Instanton Effects in Orbifold ABJM Theory

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Contents

Message

The Exact Instanton Expansion
of The ABJM Partition Function
Has Been Found!

Caution:

Not Solved Completely Logically
(Partial Assist From Numerical Method)

Why ABJM Interesting?

ABJM Theory Describes Multiple M2-branes on

$$C^4/Z_k \rightarrow R_{>0} \times S^1 \times CP^3 \quad (k \rightarrow \infty)$$

- log (Partition Function) = M2 DOF = $N^{3/2}$ + ...

[Drukker-Marino-Putrov]

- Understanding Membrane Interactions
- A prototype of AdS/CFT

Why ABJM Exact PF Interesting?

- A completion of $N^{3/2}$ DOF

  (Understanding Membrane Interaction)
- Perturbative Corrections Sum Up To Airy

  $Z(N) \sim \mathrm{Ai}[N] \sim \exp(N^{3/2})$ [Fuji-Hirano-M]
- Poles from Coefficients of Two Instantons Cancel Between Themselves [Hatsuda-M-Okuyama]
- Completely by Refined Topological Strings

  [Hatsuda-Marino-M-Okuyama]

Question

So Far So Good for ABJM

- How General, Properties of ABJM?

 - More Instantons, More Cancellations?

 - Exact Expansion for General Theories?

Contents

ABJM Matrix Model

- Partition Function of ABJM Theory
- Due to SUSY, Localized to Matrix Model

$N_1 = N_2 = N$

$$Z(N) = \frac{1}{N_1! N_2!} \int \prod_{i=1}^{N_1} \frac{d\mu_i}{2\pi} \prod_{k=1}^{N_2} \frac{d\nu_k}{2\pi} e^{-(\sum \mu_i^2 - \sum \nu_k^2)/2g_s}$$

$$\times \frac{\prod_{i<j} \left(2 \sinh \frac{\mu_i - \mu_j}{2}\right)^2 \prod_{k<l} \left(2 \sinh \frac{\nu_k - \nu_l}{2}\right)^2}{\prod_{i,k} \left(2 \cosh \frac{\mu_i - \nu_k}{2}\right)^2}$$

$g_s = 2\pi i / k$

[Pestun, Kapustin-Willett-Yaakov]

't Hooft Expansion

- $N \to \infty$ Expansion with $\lambda = N/k$ Fixed
- Genus Expansion [Drukker-Marino-Putrov]

$$\log[Z(N)] = N^2 F_0(\lambda) + N^0 F_1(\lambda) + N^{-2} F_2(\lambda) + N^{-4} F_3(\lambda) + \ldots$$

$$= N^2[F_0^{\mathrm{pert}}(\lambda) + \#\, e^{-2\pi\sqrt{2\lambda}} + \#\, e^{-4\pi\sqrt{2\lambda}} + \ldots]$$

$$+ N^0[F_1^{\mathrm{pert}}(\lambda) + \#\, e^{-2\pi\sqrt{2\lambda}} + \#\, e^{-4\pi\sqrt{2\lambda}} + \ldots]$$

$$+ \ldots \ldots$$

Sum Up To Airy Function!

[Fuji-Hirano-M, Honda et al]

