Effective Field Theory for Spacetime Symmetry Breaking

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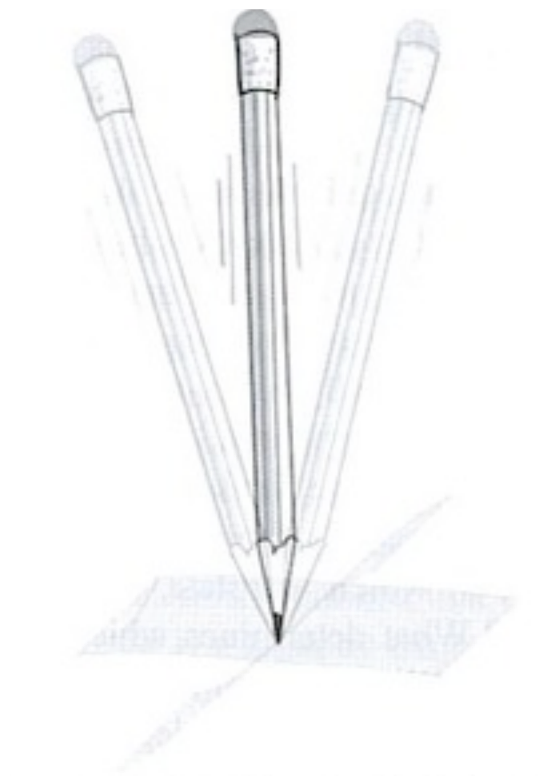
based on a paper in preparation

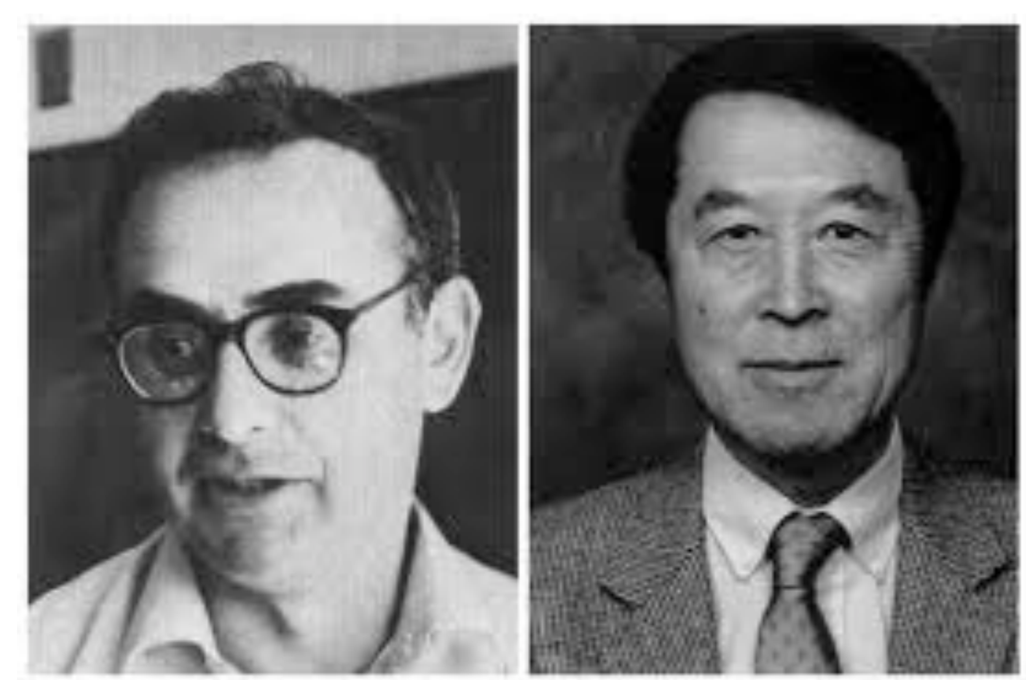
with Y. Hidaka (RIKEN) and G. Shiu (HKUST)

