Quantum corrections for a string world sheet in AdS/CFT correspondence

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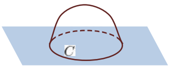
1. Introduction and Motivation

◆ Wilson loop VEV (I_J : modified Bessel function)

$$\langle W(C)\rangle = \frac{2}{\sqrt{\lambda'}} I_1(\sqrt{\lambda'}) \sim \frac{2}{\sqrt{\lambda'}}\frac{1}{\sqrt{2\pi\sqrt{\lambda'}}} e^{\sqrt{\lambda'}} \quad (\lambda' = \lambda \tanh^2 \sigma_0)$$

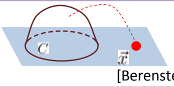


$$e^{-S} = e^{\sqrt{\lambda'}}$$

[Rey-Yee '98; Maldacena '98] [Drukker '06]

◆ Correlation function (a : radius of C, ℓ : distance)

$$\frac{\langle W(C)\mathcal{O}_J(\vec{x})\rangle}{\langle W(C)\rangle} = \frac{1}{2N}\frac{a^J}{\ell^{2J}}\sqrt{J\lambda'}\frac{I_J(\sqrt{\lambda'})}{I_1(\sqrt{\lambda'})} \sim \frac{1}{2N}\frac{a^J}{\ell^{2J}}\sqrt{J\lambda'}$$



$$\frac{1}{e^{-S}}\frac{\delta}{\delta s_0^J(\vec{x})} e^{-S}\Big|_{s_0^J=0} = \frac{1}{2N}\frac{a^J}{\ell^J}\sqrt{J\lambda'}$$

[Berenstein-Corrado-Fischler-Maldacena '98] [Semenoff-Young '06]

world sheet fluctuations ⇒ further agreement ?

2. 1/4BPS Wilson loop and gravity dual

◆ 1/4 BPS Wilson loop [Drukker '06]

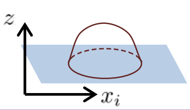
$$W(C) = \frac{1}{N}\mathrm{trP}\exp\oint\left(iA_i(x(\tau))\dot{x}_i(\tau) + |\dot{x}(\tau)|\Theta_I(\tau)\Phi_I(x(\tau))\right)$$

$$x_i(\tau) = (\,a\cos\tau\,,\,a\sin\tau\,,\,0\,,\,0\,)$$

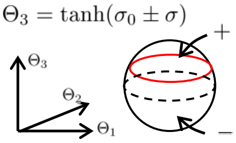
$$\Theta_I(\tau) = (\mathrm{sech}\sigma_0\cos\tau\,,\mathrm{sech}\sigma_0\sin\tau\,,\tanh\sigma_0\,,0\,,0\,,0)$$

◆ gravity dual

$$z = a\tanh\sigma$$
$$x_1 = a\,\mathrm{sech}\sigma\cos\tau$$
$$x_2 = a\,\mathrm{sech}\sigma\sin\tau$$



$$\Theta_1 = \mathrm{sech}(\sigma_0 \pm \sigma)\cos\tau$$
$$\Theta_2 = \mathrm{sech}(\sigma_0 \pm \sigma)\sin\tau$$
$$\Theta_3 = \tanh(\sigma_0 \pm \sigma)$$



$e^{-S}\big|_\pm = e^{\pm\sqrt{\lambda'}}$ ($\pm$) solutions ⇔ two saddle points

3. Zero modes and broken zero modes

◆ zero modes for $\sigma_0 = 0$ [Drukker '06]

$\Theta_1 = \mathrm{sech}\sigma\cos\tau$ $\quad\Theta_3 = \tanh\sigma\cos\alpha$

$\Theta_2 = \mathrm{sech}\sigma\sin\tau$ $\quad\Theta_4 = \tanh\sigma\sin\alpha\cos\beta$

$\Theta_5 = \tanh\sigma\sin\alpha\sin\beta\cos\gamma$

$\Theta_6 = \tanh\sigma\sin\alpha\sin\beta\sin\gamma$

(α,β,γ) zero modes (S^3)

$$\langle W(C)\rangle = 1 \quad\Leftrightarrow\quad \int d\Omega_3 e^{-S(\alpha,\beta,\gamma)} = \int d\Omega_3 e^0 = 1$$

◆ broken zero modes for $\sigma_0 \sim 0$

No zero modes, but the broken zero modes give non-negligible contribution.

$$Z = \int d\Omega_3 e^{-S(\alpha,\beta,\gamma)} = \int d\Omega_3 e^{\cos\alpha\sqrt{\lambda'}} = \frac{2}{\sqrt{\lambda'}}I_1(\sqrt{\lambda'})$$

$$(\sigma_0 \ll 1,\quad \lambda \gg 1,\quad \lambda' = \lambda\tanh^2\sigma_0\ :\ \text{finite}\ )$$