Worldsheet Instanton

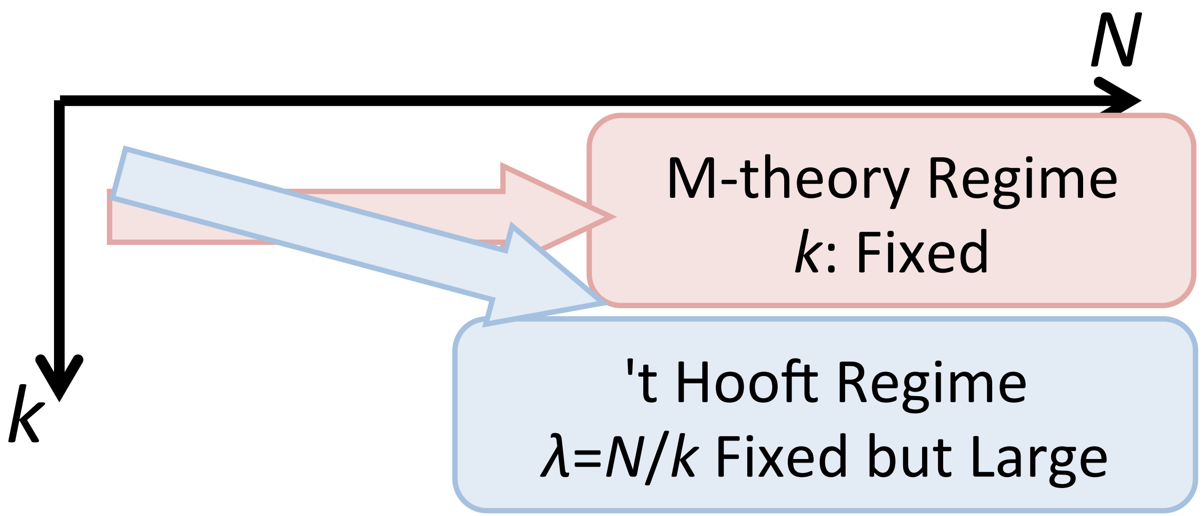
$$e^{-2\pi\sqrt{2\lambda}} = \exp[- T_{F1}\,\mathrm{Area}(\mathbf{CP}^1)]$$

F-String Wrapping $\mathbf{CP}^1 \subset \mathrm{CP}^3$

[Cagnazzo-Sorokin-Wulff, DMP]

M-theory Expansion [Herzog-Klebanov-Pufu-Tesileanu]

- M-theory Background: C^4/Z_k
- $N \to \infty$ with k Fixed
- Close To 't Hooft Limit But Not Exactly



Fermi Gas Formalism [Marino-Putrov]

- Rewriting Partition Function
- Regarding as Fermi Gas with Hamiltonian e^{-H}

$$Z(N) = (N!)^{-1} \sum_{\sigma\in S(N)} (-1)^{\sigma} \int \Pi_i \, dq_i \langle q_i | e^{-H} | q_{\sigma(i)} \rangle$$

$$e^{-H} \sim (2 \cosh q/2)^{-1} (2 \cosh p/2)^{-1}$$

$$[q,p] = i\, 2\pi k$$

- Introducing Grand Potential

$$e^{J(\mu)} = \sum_{N=0}^{\infty} Z(N)\, e^{\mu N} = \det (1 + e^{\mu}\, e^{-H})$$

WKB (small k) expansion

$$J(\mu) = k^{-1} J_0(\mu) + k^1 J_1(\mu) + k^3 J_2(\mu) + k^5 J_3(\mu) + \ldots$$

$$= k^{-1} [\, J_0^{\text{pert}}(\mu) + (\#\mu^2+\#\mu+\#)e^{-2\mu} + (\ldots)\, e^{-4\mu} + \ldots \,]$$

$$+\, k\, [\, J_1^{\text{pert}}(\mu) + (\#\mu^2+\#\mu+\#)e^{-2\mu} + (\ldots)\, e^{-4\mu} + \ldots \,]$$

$$+ \quad \ldots \ldots$$

$$= (\#\mu^3+\#\mu+\#) + (\#\mu^2+\#\mu+\#)e^{-2\mu} + (\ldots)e^{-4\mu} + \ldots$$

Airy Function Reproduced

[Marino-Putrov]

