F}_{ij} \lambda^i (\psi + 2\not{F}) \lambda^j$$

+ (terms including  $\psi_{\mu I}$  or  $\eta$ ).

$$\mathcal{F}_i = \frac{\partial \mathcal{F}}{\partial \phi^i}, \quad \mathcal{F}_{ij} = \frac{\partial^2 \mathcal{F}}{\partial \phi^i \partial \phi^j}, \quad \mathcal{F}_{ijk} = \frac{\partial^3 \mathcal{F}}{\partial \phi^i \partial \phi^j \partial \phi^k}$$

► Prepotential: gauge invariant homogeneous cubic polynomial in  $\phi$

►  $i, j, k$  run over ordinary vector multiplets and "the central charge vector multiplet"

$$(\phi, A_\mu, \lambda, D_a)^{i=0} = (1, 2ia_\mu, 0, -2t_a)$$

► If  $\mathcal{F}$  includes only dynamical  $\phi$ ,

▷ The theory is conformal (CS)

▷ Symmetries of the background restrict  $v^{\mu\nu} + 2f^{\mu\nu}$

► If  $\mathcal{F}$  includes  $\phi^0$ ,

▷ The theory is not conformal (YM, FI)

▷ Symmetries of the background restrict **not only  $v^{\mu\nu} + 2f^{\mu\nu}$  but also  $f^{\mu\nu}$**

## Example: $S^5$

$S^5$ :  $S^1$  fibration over  $\mathbb{C}P_2$

$$ds^2 = ds_{\mathbb{C}P_2}^2 + e^{\hat{5}} e^{\hat{5}}, \quad ds_{\mathbb{C}P_2}^2 = e^{\hat{m}} e^{\hat{m}}, \quad e^{\hat{5}} = r(dx^5 + \mathcal{V})$$

$SO(6)$  invariance  $\longrightarrow$  impose  $v^{\mu\nu} + 2f^{\mu\nu} = 0$

$$V_{\hat{m}}^a = -\frac{3i}{2}\mathcal{V}_m \delta^{a3}, \quad V_{\hat{5}}^a = \frac{3i}{2r} \delta^{a3}, \quad \longrightarrow \text{flat connection, gauged away}$$

$$v_{\hat{m}\hat{5}} = 0, \quad f_{\hat{m}\hat{n}} J_{\hat{m}\hat{n}}^a + 2t_a = -\frac{i}{r} \delta^{a3}$$

$$P \equiv C - 20t_a t_a - 4f_{\mu\nu} v^{\mu\nu} - 6f_{\mu\nu} f^{\mu\nu} = \frac{5}{r^2}$$

If  $\mathcal{F}$  includes  $\phi^0 \phi^i \phi^j$ , also impose  $f^{\mu\nu} = 0$

$$f_{\hat{m}\hat{n}} = 0, \quad t_a = -\frac{i}{2r} \delta^{a3}$$

This agree with [Hosomichi-Seong-Terashima '12]



## Example: $S^4 \times \mathbb{R}$

$$ds^2 = ds_{S^4}^2 + (dx^5)^2$$

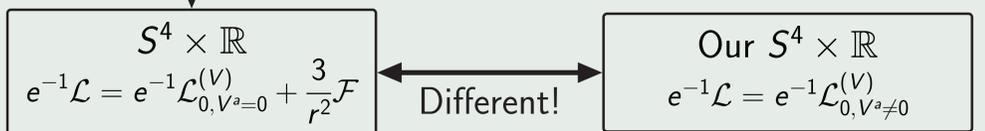
$SO(5)$  invariance  $\longrightarrow v^{\mu\nu} + 2f^{\mu\nu} = 0 / v^{\mu\nu} = f^{\mu\nu} = 0$

$$v_{\hat{m}\hat{5}} = 0, \quad V^a = \frac{1}{4}\omega_{\hat{p}\hat{q}}^{S^4} J_{\hat{p}\hat{q}}^a, \quad P = 0$$

$$f_{\hat{m}\hat{n}} J_{\hat{m}\hat{n}}^a + 2t_a = 0 / f_{\hat{m}\hat{n}} = t_a = 0$$

Flat background

Weyl rescaling



[Pan '13]

►  $S^4$  does not admit an almost complex structure

► It is necessary to turn on a nontrivial  $Sp(1)_R$  flux for the existence of  $J_{\hat{m}\hat{n}}^a$

## Example: $S^3 \times \Sigma$

$\Sigma$ : a Riemann surface,  $S^3$ :  $S^1$  fibration over  $S^2$

$$ds^2 = ds_{\Sigma}^2 + ds_{S^2}^2 + e^{\hat{5}} e^{\hat{5}},$$

$$ds_{\Sigma}^2 = e^{\hat{1}} e^{\hat{1}} + e^{\hat{2}} e^{\hat{2}}, \quad ds_{S^2}^2 = e^{\hat{3}} e^{\hat{3}} + e^{\hat{4}} e^{\hat{4}}, \quad e^{\hat{5}} = r(dx^5 + \mathcal{V})$$

$SO(4)$  invariance  $\longrightarrow v^{\mu\nu} + 2f^{\mu\nu} = 0 / v^{\mu\nu} = f^{\mu\nu} = 0$

except for  $(\hat{1}, \hat{2})$ -component

$$v_{\hat{m}\hat{5}} = 0, \quad v_{\hat{1}\hat{2}} + 2f_{\hat{1}\hat{2}} = \frac{1}{r}, \quad V_{\hat{m}=1,2}^a = -\frac{i}{2}\delta^{a3}\omega_{\hat{m}\hat{1}\hat{2}}^{\Sigma}$$

$$V_{\hat{m}=3,4}^a = i\delta^{a3}\mathcal{V}_{\hat{m}}, \quad V_{\hat{5}}^a = -\frac{i}{r}\delta^{a3} \longrightarrow \text{flat connection on } S^3, \text{ gauged away}$$

$$f_{\hat{m}\hat{n}} J_{\hat{m}\hat{n}}^a + 2t_a = 0 / f_{\hat{1}\hat{2}} = -it_3$$

If we take

$$\mathcal{F} = \frac{1}{2g_{\text{YM}}^2} \phi^0 \text{tr}(\phi)^2, \quad f_{\hat{1}\hat{2}} = -it_3 = \frac{1}{2r},$$

we obtain the SYM action in [Fukuda-Kawano-Matsumiya '12]

## Summary and future directions

► Constructed the 5d SUSY backgrounds

► Showed that the partition function is not affected from the local dof of the background fields

► Realized several known backgrounds

► New backgrounds?

► Relax symplectic Majorana condition?

► Global issues?

► Isometry along 5th direction

$\longrightarrow$  relation to 4d  $\mathcal{N} = 2$  supergravity?