@YITP workshop July 22nd 2014

1. Introduction

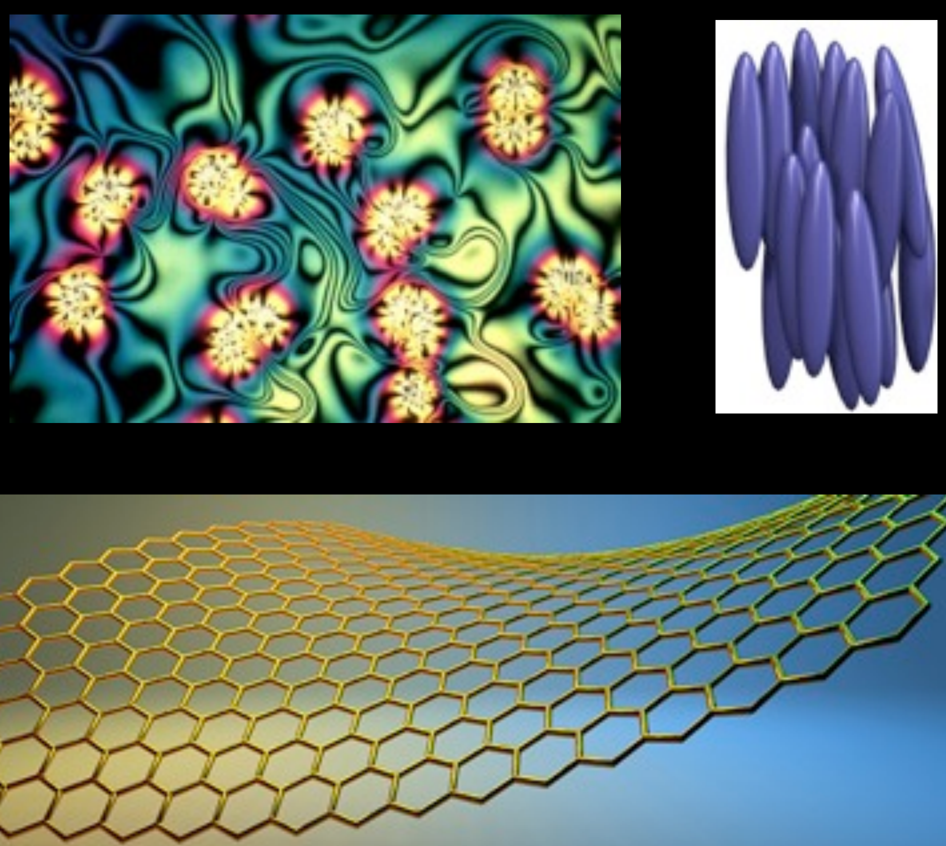


symmetry breaking in physics

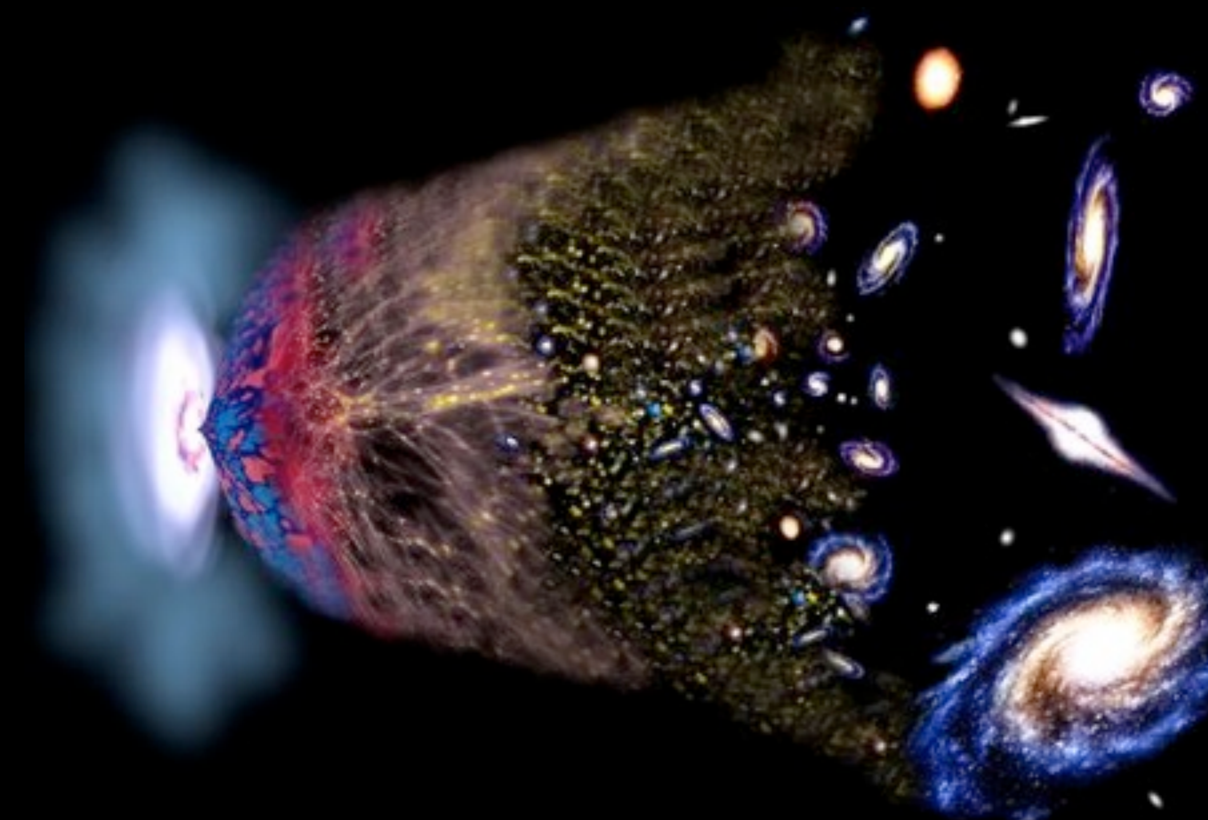


spacetime symmetry breaking

condensed matter



cosmology



various phases of liquid crystal

nematic

smectic A

smectic C

figs: wikipedia

inhomogeneous chiral condensate (in QCD phase diagram?)

$$\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\psi\rangle = \Delta(z)$$

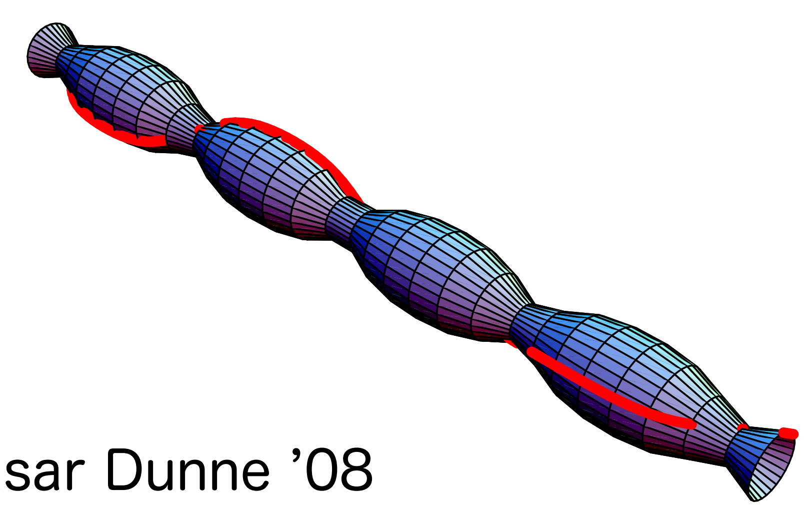
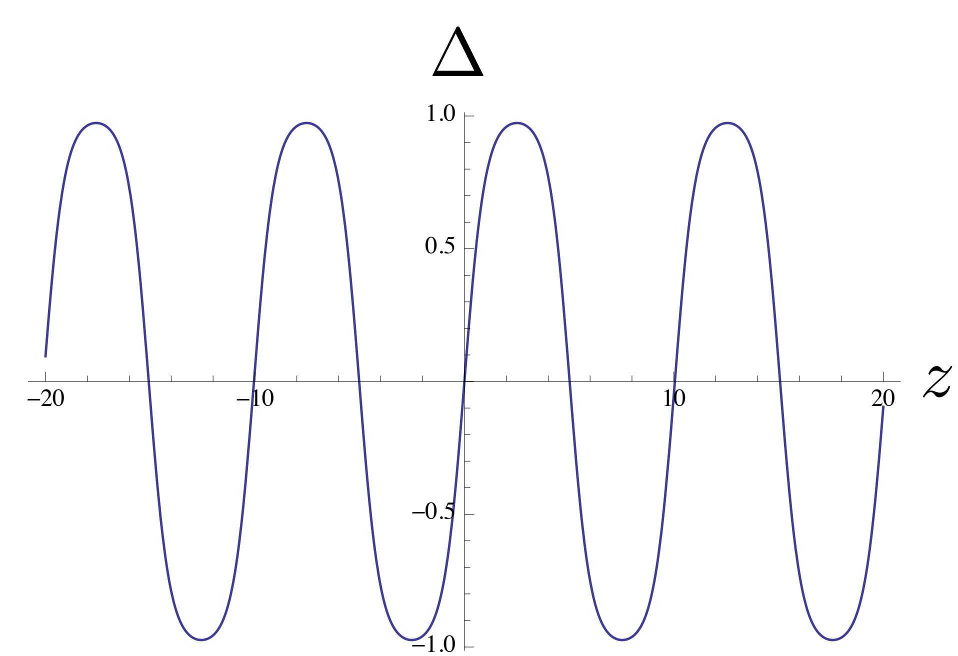