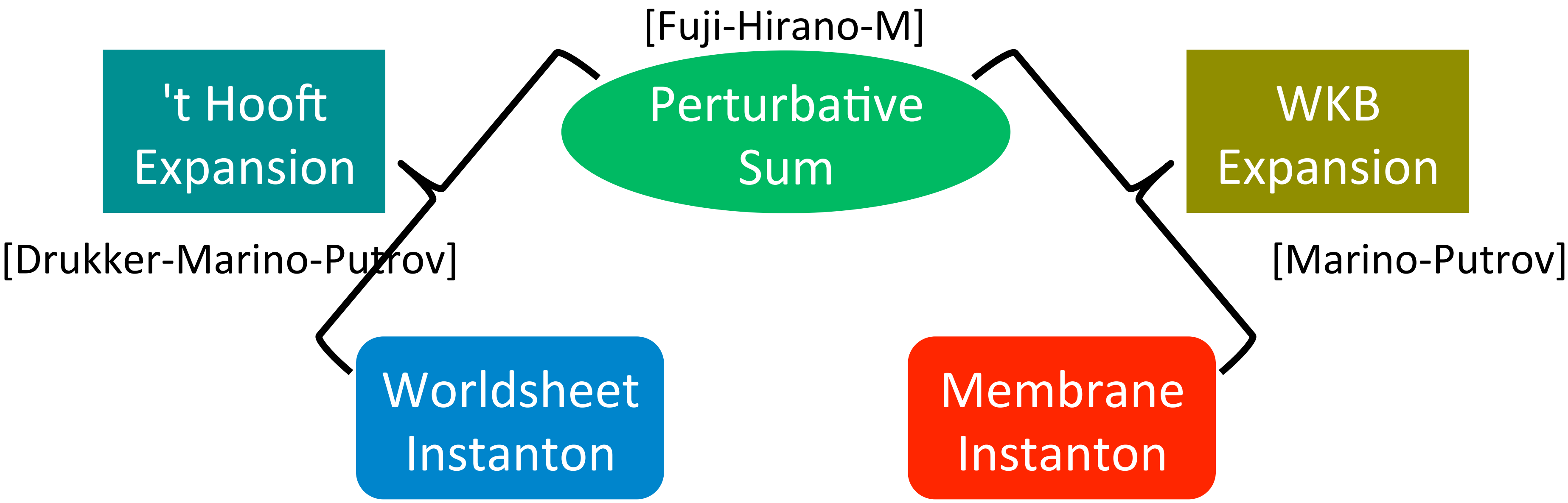
Membrane Instanton

$$e^{-2\mu} \approx e^{-\pi\sqrt{2Nk}} = \exp[-\, T_{D2}\, \text{Area}(\mathbf{RP}^3)]$$

