D-brane on Poisson manifold and Generalized Geometry

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Abstract

The properties of the D-brane fluctuations are investigated using the two types of deformation of the Dirac structure, based on the B-transformation and the β-transformation, respectively.
The former gives the standard gauge theory with 2-form field strength.
The latter gives a non-standard gauge theory with bivector field strength on the Poisson manifold and the vector field as a gauge potential, where the gauge symmetry is a diffeomorphism generated by the Hamiltonian vector field.
The map between the two gauge theories is constructed with the help of Moser's Lemma and the Magnus expansion.
The relation to gauge theory on noncommutative D-brane is also investigated.

Motivations

Seiberg-Witten Map

D-brane effective theory can be described by both
-Ordinary gauge theory,
-Noncommutative gauge theory,
reflecting the regularization schemes
-Pauli-Villars regularization,
-Point-splitting regularization, respectively.

As far as the effective theory is well-defined, they must be identified by field redefinition [`99 Seiberg, Witten]: Seiberg-Witten map

Ordinary gauge th. ⟺ Noncomm. gauge th.

Noncommutative D-brane (D-brane from D-instanton)

Coherent boundary state

$$|B\rangle = \int [d\xi] \exp\left[\frac{i}{2}\int d\sigma \xi^\alpha \partial_\sigma \xi^\beta \omega_{\alpha\beta} - i\int d\sigma P_\alpha(\sigma)\xi^\alpha\right]|B\rangle_{-1} \quad \text{with } \hat{X}^\mu(\sigma)|B\rangle_{-1} = 0$$

can be interpreted as both [`99 Ishibashi, `99 Okuyama]
-D-brane with gauge field $F_{\alpha\beta} = \omega_{\alpha\beta}$
-D-instantons on noncommutative plane $[\xi^\alpha, \xi^\beta] = i\theta^{\alpha\beta}$

Furthermore, gauge field strength can be interpreted as V.E.V. of B-filed [`99 Ishibashi, Iso, Kawai, Kitazawa] $\langle B_{\alpha\beta}\rangle = \omega_{\alpha\beta}$

D-brane with/in Gauge field/B-field ⟺ $\omega^{-1} = \theta$ D-instantons on Noncomm. plane

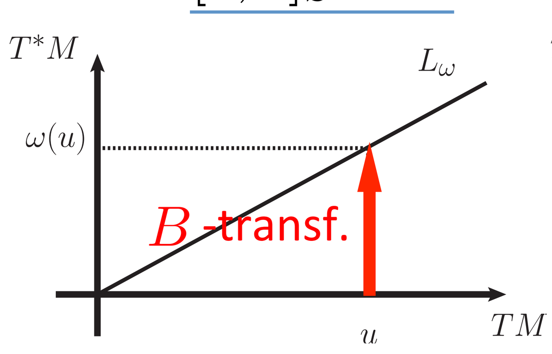
Generalized Geometry [`03 Hitchin, `04 Gualtieri]

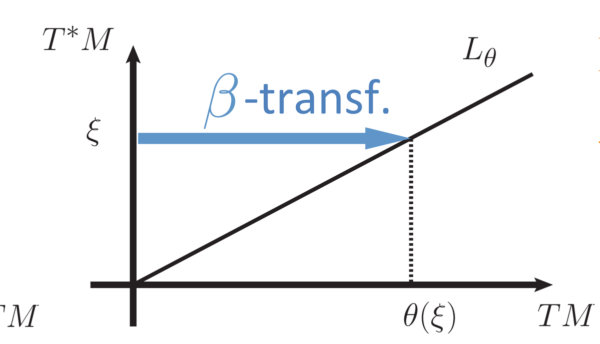
Generalized tangent bundle $E = TM \oplus T^*M$
Generalized section: vector + 1-form $v + \xi = v^i\partial_i + \xi_i dx^i$
-Dorfman bracket

$$[v+\xi, w+\eta]_D = \mathcal{L}_v w + \mathcal{L}_v \eta - \iota_w d\xi =: \mathcal{L}_{v+\xi}(w+\eta)$$