fig: Basar Dunne '08

chiral spiral (complex)



real kink

various phases of liquid crystal

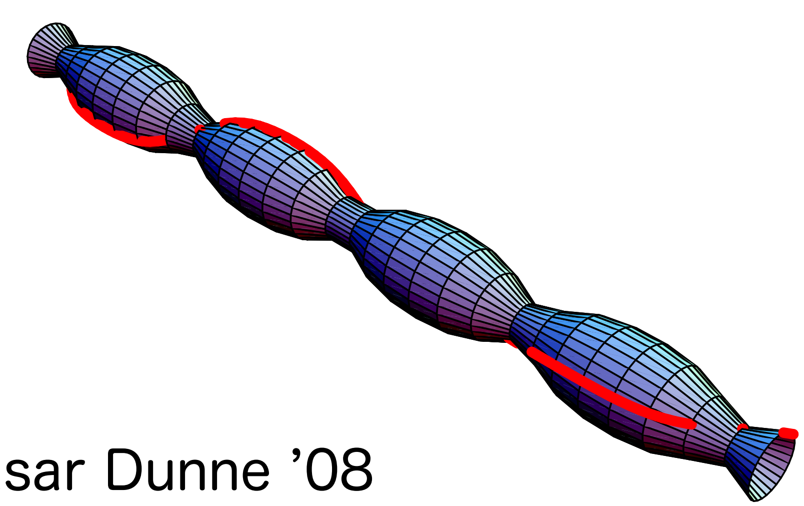
kipedia

would like to understand low-energy dynamics

- # of gapless modes? their dispersion relations?
- from (spacetime) symmetry point of view?

in ?)

$\langle\psi\psi\rangle + i\langle\psi i\gamma\,\psi\rangle = \Delta(z)$



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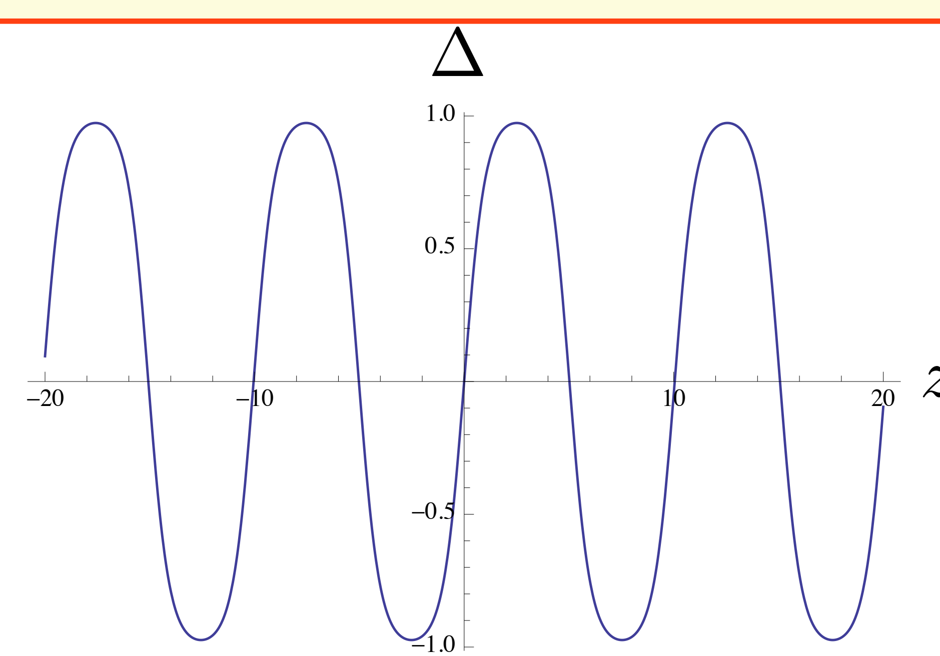
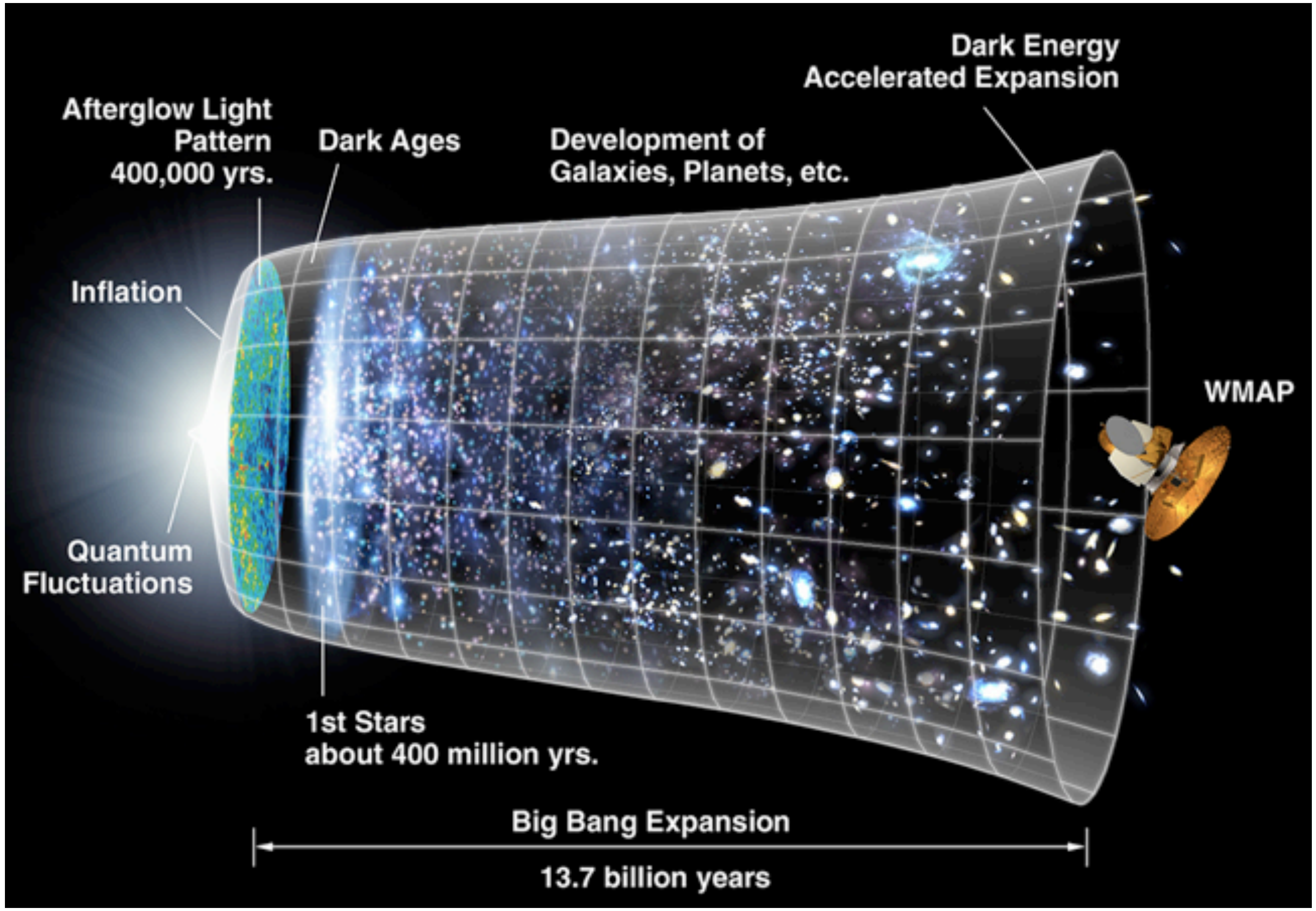