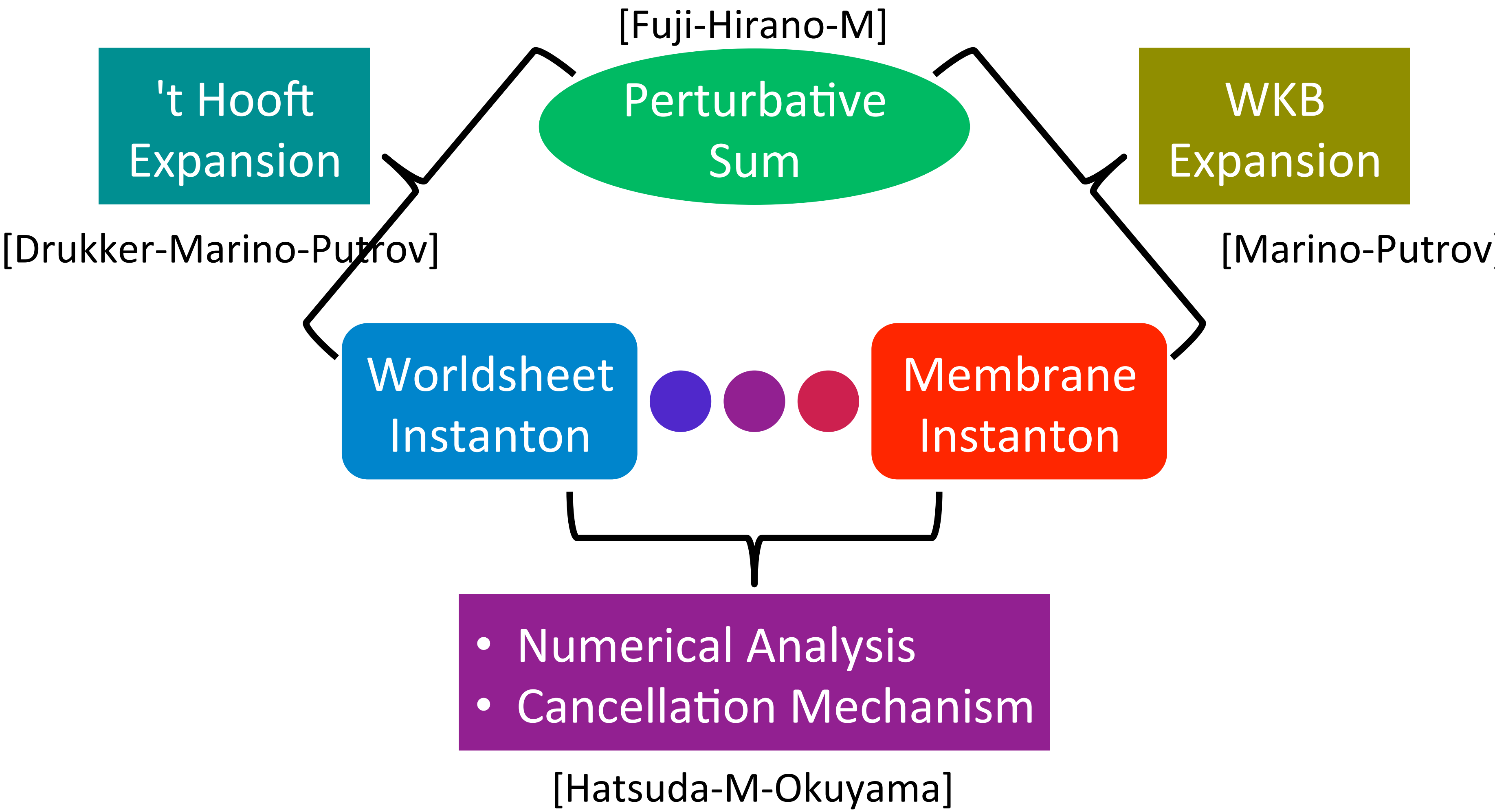
D2-brane Wrapping $\mathbf{RP}^3 \subset CP^3$

[Drukker-Marino-Putrov]

Short Summary



Furthermore



Bound States [Hatsuda-M-Okuyama 1301]

MB WS

Bound States $e^{-2l\mu-4m\mu/k}$ Taken Care Of

By WS instantons with Chemical Potential Redef

$$\mu \to \mu_{\mathrm{eff}}$$

$$\mu_{\mathrm{eff}} = \mu + \#\, e^{-2\mu} + \#\, e^{-4\mu} + \ldots$$

Cancelation Mechanism [Hatsuda-M-Okuyama 1211]

$J_{k=1}(\mu)$ = [# μ^2 + # μ + #] $e^{-4\mu}$ + [# μ^2 + # μ + #] $e^{-8\mu}$ + [# μ^2 + # μ + #] $e^{-12\mu}$ + ...

$J_{k=2}(\mu)$ = [# μ^2 + # μ + #] $e^{-2\mu}$ + [# μ^2 + # μ + #] $e^{-4\mu}$ + [# μ^2 + # μ + #] $e^{-6\mu}$ + ...

$J_{k=3}(\mu)$ = [#] $e^{-4\mu/3}$ + [#] $e^{-8\mu/3}$ + [# μ^2 + # μ + #] $e^{-4\mu}$ + ...

$J_{k=4}(\mu)$ = [#] $e^{-\mu}$ + [# μ^2 + # μ + #] $e^{-2\mu}$ + [#] $e^{-3\mu}$ + ...

...

$J_{k=6}(\mu)$ = [#] $e^{-2\mu/3}$ + [#] $e^{-4\mu/3}$ + [# μ^2 + # μ + #] $e^{-2\mu}$ + ...