generates diffeomorphism + gauge transformation of B-field
-Canonical inner product $\langle u+\xi, v+\eta\rangle = \iota_u\eta + \iota_v\xi$
is invariant under $O(D,D)$ transformation
-Dirac structure
is special sub-bundle $L \subset E = TM \oplus T^*M$
s.t. -Maximally isotropic $\langle u+\xi, v+\eta\rangle = 0$ $\dim L = D$
-Involutive $u+\xi,\ v+\eta \in \Gamma(L)$
e.g. -Tangent bundle TM ($[\cdot,\cdot]_D \to [\cdot,\cdot]$)
-Cotangent bundle T*M ($[\cdot,\cdot]_D = 0$)
-B-transformed TM $L_\omega = \{u + \omega(u) | u \in TM\}$
Dirac str. iff ω is symplectic form $d\omega = 0$
-β-transformed T*M $L_\theta = \{\xi + \theta(\xi) | \xi \in T^*M\}$
Dirac str. iff θ is Poisson bivector $\theta = \frac{1}{2}\theta^{ij}\partial_i \wedge \partial_j$

$$[\theta,\theta]_S = 0 \iff \theta^{il}\partial_l\theta^{jk} + \theta^{jl}\partial_l\theta^{ki} + \theta^{kl}\partial_l\theta^{ij} = 0$$





If we set $L_0 = L_\omega = L_\theta$

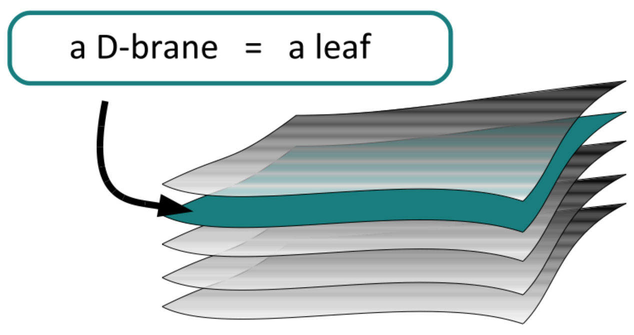
$$\begin{cases} u = \theta(\xi) \\ \xi = \omega(u) \end{cases} \Rightarrow \theta = \omega^{-1}$$

Dirac structure described in terms of

Tangent bundle TM: Symplectic structure ⟺ $\omega^{-1} = \theta$ Cotangent bundle T*M: Poisson structure