real kink

cosmology

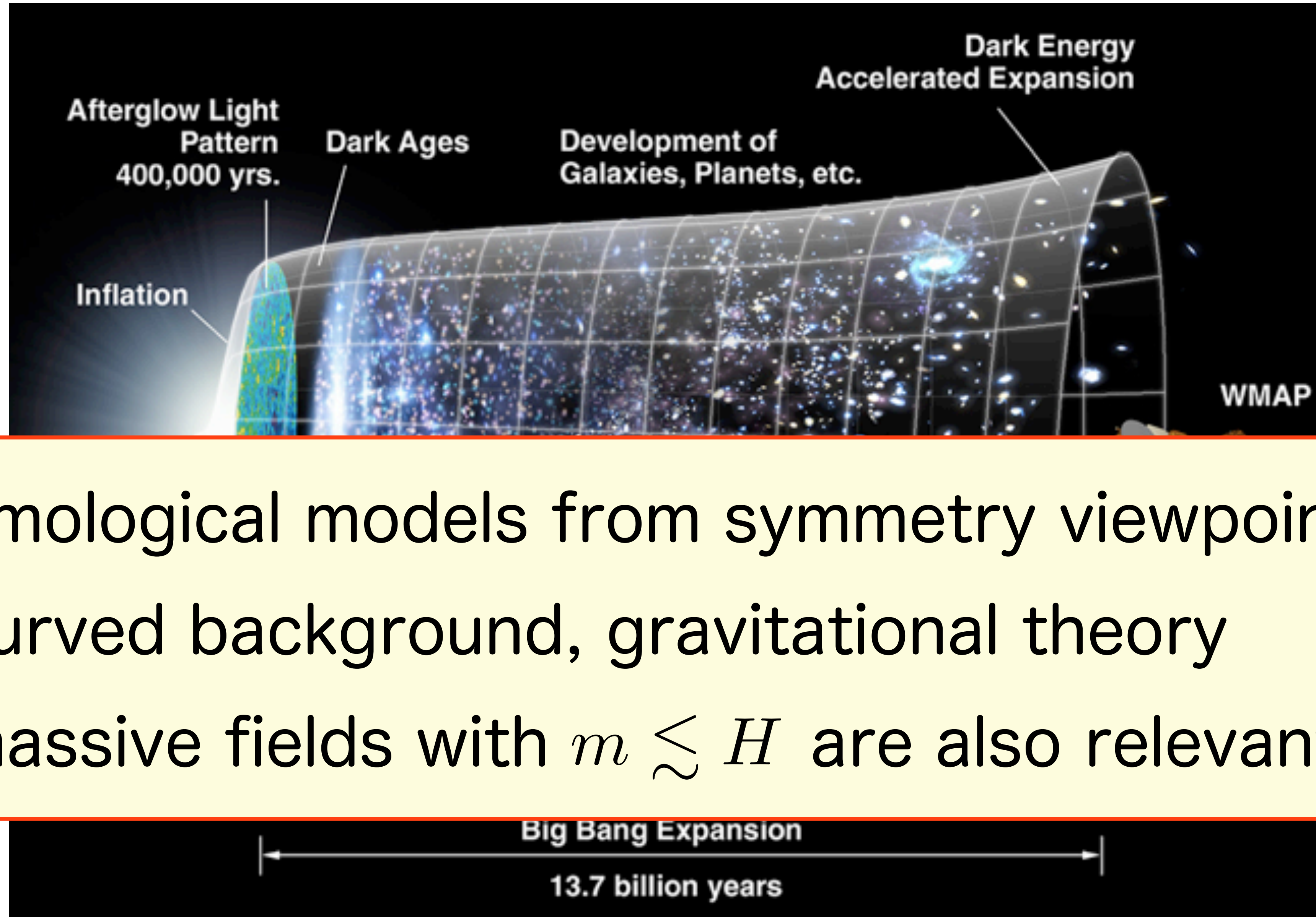


- cosmic expansion breaks time translation generically
- various models for inflation

ex. anisotropic inflation: rotation is also broken

ex. gaugeflation: internal SU(2) x rotation → diagonal SU(2)

cosmology



cosmological models from symmetry viewpoint

- curved background, gravitational theory
- massive fields with $m \lesssim H$ are also relevant

- cosmic expansion breaks time translation generically
- various models for inflation

ex. anisotropic inflation: rotation is also broken

ex. gaugeflation: internal SU(2) x rotation $\to$ diagonal SU(2)

coset construction

coset construction for internal symmetry breaking

consider an internal symmetry breaking $G \to H$

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{a} \quad \begin{cases} \mathfrak{h}: \text{ residual symmetry} \\ \mathfrak{a}: \text{ broken symmetry} \end{cases}$$

- NG modes π^a = coordinates of G/H

$$\Omega = e^{\pi^a(x) T_a} \quad \text{with} \quad T_a \in \mathfrak{a} \quad \text{(broken symmetry)}$$

- ingredients of effective action:

Maurer-Cartan one form $J_\mu = \Omega^{-1} \partial_\mu \Omega$

- effective action is local right H invariant

※ coset construction provides general effective action

extension to spacetime symmetry breaking

ex. conformal symmetry breaking (conformal → Poincare)

broken symmetry: dilatation D and special conformal K_μ

MC form: $J_\mu = \Omega^{-1}\partial_\mu\Omega$ with $\Omega = e^{x^\mu P_\mu} e^{\phi D} e^{\xi^\mu K_\mu}$

- introduce two types of “NG modes”

  ϕ : dilaton, ξ^μ : spurious field to be removed

- global symmetry picture leads to wrong NG mode counting

  ※ NG modes = local transformations of order parameters

- remove ξ^μ by imposing the inverse Higgs constraints

motivation

coset construction:

- has been applied to various condensed matter systems
- captures a certain aspects of spacetime symmetry breaking

however, its understanding seems not complete

- no proof that coset construction provides general action
- appearance of spurious NG mode may not be attractive

would like to have an approach

- without spurious NG mode from the beginning
- appropriate to curved spacetime & gravitational theory

$\rightarrow$ effective theory based on a local symmetry picture

plan of my talk:

1. Introduction ✔

2. Basic strategy

3. Case study 1: scalar domain walls

4. Case study 2: vector domain walls

5. Summary and discussion

2. Basic strategy

coset construction from gauge symmetry breaking effective action for massive gauge boson A_μ :

$$\int d^4x \, \mathrm{tr}\left[-\frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} - \frac{v^2}{2}A_{\mathfrak{a}\,\mu}\,A^{\mu}_{\mathfrak{a}} + \dots\right] \quad \text{with} \quad A_{\mathfrak{a}\mu} \in \mathfrak{a}$$

- g : gauge coupling, v : order parameter
- NG modes are eaten by gauge boson (unitary gauge)

😊 dynamical dof = gauge field only

introduce NG modes by Stuckelberg method:

$$A_\mu \to A'_\mu = \Omega^{-1}A_\mu\Omega + \Omega^{-1}\partial_\mu\Omega \quad \text{with} \quad \Omega = e^{\pi^a(x)T_a}$$

※ global symmetry limit can be obtained by setting $A_\mu = 0$

$A_\mu \to J_\mu = \Omega^{-1}\partial_\mu\Omega$ in the unitary gauge effective action

$$\int d^4x \, \mathrm{tr}\left[-\frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} - \frac{v^2}{2}A_{\mathfrak{a}\,\mu}\,A^{\mu}_{\mathfrak{a}} + \dots\right] \to \int d^4x \, \mathrm{tr}\left[-\frac{v^2}{2}J_{\mathfrak{a}\,\mu}\,J^{\mu}_{\mathfrak{a}} + \dots\right]$$

gauge field is the only dof in the unitary gauge

↓

can be thought of as a proof for completeness of coset construction

unitary gauge is convenient to find general ingredients for EFT

the most careful way to construct the general effective action will be

1. gauge the (broken) global symmetry
2. write down the unitary gauge effective action
3. introduce NG modes by Stuckelberg method
   and decouple the gauge sector

local properties of spacetime symmetry

local properties of spacetime symmetry

consider a spacetime symmetry associated with $x'^{\mu} = x^{\mu} - \epsilon^{\mu}(x)$

its local properties around a point $x^{\mu} = x^{\mu}_{*}$ can be read off as

$$\epsilon^{\mu}(x) = \epsilon^{\mu}(x_{*}) + (x^{\nu} - x^{\nu}_{*})\nabla_{\nu}\epsilon^{\mu}(x) + \dots$$

- 1st term: shift of coord. system (translation)
- 2nd term: deformations of coord. system

$$\nabla_{\mu}\epsilon^{\nu} = \delta^{\nu}_{\mu}\lambda + s_{\mu}{}^{\nu} + \omega_{\mu}{}^{\nu}$$

- trace part λ : isotropic rescaling
- symmetric traceless $s_{\mu\nu}$: anisotropic rescaling
- antisymmetric $\omega_{\mu\nu}$: Lorentz transformation

ex. special conformal on Minkowski space

$$\nabla_{\mu}\epsilon^{\nu} = 2\delta^{\nu}_{\mu}(b \cdot x) + 2(b_{\mu}x^{\nu} - b_{\nu}x^{\mu})$$

locally, a combination of Poincare & isotropic rescaling

relativistic symmetry	diffeomorphism	local Lorentz	local Weyl
translation	✓		
isometry	✓	✓	
conformal	✓	✓	✓

Table 1: Embedding of spacetime symmetry in relativistic systems.

nonrelativistic symmetry	foliation preserving	local rotation	local anisotropic Weyl
translation	✓		
Galilean	✓	✓	
Schrodinger	✓	✓	✓
Galilean conformal	✓	✓	✓

Table 2: Embedding of spacetime symmetry in nonrelativistic systems.

as the local decomposition suggests,

any spacetime symmetry transformation can be embedded

in diffeomorphism, local Lorentz, local (an)isotropic Weyl

gauging spacetime symmetry

gauging spacetime symmetry

global spacetime symmetry $\in$ diffeo x local Lorentz x local Weyl

- diffeo & local Lorentz

  can be gauged by introducing curved spacetime action

$$\int d^4x\,\mathcal{L}[\Phi, \partial_m \Phi] \to \int d^4x\sqrt{-g}\,\mathcal{L}[\Phi, e^\mu_m \nabla_\mu \Phi]$$

- Weyl symmetry

  1. Ricci gauging (not necessarily possible)

     introduce a local Weyl invariant curved spacetime action

  2. Weyl gauging (always possible)

     gauge global Weyl symmetry by introducing a gauge field W_μ

EFT recipe

diffeomorphism	local Lorentz	local Weyl	internal gauge
spacetime dependence	spin	scaling dimension	internal charge
metric $g_{\mu\nu}$	vierbein e_μ^m	Weyl gauge field W_μ	gauge field A_μ

symmetry breaking pattern based on local symmetries:

can be classified by condensation patterns $\langle\Phi^A(x)\rangle = \bar{\Phi}^A(x)$

once symmetry breaking patterns are given or identified,

we construct the effective action in the following way:

1. gauge the (broken) global symmetry
2. write down the unitary gauge effective action
3. introduce NG modes by Stuckelberg method
   and decouple the gauge sector

plan of my talk:

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2. Basic strategy ✔
3. Case study 1: scalar domain walls
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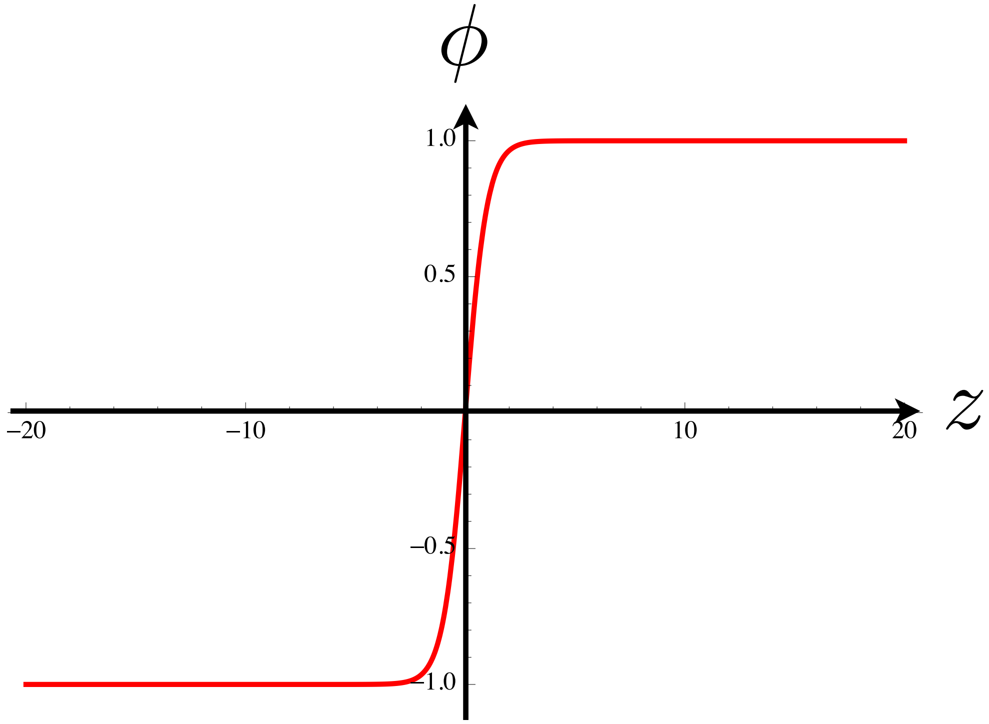
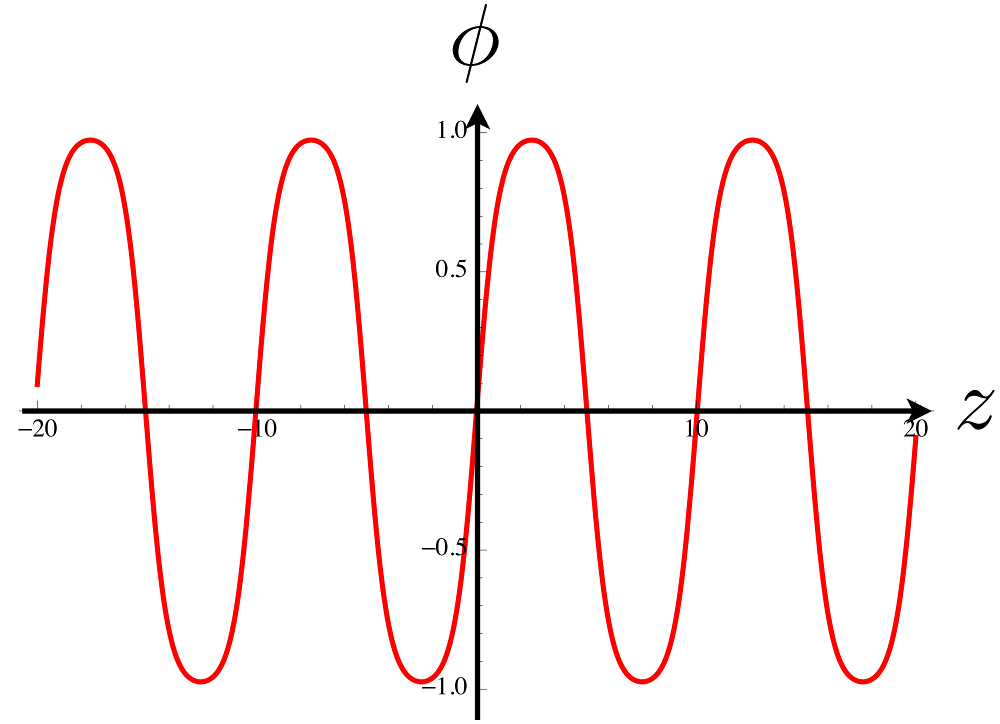
3. Scalar domain-walls

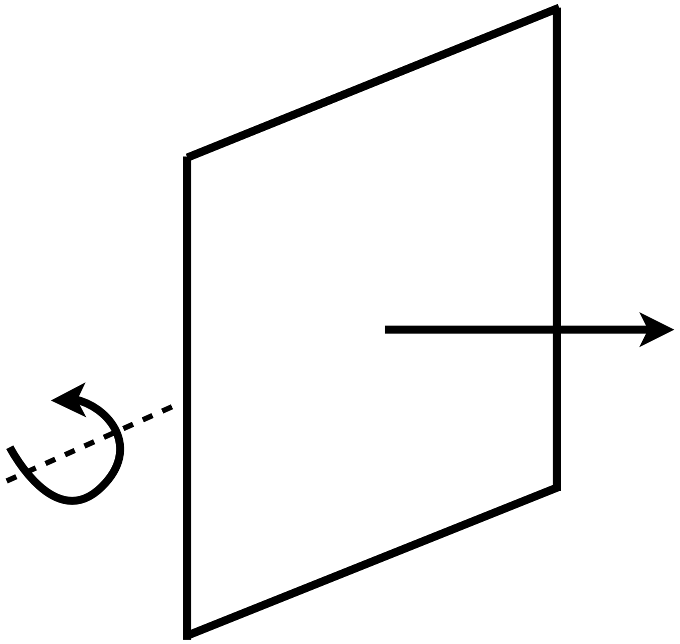
domain-wall configurations of a real scalar