4. Explicit form of broken zero modes

◆ An explicit form of broken zero modes

$$\Theta_1 = f(\alpha,\sigma)\cos\tau$$
$$\Theta_2 = f(\alpha,\sigma)\sin\tau$$
$$\Theta_3 = f(\alpha,\sigma)(\cosh\sigma_0\sinh\sigma\cos\alpha + \sinh\sigma_0\cosh\sigma)$$
$$\Theta_4 = f(\alpha,\sigma)\sinh\sigma\sin\alpha\cos\beta$$
$$\Theta_5 = f(\alpha,\sigma)\sinh\sigma\sin\alpha\sin\beta\cos\gamma$$
$$\Theta_6 = f(\alpha,\sigma)\sinh\sigma\sin\alpha\sin\beta\sin\gamma$$
$$f(\alpha,\sigma) = \left(\cosh\sigma_0\cosh\sigma + \sinh\sigma_0\sinh\sigma\cos\alpha\right)^{-1}$$

◆ Properties

- ✓ $(\tau,\sigma,\alpha,\beta,\gamma)$ form an S^5 coordinate system.
- ✓ Virasoro constraints satisfied.
- ✓ Boundary conditions at $\sigma = 0$ satisfied.
- ✓ It reduces to the classical solution at $\alpha = \pm\pi$.
- ✓ It reduces to the zero mode at $\sigma_0 = 0$.

5. Correlation function

[Related work includes: Berenstein-Corrado-Fischler-Maldacena '98; Semenoff-Young '06; Giombi-Ricci-Trancanelli '06]

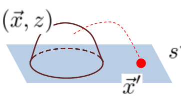
◆ local operator with R charge

$$\mathcal{O}_J = \frac{(2\pi)^J}{\sqrt{J\lambda^J}}\mathrm{tr}(\Phi_3 + i\Phi_4)^J$$

◆ gravity dual of the correlation functions

$$\frac{1}{Z}\int d\Omega_3 \frac{\delta}{\delta s_0^J(\vec{x})}\exp\left[-\frac{1}{4\pi\alpha'}\int d^2\sigma(g_{MN}+h_{MN})\partial_a X^M\partial_a X^N\right]\Bigg|_{s_0^J=0}$$

$$h^{\mathrm{AdS}}_{\mu\nu} = \left[\frac{4}{J+1}D_{(\mu}D_{\nu)} - \frac{6J}{5}g^{\mathrm{AdS}}_{\mu\nu}\right]s^J\mathcal{Y}_J\,,\quad h^{\mathrm{S}}_{\alpha\beta} = 2Jg^{\mathrm{S}}_{\alpha\beta}s^J\mathcal{Y}_J$$



$$s^J(\vec{x},z) = c\int d^4x'\left[\frac{z}{z^2+(\vec{x}-\vec{x}')^2}\right]^J s_0^J(\vec{x}')$$

Our Result:
$$\frac{1}{2N}\frac{a^J}{\ell^{2J}}\sqrt{J\lambda'}\frac{I_J(\sqrt{\lambda'})}{I_1(\sqrt{\lambda'})}\left(1 - \frac{J+2}{\sqrt{\lambda'}}\frac{I_{J+1}(\sqrt{\lambda'})}{I_J(\sqrt{\lambda'})}\right)$$
$$(\sigma_0 \ll 1,\quad \lambda \gg 1,\quad \lambda' = \lambda\tanh^2\sigma_0\ :\ \text{finite}\ )$$

6. Discussions and future directions

◆ Discussions

- Our results agree with the gauge theory results in the limit $J \ll \sqrt{\lambda'}$. Since $J^2/\sqrt{\lambda'}$ is the natural parameter in the asymptotic expansion of $I_J(\sqrt{\lambda'})$, this limit still allows non-trivial situation.
- Numerical samples by Mathematica ($\sqrt{\lambda'} = t = 10^4$)

J	2	10	100	1000
$\frac{e^t}{\sqrt{2\pi t}}\frac{1}{I_J(t)}$	1.00019	1.005	1.6487	4.986×10^{21}
$1 - \frac{J+2}{t}\frac{I_{J+1}(t)}{I_J(t)}$	0.9996	0.9988	0.9899	0.9093

- Since the R-charge conservation is not respected, an exact (finite $\sqrt{\lambda'}$) agreement is not expected.

◆ Future directions

- Improvement by using string solutions with R-charge.
- Application to the case with higher genus .