Settings

D-brane as Dirac Structure

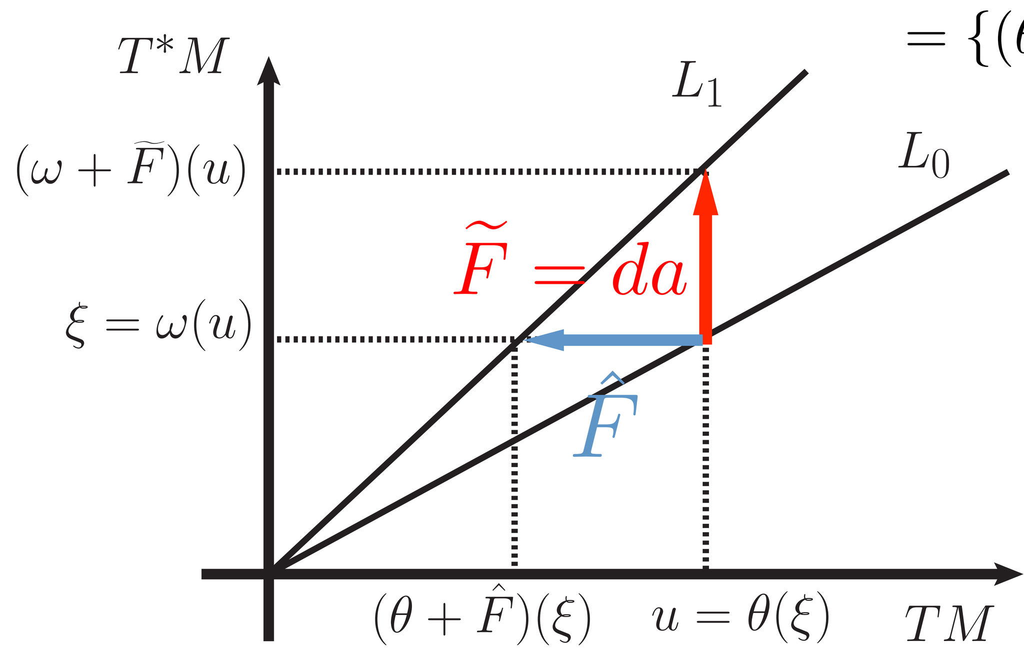


D-brane:
a leaf of foliation generated by Dirac structure [`12 Asakawa, Sasa, Watamura]
-Tangent bundle TM: *Flat D9-brane*
-Cotangent bundle T*M: *D-instanton*
-B-transformed TM: *Flat D9 in B-field backgrounds* $\langle B\rangle = \omega$
-β-transformed T*M: *"D9 on Poisson mfd."*
D9 consists of D-instatons on Poisson mfd.

D-brane fluctuation (D9-brane in $\langle B\rangle = \omega$ for simplicity)

implies replacement of DBI action $\sqrt{g+B} \to \sqrt{g+(B+F)}$
is identified with deformation of Dirac structure

$$\omega \to \omega + \tilde{F} = \omega + da \qquad L_0 \to L_1 = \{u + (\omega+\tilde{F})(u) | u \in TM\} = \{(\theta+\hat{F})(\xi) + \xi | \xi \in T^*M\}$$



Open-Closed relation

$$\theta + \hat{F} = (\omega + \tilde{F})^{-1}$$

[`13 Jurco, Schupp, Vysoky]

$$[\theta+\hat{F}, \theta+\hat{F}]_S = 0$$

⟺ Maurer-Cartan like eq.

$$d_\theta\hat{F} + \frac{1}{2}[\hat{F},\hat{F}]_S = 0$$

with $d_\theta\hat{F} \equiv [\theta,\hat{F}]_S$

Results

Description of D-brane fluctuation in terms of

Tangent bundle TM		⟺	Cotangent bundle T*M	
a	1-form	gauge field	Φ	Vector field
$\tilde{F}$	2-form	field strength	$\hat{F}$	Bi-vector field
$\tilde{F} = da$			$\hat{F} = e^{-\mathcal{L}_\Phi}\theta - \theta$	
$d\tilde{F} = 0$		Bianchi identity	$d_\theta\hat{F} + \frac{1}{2}[\hat{F},\hat{F}]_S = 0$	
$a \to a + d\lambda$		Gauge transformation	$e^{-\mathcal{L}_\Phi} \to e^{-\mathcal{L}_\Phi}e^{-\mathcal{L}_{d_\theta f}}$	

Gauge invariance: Volume preserving diffeomorphism

$$\Phi \to \Phi + d_\theta f + \cdots \qquad d_\theta f = \theta^{ij}(\partial_j f)\partial_i \text{ ···Hamiltonian vector field}$$

Seiberg-Witten like relation

$$\Phi(a + d\lambda) = \Phi'(a,\lambda) \sim \Phi(a) + d_\theta f(a,\lambda) + \cdots$$

Conclusions & Discussions

Seiberg-Witten Map
-key property of the D-brane effective theory
-classically, Moser's Lemma plays a role
Generalized Geometry
-candidate to formulate stringy geometry
-D-brane is identified with Dirac structure
We found
-gauge theory with Hamiltonian vector field gauge invariance
＊but we have not yet constructed its action
-interpretation of Moser's lemma
in generalized geometry framework

Remark

$$H \longleftarrow B + F \longleftarrow A$$
$$R? \longleftarrow \theta + \hat{F} \longleftarrow \Phi$$

Constructions/Investigations of
-Nonabelian gauge theory version i.e. Dirac structure of multiple D-branes
Applications to
-Extended generalized geometry, M2M5 system, ... e.g. [Jurco, et.al.], ...
-Nongeometric/Nonassociative background e.g. [Blumenhagen, et.al.], ...