single domain wall



multi domain walls

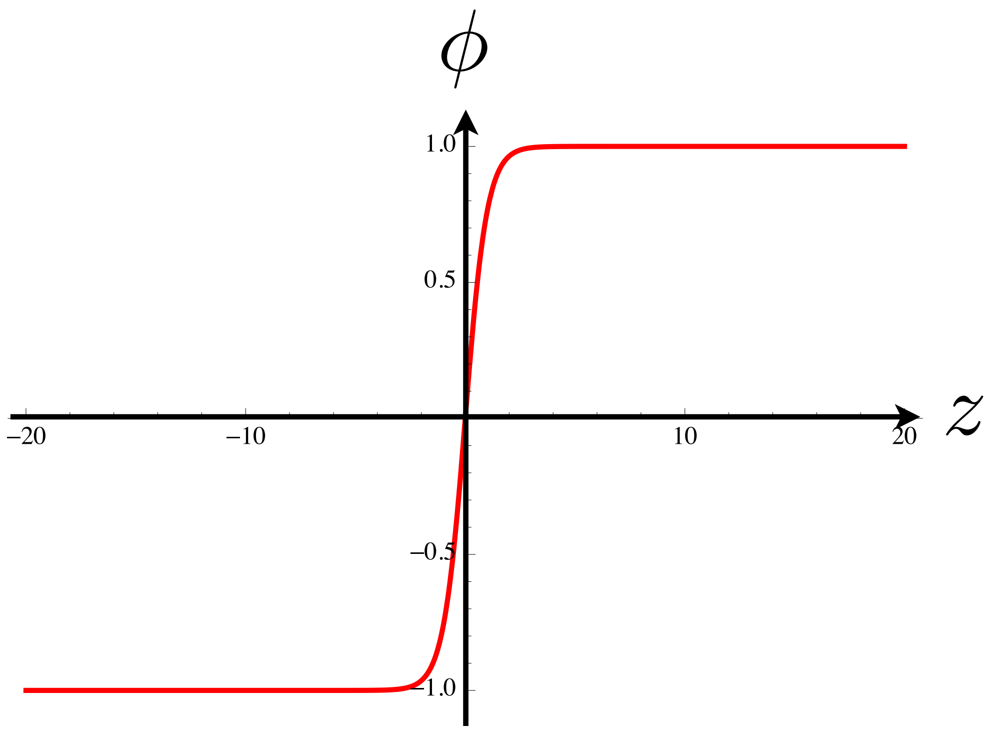


symmetry breaking

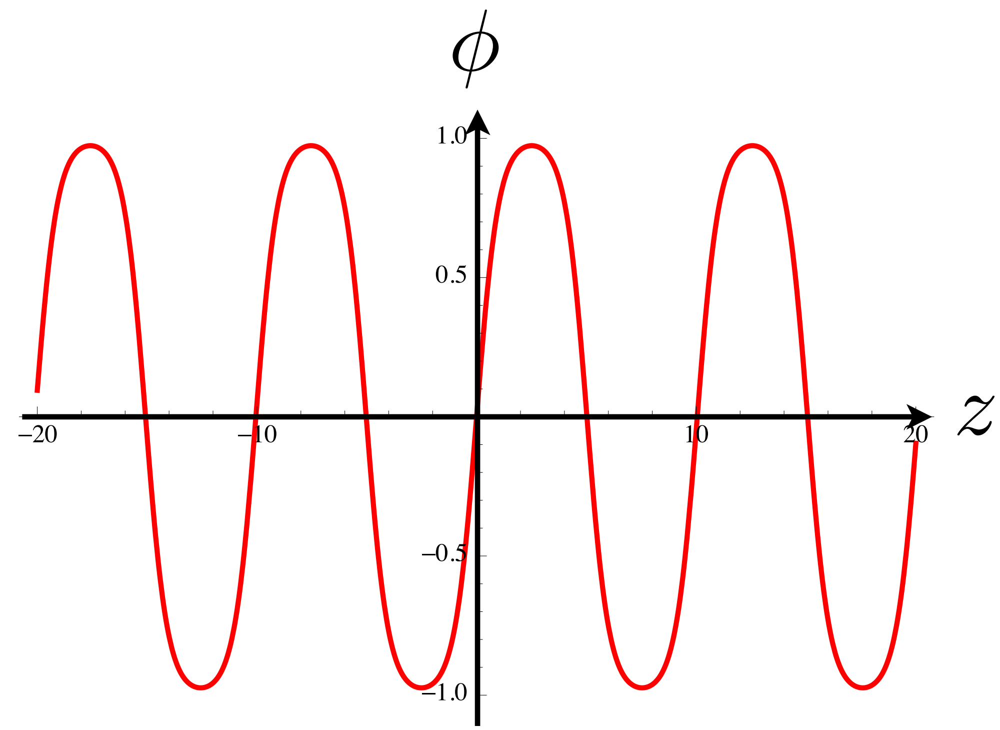
in global sense:

translation and Lorentz invariance are broken

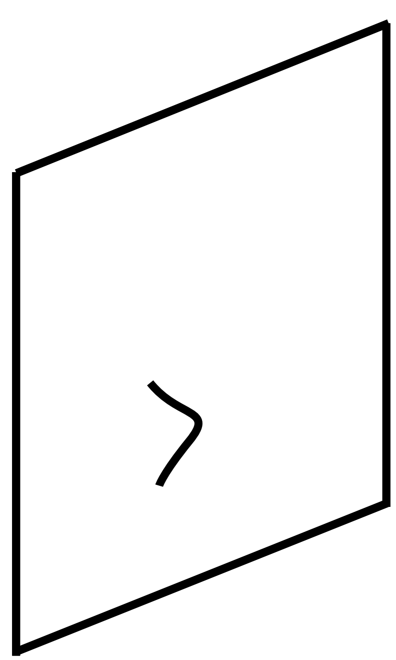
domain-wall configurations of a real scalar



single domain wall



multi domain walls



symmetry breaking

in global sense:

translation and Lorentz invariance are broken

in local sense: only z-diffeo is broken

full diffeo → (1+2)-dim diffeo

domain-wall configurations of a real scalar

- unitary gauge action (cf. EFT for inflation ['07 Cheung et al.])

  dof = metric $g_{\mu\nu}$, residual symmetry = (1+2)-dim diffeo

$$S = \int d^4x\sqrt{-g}\left[\alpha(z) + \beta(z)g^{zz}(x) + \gamma(z)(g^{zz} - 1)^2 + \ldots\right]$$

- action for NG modes

  1. Stuckelberg method: $z \to z + \pi(x)$

  2. decouple the gauge sector ⇔ to set $g_{\mu\nu} = \eta_{\mu\nu}$

$$S = \int d^4x \left[\alpha(z+\pi) + \beta(z+\pi)(1 + 2\partial_z\pi + \partial_\mu\pi\partial^\mu\pi) + \gamma(z+\pi)(2\partial_z\pi + \ldots)^2 + \ldots\right]$$

  3. background (bulk) eom $\to$ $\alpha(z) = \beta(z)$

domain-wall configurations of a real scalar

- unitary gauge action (cf. EFT for inflation ['07 Cheung et al.])

  dof = metric $g_{\mu\nu}$, residual symmetry = (1+2)-dim diffeo

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  1. Stuckelberg method: $z \to z + \pi(x)$

  2. decouple the gauge sector ⇔ to set $g_{\mu\nu} = \eta_{\mu\nu}$

$$S = \int d^4x \left[\alpha(z)\partial_\mu\pi\partial^\mu\pi + 4\gamma(z)(\partial_z\pi)^2 + \mathcal{O}(\pi^3)\right] + \int d^3x \left[\alpha(z)\pi + \mathcal{O}(\pi^2)\right]_{z=-\infty}^{z=\infty}$$

