Supersymmetric backgrounds from 5d $\mathcal{N}=1$ supergravity

Hiroki Matsuno (Tokyo Institute of Technology)

Based on JHEP **1407** (2014) 055 [arXiv:1404.0210] with Yosuke Imamura

Introduction

The construction of rigid SUSY on Euclidean curved backgrounds

↓ Computation by localization

S^4 [Pestun '07], S^3 [Kapustin-Willett-Yaakov '09], $\cdots$

Nonperturbative properties of SUSY field theory and dualities

The systematic construction of SUSY on curved backgrounds

[Festuccia-Seiberg '11]

1. Start from off-shell SUGRA, and set the gravity multiplet as background
2. SUSY condition: $\delta_Q\psi_\mu = 0$

- By solving $\delta_Q\psi_\mu = 0$, we obtain backgrounds which admit rigid SUSY
  - 4d: Hermitian manifold or (squashed) S^4 [Klare-Tomasiello-Zaffaroni, Dumitrescu-Festuccia-Seiberg, Dumitrescu-Festuccia '12]
  - 3d: Almost contact metric structure with a certain integrability condition [Closset-Dumitrescu-Festuccia-Komargodski '12]
- Using this formulation, we can know whether the deformation of the backgrounds corresponds the Q-exact deformation of the action or not

  ⟶ Parameter dependence of 4d/3d partition function

[Closset-Dumitrescu-Festuccia-Komargodski '13]

How about in 5d?

Motivation to 5d: the close relation to 6d $(2,0)$ theory

- KK mode in 6d is conjectured as instanton in 5d [Douglas, Lambert '10]
- N^3 behavior of S^5 partition function [Kallen-Minahan-Nedelin-Zabzine '12]
- and also for $S^3\times\Sigma$, $S^2\times M_3$, $\ldots$

5d $\mathcal{N}=1$ Weyl multiplet [Kugo-Ohashi '00]

	fields	dof	$Sp(1)_R$		
bosons	vielbein	10	**1**	$e_\mu^{\hat\nu}$	
	$U(1)_Z$ gauge field	4	**1**	a_μ	(field strength $f_{\mu\nu}$)
	anti-sym. tensor	10	**1**	$v^{\mu\nu}$	
	$Sp(1)_R$ triplet scalars	3	**3**	t_a	
	$Sp(1)_R$ gauge field	12	**3**	V_μ^a	
	scalar	1	**1**	C	
fermions	gravitino	32	**2**	$\psi_{I\mu\alpha}$	
	fermion	8	**2**	$\eta_{I\alpha}$	

$$\delta_Q(\xi)\psi_\mu = D_\mu\xi - f_{\mu\nu}\gamma^\nu\xi + \frac{1}{4}\gamma_{\mu\rho\sigma}v^{\rho\sigma}\xi - t\gamma_\mu\xi$$
$$\delta_Q(\xi)\eta = -2\gamma_\nu\xi D_\mu v^{\mu\nu} + \xi C + 4(\not{D}t)\xi + 8(\not{f} - \not{v})t\xi + \gamma^{\mu\nu\rho\sigma}\xi f_{\mu\nu}f_{\rho\sigma}$$

D_μ: covariant derivative for Lorentz $Sp(2)_L = SO(5)_L$, R-sym. $Sp(1)_R = SU(2)_R$ and central charge sym. $U(1)_Z$

- We must solve not only $\delta_Q\psi_\mu = 0$ but also $\delta_Q\eta = 0$
- Partial analysis, which focused on $\delta_Q\psi_\mu = 0$ was done in [Pan '13]

SUSY backgrounds

We assume the existence of one symplectic Majorana spinor $\xi_{I\alpha}$ which is a SUSY transformation parameter

Bilinears of ξ:

$$S = (\xi\xi), \quad R^\mu = (\xi\gamma^\mu\xi), \quad J^a_{\mu\nu} = \frac{1}{S}(\xi\tau^a\gamma_{\mu\nu}\xi)$$

Define the 5th direction: $R^\mu\partial_\mu = \partial_5$, $m, n, \ldots = 1, \ldots, 4$

We can locally take

$$e^{\hat m} = e_n^{\hat m}dx^n, \quad e^{\hat 5} = S(dx^5 + \mathcal{V}_m dx^m)$$