WS(1) WS(2) WS(3)

↑ ↑ ↑

Free Energy of Topological String

Cancelation Mechanism [Hatsuda-M-Okuyama 1211]

(Raison d'Etre for M-theory!?)

$$J_{k=1}(\mu) = [\#\mu^2+\#\mu+\#]e^{-4\mu} + [\#\mu^2+\#\mu+\#]e^{-8\mu} + [\#\mu^2+\#\mu+\#]e^{-12\mu} + \ldots$$

$$J_{k=2}(\mu) = [\#\mu^2+\#\mu+\#]e^{-2\mu} + [\#\mu^2+\#\mu+\#]e^{-4\mu} + [\#\mu^2+\#\mu+\#]e^{-6\mu} + \ldots$$

$$J_{k=3}(\mu) = [\#]e^{-4\mu/3} + [\#]e^{-8\mu/3} + [\#\mu^2+\#\mu+\#]e^{-4\mu} + \ldots$$

$$J_{k=4}(\mu) = [\#]e^{-\mu} + [\#\mu^2+\#\mu+\#]e^{-2\mu} + [\#]e^{-3\mu} + \ldots$$

...

$$J_{k=6}(\mu) = [\#]e^{-2\mu/3} + [\#]e^{-4\mu/3} + [\#\mu^2+\#\mu+\#]e^{-2\mu} + \ldots$$

WS(1) WS(2) WS(3)

MB(2) MB(1)

↑ ↑ ↑

Free Energy of Topological String

All Explicitly In Topological Strings

[..., Hatsuda-M-Marino-Okuyama]

$$J(\mu)=J^{\text{pert}}(\mu^{\text{eff}})+J^{\text{WS}}(\mu^{\text{eff}})+J^{\text{MB}}(\mu^{\text{eff}})$$

$$J^{\text{pert}}(\mu)=C\mu^3/3+B\mu+A$$

$$J^{\text{WS}}(\mu^{\text{eff}})=F_{\text{top}}(T_1^{\text{eff}},T_2^{\text{eff}},\lambda)$$

$$J^{\text{MB}}(\mu^{\text{eff}})=(2\pi \text{i})^{-1}\,\partial_\lambda[\lambda F_{\text{NS}}(T_1^{\text{eff}}/\lambda,T_2^{\text{eff}}/\lambda,1/\lambda)]$$

$$F_{\text{top}}(T_1,T_2,\tau) = \ldots$$

$$F_{\text{NS}}(T_1,T_2,\tau) = \ldots$$

$$C=2/\pi^2 k,\ B=\ldots,\ A=\ldots$$

$$T_1^{\text{eff}}=4\mu^{\text{eff}}/k-\text{i}\pi$$

$$T_2^{\text{eff}}=4\mu^{\text{eff}}/k+\text{i}\pi$$

$$\lambda=2/k$$

$$\mu^{\text{eff}}=\begin{cases}\mu-(-1)^{k/2}2e^{-2\mu}{}_4F_3(1,1,3/2,3/2;2,2,2;(-1)^{k/2}16e^{-2\mu}) & k=\text{even}\\ \mu+e^{-4\mu}{}_4F_3(1,1,3/2,3/2;2,2,2;-16e^{-4\mu}) & k=\text{odd}\end{cases}$$

All Explicitly In Topological Strings

[..., Hatsuda-M-Marino-Okuyama]

$$J(\mu)=J^{\mathrm{pert}}(\mu^{\mathrm{eff}})+J^{\mathrm{WS}}(\mu^{\mathrm{eff}})+J^{\mathrm{MB}}(\mu^{\mathrm{eff}})$$

$$J^{\mathrm{pert}}(\mu)=C\mu^3/3+B\mu+A$$

$$J^{\mathrm{WS}}(\mu^{\mathrm{eff}})=F_{\mathrm{top}}(T_1^{\mathrm{eff}},T_2^{\mathrm{eff}},\lambda)$$

$$J^{\mathrm{MB}}(\mu^{\mathrm{eff}})=(2\pi i)^{-1}\,\partial_\lambda[\lambda F_{\mathrm{NS}}(T_1^{\mathrm{eff}}/\lambda,T_2^{\mathrm{eff}}/\lambda,1/\lambda)]$$