$J^a_{\mu\nu}$ satisfies

$$J^a_{\hat m\hat 5} = 0, \quad -\frac{1}{2}\epsilon^{(4)}_{\hat m\hat n\hat p\hat q}J^a_{\hat p\hat q} = J^a_{\hat m\hat n}, \quad J^a_{\hat m\hat p}J^b_{\hat p\hat n} = \delta_{ab}\delta_{\hat m\hat n} + i\epsilon_{abc}J^c_{\hat m\hat n}$$

By $\delta_Q\psi_\mu = \delta_Q\eta = 0$, all fields (including $e^{\hat m}$, S and $\mathcal{V}_m$) are x^5-independent and, up to gauge transformation,

$$a_5 = \frac{1}{2}S,$$
$$v_{\hat p\hat q} = \epsilon^{(4)}_{\hat p\hat q\hat m\hat n}\left(\frac{S}{4}\mathcal{W}_{\hat m\hat n} - f_{\hat m\hat n} + t_a J^a_{\hat m\hat n}\right),$$
$$V^a_{\hat m} = \frac{1}{4}\omega^{(4)}_{\hat m\hat p\hat q}J^a_{\hat p\hat q} + \frac{1}{2}J^a_{\hat m\hat p}v^{\hat p\hat 5},$$
$$V^a_{\hat 5} = \frac{1}{2}J^a_{\hat m\hat n}\left(f_{\hat m\hat n} - \frac{S}{2}\mathcal{W}_{\hat m\hat n}\right) + t_a,$$
$$C = 2D^{(4)}_{\hat m}v^{\hat m\hat 5} + 4t_a J^a_{\hat m\hat n}f_{\hat m\hat n} + 32t_a t_a - \epsilon^{(4)}_{\hat m\hat n\hat p\hat q}\left(f^{\hat m\hat n} - \frac{S}{2}\mathcal{W}^{\hat m\hat n}\right)\left(f^{\hat p\hat q} - \frac{S}{2}\mathcal{W}^{\hat p\hat q}\right)$$

($\mathcal{W}$: field strength of $\mathcal{V}$)

Independent fields are $e_n^{\hat m}$, S, $\mathcal{V}_m$, $a_{\hat m}$, $v^{\hat m\hat 5}$ and t^a

(Similar analysis can be done for background vector multiplets)

Q-exact deformations

A small deformation of the background gives the change of the action

$$S_1 = \int d^5x\sqrt{g}\Big[-\delta e_\mu^{\hat\nu}T^\mu_{\hat\nu} + \delta V^a_\mu R^\mu_a + (\delta\psi_\mu S^\mu) - \delta a_\mu J^\mu + \delta v^{\mu\nu}M_{\mu\nu} + \delta C\Phi + (\delta\eta\chi) + \delta t_a X_a\Big]$$

($T^\mu_{\hat\nu}$, R^μ_a, $\ldots$: components of the supercurrent multiplet)

On the other hand, a Q-exact deformation which can be regarded as a change of background fields generally takes the form

$$S_Q(\xi; H_\mu, K) = \delta_Q(\xi)\int\sqrt{g}d^5x\,[H_\mu S^\mu + K\chi]$$

For example, by using the transformation law for χ,

$$S_Q(\xi; 0, \frac{4}{S}k^a\xi\tau_a) = \int d^5x\sqrt{g}\left(k^a R_a^{\hat 5} - 2k^a J^a_{\hat m\hat n}M^{\hat m\hat n} + \left(4k^a J^a_{\hat m\hat n}f^{\hat m\hat n} + 64k^a t_a\right)\Phi + k^a X_a\right)$$

This can be realized as the deformation $\delta t_a = k^a$ and associated deformations of dependent fields

Similarly, we obtain the result that all SUSY-preserving local deformations give Q-exact deformations and do not change the partition function

(We obtain similar results for background vector multiplets)

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Vector multiplet and Lagrangian [Kugo-Ohashi '00]

	fields	dof	$Sp(1)_R$		
bosons	gauge field	4	**1**	A_μ	(field strength $F_{\mu\nu}$)
	scalar	1	**1**	ϕ	
	auxiliary fields	3	**3**	D_a	
fermions	gaugino	8	**2**	$\lambda_{I\alpha}$	
	"prepotential"			$\mathcal{F}(\phi)$	

The Lagrangian for vector multiplets in 5d $\mathcal{N}=1$ SUGRA is

$$e^{-1}\mathcal{L}^{(V)}_{\text{SUGRA}} = e^{-1}\mathcal{L}^{(V)}_0 + e^{-1}\mathcal{L}^{(V)}_1,$$

$$\begin{aligned} e^{-1}\mathcal{L}^{(V)}_0 &= -\frac{1}{2}\mathcal{F}_i[\lambda,\lambda]^i \\ &\quad + \mathcal{F}_{ij}\left(\frac{1}{4}F^i_{\mu\nu}F^{\mu\nu j} + \frac{1}{2}D_\mu\phi^i D^\mu\phi^j - \frac{1}{2}D^i_a D^j_a - \frac{1}{2}\lambda^i \not{D}\lambda^j\right) \\ &\quad + \mathcal{F}_{ijk}\left(\frac{i}{6}[\text{CS}]^{ijk}_5 + \frac{1}{4}\lambda^i\left(i\not{F}^j + D^j\right)\lambda^k\right), \end{aligned}$$

$$\begin{aligned} e^{-1}\mathcal{L}^{(V)}_1 &= \mathcal{F}\left(C - 20t_a t_a - 4f_{\mu\nu}v^{\mu\nu} - 6f_{\mu\nu}f^{\mu\nu}\right) \\ &\quad - i\mathcal{F}_i F^i_{\mu\nu}\left(v^{\mu\nu} + 2f^{\mu\nu}\right) + \frac{1}{4}\mathcal{F}_{ij}\lambda^i\left(\not{v} + 2\not{f}\right)\lambda^j \\ &\quad + (\text{terms including } \psi_{\mu I} \text{ or } \eta). \end{aligned}$$

$$\mathcal{F}_i = \frac{\partial\mathcal{F}}{\partial\phi^i}, \quad \mathcal{F}_{ij} = \frac{\partial^2\mathcal{F}}{\partial\phi^i\partial\phi^j}, \quad \mathcal{F}_{ijk} = \frac{\partial^3\mathcal{F}}{\partial\phi^i\partial\phi^j\partial\phi^k}$$

- Prepotential: gauge invariant homogeneous cubic polynomial in ϕ
- i,j,k run over ordinary vector multiplets and "the central charge vector multiplet"
$$(\phi, A_\mu, \lambda, D_a)^{i=0} = (1, 2ia_\mu, 0, -2t_a)$$
- If $\mathcal{F}$ includes only dynamical ϕ,
  - The theory is conformal (CS)
  - Symmetries of the background restrict $v^{\mu\nu} + 2f^{\mu\nu}$
- If $\mathcal{F}$ includes ϕ^0,
  - The theory is not conformal (YM, FI)
  - Symmetries of the background restrict not only $v^{\mu\nu} + 2f^{\mu\nu}$ but also $f^{\mu\nu}$

Example: S^5

S^5: S^1 fibration over $\mathbb{C}P_2$

$$ds^2 = ds^2_{\mathbb{C}P_2} + e^{\hat{5}}e^{\hat{5}}, \quad ds^2_{\mathbb{C}P_2} = e^{\hat{m}}e^{\hat{m}}, \quad e^{\hat{5}} = r\left(dx^5 + \mathcal{V}\right)$$

$SO(6)$ invariance $\longrightarrow$ impose $v^{\mu\nu} + 2f^{\mu\nu} = 0$

$$V^a_{\hat{m}} = -\frac{3i}{2}\mathcal{V}_m\delta^{a3}, \quad V^a_{\hat{5}} = \frac{3i}{2r}\delta^{a3}, \quad \longrightarrow \text{flat connection, gauged away}$$

$$v_{\hat{m}\hat{5}} = 0, \quad f_{\hat{m}\hat{n}}J^a_{\hat{m}\hat{n}} + 2t_a = -\frac{i}{r}\delta^{a3}$$

$$P \equiv C - 20t_a t_a - 4f_{\mu\nu}v^{\mu\nu} - 6f_{\mu\nu}f^{\mu\nu} = \frac{5}{r^2}$$