$F(T_1,T_2,\tau_1,\tau_2)$: Free Energy of Refined Top Strings

T_1,T_2: Kahler Moduli

τ_1,τ_2: Coupling Constants

Topological Limit $F_{\mathrm{top}}(T_1,T_2,\tau) = \lim_{\tau_1\to\tau,\tau_2\to-\tau} F(T_1,T_2,\tau_1,\tau_2)$

NS Limit $F_{\mathrm{NS}}(T_1,T_2,\tau) = \lim_{\tau_1\to\tau,\tau_2\to 0} 2\pi i\tau_2 F(T_1,T_2,\tau_1,\tau_2)$

All Explicitly In Topological Strings

[..., Hatsuda-M-Marino-Okuyama]

$$J(\mu)=J^{\mathrm{pert}}(\mu^{\mathrm{eff}})+J^{\mathrm{WS}}(\mu^{\mathrm{eff}})+J^{\mathrm{MB}}(\mu^{\mathrm{eff}})$$

$$J^{\mathrm{pert}}(\mu)=C\mu^3/3+B\mu+A$$

$$J^{\mathrm{WS}}(\mu^{\mathrm{eff}})=F_{\mathrm{top}}(T_1^{\mathrm{eff}},T_2^{\mathrm{eff}},\lambda)$$

$$J^{\mathrm{MB}}(\mu^{\mathrm{eff}})=(2\pi\mathrm{i})^{-1}\,\partial_\lambda[\lambda F_{\mathrm{NS}}(T_1^{\mathrm{eff}}/\lambda,T_2^{\mathrm{eff}}/\lambda,1/\lambda)]$$

$$F(T_1,T_2,\tau_1,\tau_2)=\sum_{j_L,j_R}\sum_n\sum_{d_1,d_2}N_{j_L,j_R}^{d_1,d_2}\,\chi_{j_L}(q_L)\,\chi_{j_R}(q_R)\,e^{-n(d_1T_1+d_2T_2)}\,/[n(q_1^{n/2}-q_1^{-n/2})(q_2^{n/2}-q_2^{-n/2})]$$

$$q_1=e^{2\pi i\tau_1}\quad q_2=e^{2\pi i\tau_2}\quad q_L=e^{\pi i(\tau_1-\tau_2)}\quad q_R=e^{\pi i(\tau_1+\tau_2)}$$

$N_{j_L,j_R}^{d_1,d_2}$: BPS Index of local P^1 x P^1
(Gopakumar-Vafa or Gromov-Witten invariants)

Looking Back, What Makes ABJM Solvable?

- Max SUSY (N=6) !?
- Relations to Topological Strings?

In Fact, ABJ (N=6 & Topological Strings)

Solved Similarly.

[Awata-Hirano-Shigemori, Honda, Matsumoto-M, Honda-Okuyama, Kallen]

Generalizations

- ABJM only Two Types of Instantons

→ More Non-Trivial Cancellations

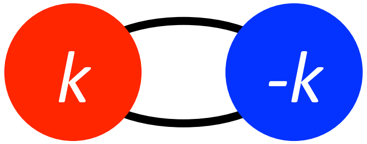
To Understand Membrane Moduli Space

- What If Relation To Topological Strings Is Unclear?