If $\mathcal{F}$ includes $\phi^0\phi^i\phi^j$, also impose $f^{\mu\nu} = 0$

$$f_{\hat{m}\hat{n}} = 0, \quad t_a = -\frac{i}{2r}\delta^{a3}$$

This agree with [Hosomichi-Seong-Terashima '12]

Flat background —(Weyl rescaling)→ Our S^5: $e^{-1}\mathcal{L} = e^{-1}\mathcal{L}^{(V)}_{0,V^a=0} + \frac{5}{r^2}\mathcal{F}$

Example: $S^4 \times \mathbb{R}$

$$ds^2 = ds^2_{S^4} + \left(dx^5\right)^2$$

$SO(5)$ invariance $\longrightarrow$ $v^{\mu\nu} + 2f^{\mu\nu} = 0$ / $v^{\mu\nu} = f^{\mu\nu} = 0$

$$v_{\hat{m}\hat{5}} = 0, \quad V^a = \frac{1}{4}\omega^{S^4}_{\hat{p}\hat{q}}J^a_{\hat{p}\hat{q}}, \quad P = 0$$

$$f_{\hat{m}\hat{n}}J^a_{\hat{m}\hat{n}} + 2t_a = 0 \ / \ f_{\hat{m}\hat{n}} = t_a = 0$$

Flat background —(Weyl rescaling)→ $S^4\times\mathbb{R}$: $e^{-1}\mathcal{L} = e^{-1}\mathcal{L}^{(V)}_{0,V^a=0} + \frac{3}{r^2}\mathcal{F}$ ←Different!→ Our $S^4\times\mathbb{R}$: $e^{-1}\mathcal{L} = e^{-1}\mathcal{L}^{(V)}_{0,V^a\neq 0}$

[Pan '13]

- S^4 does not admit an almost complex structure
- It is necessary to turn on a nontrivial $Sp(1)_R$ flux for the existence of $J^a_{\hat{m}\hat{n}}$

Example: $S^3 \times \Sigma$

Σ: a Riemann surface, S^3: S^1 fibration over S^2

$$ds^2 = ds^2_\Sigma + ds^2_{S^2} + e^{\hat{5}}e^{\hat{5}},$$

$$ds^2_\Sigma = e^{\hat{1}}e^{\hat{1}} + e^{\hat{2}}e^{\hat{2}}, \quad ds^2_{S^2} = e^{\hat{3}}e^{\hat{3}} + e^{\hat{4}}e^{\hat{4}}, \quad e^{\hat{5}} = r\left(dx^5 + \mathcal{V}\right)$$

$SO(4)$ invariance $\longrightarrow$ $v^{\mu\nu} + 2f^{\mu\nu} = 0$ / $v^{\mu\nu} = f^{\mu\nu} = 0$

except for $(\hat{1},\hat{2})$-component

$$v_{\hat{m}\hat{5}} = 0, \quad v_{\hat{1}\hat{2}} + 2f_{\hat{1}\hat{2}} = \frac{1}{r}, \quad V^a_{\hat{m}=\hat{1},\hat{2}} = -\frac{i}{2}\delta^{a3}\omega^\Sigma_{\hat{m}\hat{1}\hat{2}}$$

$$V^a_{\hat{m}=\hat{3},\hat{4}} = i\delta^{a3}\mathcal{V}_{\hat{m}}, \quad V^a_{\hat{5}} = -\frac{i}{r}\delta^{a3} \longrightarrow \text{flat connection on } S^3, \text{ gauged away}$$

$$f_{\hat{m}\hat{n}}J^a_{\hat{m}\hat{n}} + 2t_a = 0 \ / \ f_{\hat{1}\hat{2}} = -it_3$$

If we take

$$\mathcal{F} = \frac{1}{2g^2_{\text{YM}}}\phi^0\text{tr}(\phi)^2, \quad f_{\hat{1}\hat{2}} = -it_3 = \frac{1}{2r},$$

we obtain the SYM action in [Fukuda-Kawano-Matsumiya '12]

Summary and future directions

- Constructed the 5d SUSY backgrounds
- Showed that the partition function is not affected from the local dof of the background fields
- Realized several known backgrounds

- New backgrounds?
- Relax symplectic Majorana condition?
- Global issues?
- Isometry along 5th direction
  $\longrightarrow$ relation to 4d $\mathcal{N}=2$ supergravity?