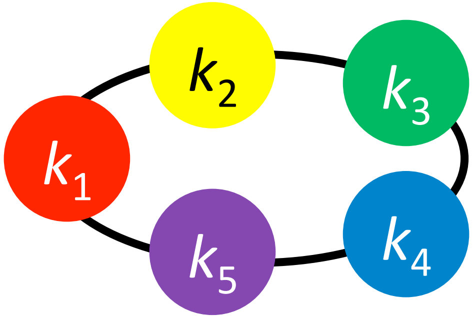
Contents

Generalizations

- ABJM in Quiver Diagram



- More General N=3 Circular Quivers ($\Sigma_i k_i = 0$)

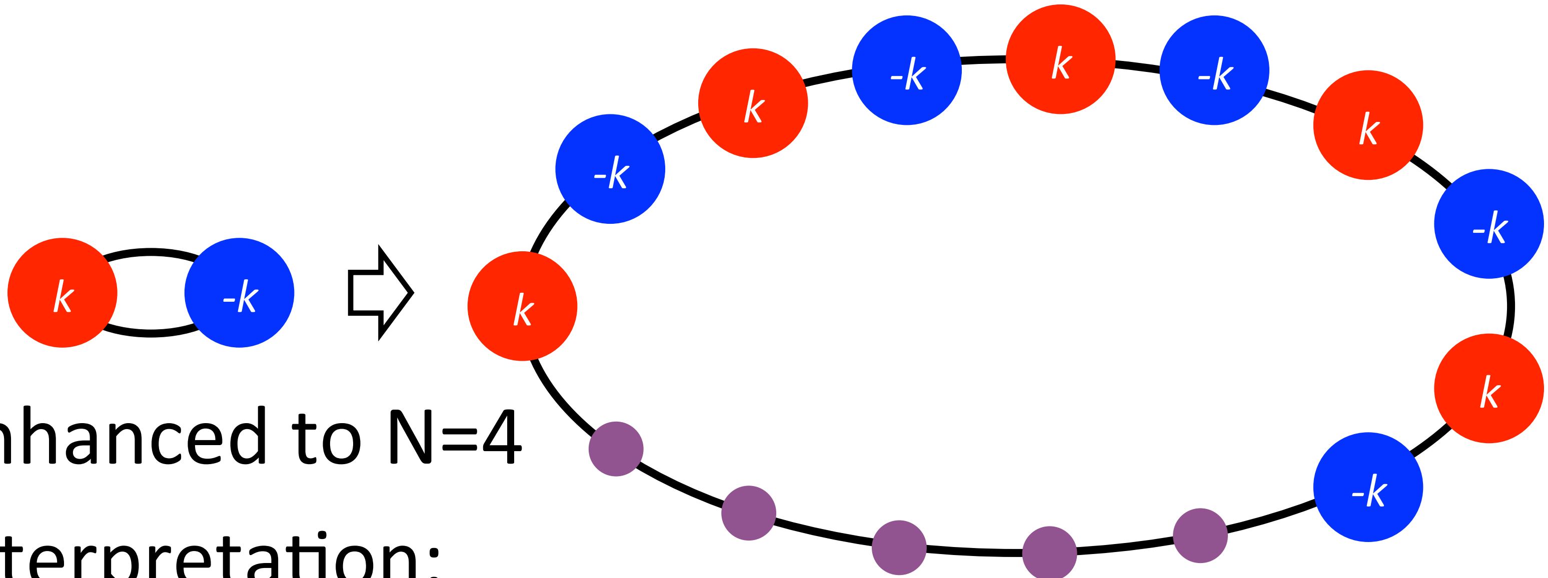


(Expected to be M2 on General Background)

Generalizations

Especially, [Gaiotto-Witten, Hosomichi-Lee[3]-Park]

$[U(N)_k \times U(N)_{-k}]^r$



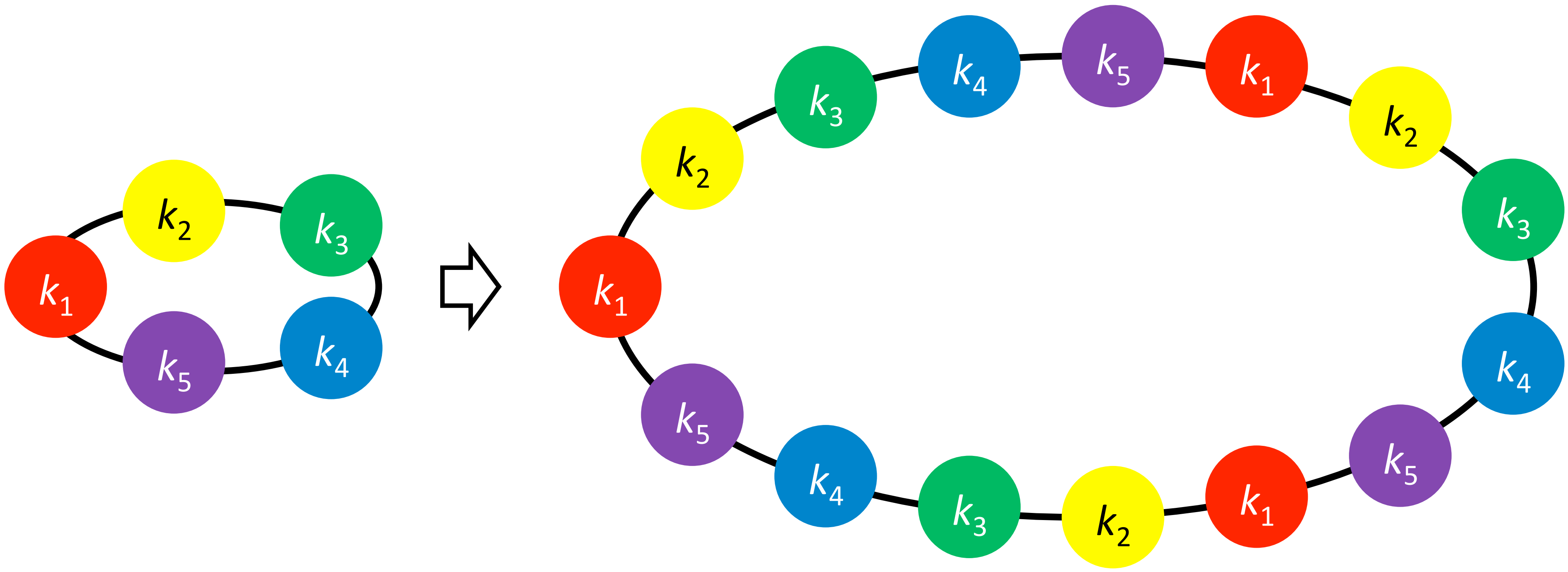
Enhanced to N=4

Interpretation:

Orbifold $C^4 / (Z_{kr} \times Z_r)$ [Benna-Klebanov-Klose-Smedback, Imamura-Kimura, Terashima-Yagi]

In Fact, More Generally

As Long As Multiple Repetition



Results [Honda-M]

- Grand Potential for Multiple Repetition

$$J_r(\mu) = \text{... in terms of } J_1(\mu)$$

- Especially, Perturbative Coefficients

$$C_r = C_1 / r^2 \qquad B_r = B_1 - \pi^2 C_1 (1 - r^{-2})/3 \qquad A_r = r\,A_1$$

Results [Honda-M]

- Orbifold ABJM Partition Function Solved

- A New Instanton Effect Originates From Pert

$$\exp(-2\pi^2 C\mu/k) = \exp(-2\mu/r)$$

- Interpretation:

D2 wrapping A New Lagragian Submfd RP^3/Z_r

More N=4 Superconformal Theories

N=4 Enhancement, Not Restricted To

$$\{k_a\} = \{k, -k, k, -k, \ldots, k, -k\}$$

But [Imamura-Kimura]

$$k_a = (k/2)\,(s_a - s_{a-1}) \qquad s_a = \pm 1$$

Many Properties Similar to ABJM

Airy Functions, Cancellation Mechanism, ...

[M-Nosaka, see Nosaka's poster]

Summary

- ABJM Partition Function Solved
- Generalizations to General N=3 Superconformal Circular Quiver Theories?
- Start with N=4

 - Orbifold Case: Solved AND Beyond

 - General Cases: Just Starting

- Hope: Solve All Superconformal Theories & Understand Membrane Moduli Space

Thank you for your attention.