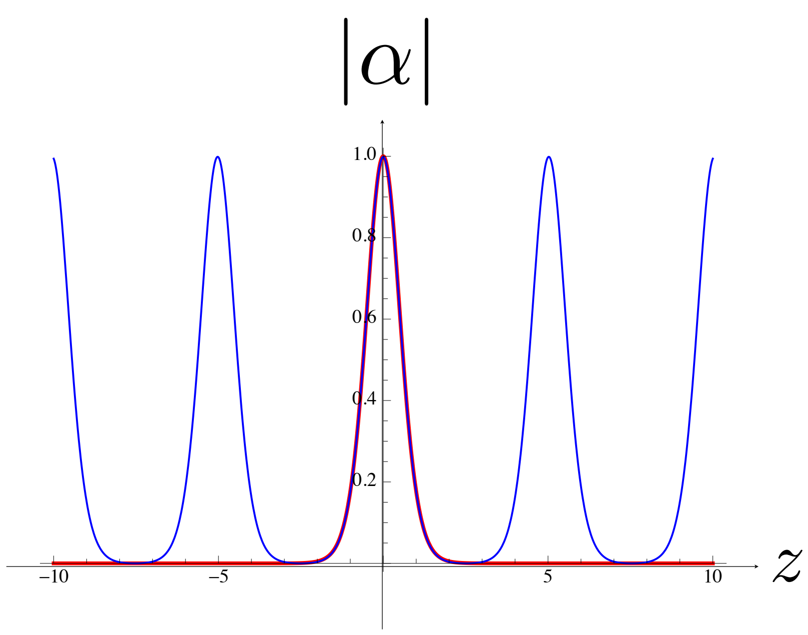
  3. background (bulk) eom → $\alpha(z) = \beta(z)$

domain-wall configurations of a real scalar

let us take a closer look at the obtained action

$$S = \int d^4x \left[\alpha(z)\partial_\mu\pi\partial^\mu\pi + 4\gamma(z)(\partial_z\pi)^2 + \mathcal{O}(\pi^3)\right] + \int d^3x \left[\alpha(z)\pi + \mathcal{O}(\pi^2)\right]_{z=-\infty}^{z=\infty}$$

- free function $\alpha(z)$ = domain-wall profile $\alpha(z) \sim V(z)$



single domain wall:

no kinetic term outside the brane

$\rightarrow$ NG mode does not propagate in the bulk

multiple domain wall:

nonvanishing $\alpha(z)$ @ boundary

$\rightarrow$ instability unless we impose $\pi(\pm\infty) = 0$

※ $\alpha = 0$ for stable backgrounds

domain-wall configurations of a real scalar

let us take a closer look at the obtained action

$$S = \int d^4x \left[\alpha(z)\partial_\mu \pi \partial^\mu \pi + 4\gamma(z)(\partial_z \pi)^2 + \mathcal{O}(\pi^3)\right]$$

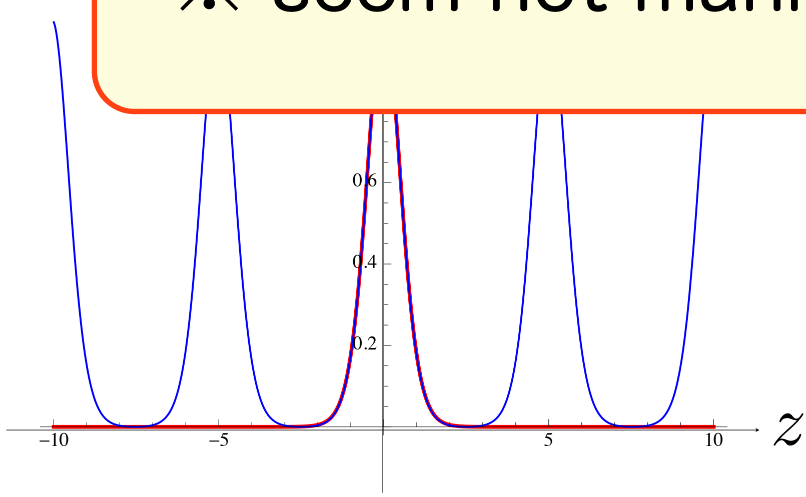
applying a similar discussion in nonrelativistic systems,

we obtain the dispersion relations $\omega^2 \sim 0 \cdot k_\parallel^2 + k_\parallel^4 + k_\perp^2$

for NG modes in inhomogeneous chiral condensates

※ seem not manifest in standard coset construction

bulk



multiple domain wall:

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plan of my talk:

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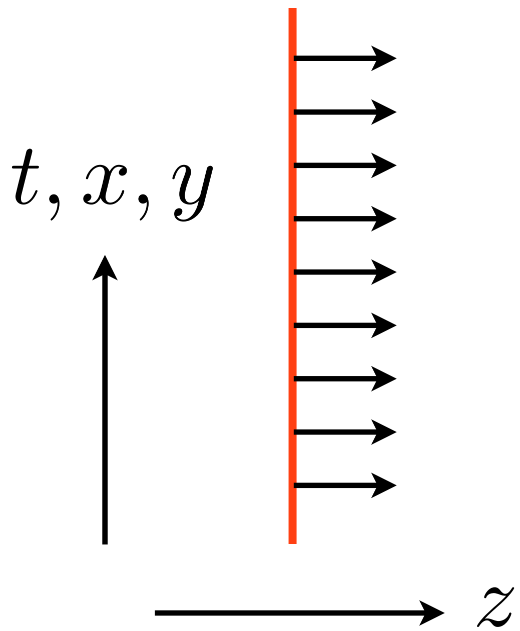
4. vector domain walls

vector domain-wall



symmetry breaking

in global sense:

translation and Lorentz invariance are broken

in local sense:

z-diffeo & z-μ local Lorentz are broken

full diffeo x local Lorentz $\rightarrow$ (1+2)-dim diffeo x local Lorentz

※ introduce $g_{\mu\nu}$ and e_μ^m to gauge spacetime symmetry

- minimal setup in the unitary gauge

dynamical dof: metric $g_{\mu\nu}$, vierbein e_μ^m

residual symmetry: (1+2)-dim diffeo x local Lorentz

vector domain-wall

decompose action schematically as $S = S_P + S_L + S_{PL}$

- S_L : breaks the local Lorentz

$$S_L = \int d^4x \sqrt{-g} \left[\alpha_1 \left(\nabla^\mu e^3_\mu \right)^2 + \alpha_2 \left(\nabla_\mu e^3_\nu - \nabla_\nu e^3_\mu \right)^2 + \alpha_3 \left(e^\nu_3 \nabla_\nu e^3_\mu \right)^2 \right]$$

vector domain-wall

decompose action schematically as $S = S_P + S_L + S_{PL}$

- S_L : breaks the local Lorentz

$$S_L = \int d^4x \left[\alpha_1 \left(\partial^{\widehat{\mu}}\xi_{\widehat{\mu}}\right)^2 + \alpha_2 \left(\partial_{\widehat{\mu}}\xi_{\widehat{\nu}} - \partial_{\widehat{\nu}}\xi_{\widehat{\mu}}\right)^2 + \left(2\alpha_2 + \alpha_3\right)\left(\partial_z \xi_{\widehat{\mu}}\right)^2\right]$$

$\rightarrow$ kinetic terms for Lorentz NG modes $\xi_{\widehat{\mu}}$ $(\widehat{\mu} = t, x, y)$

\# vector domain-wall

decompose action schematically as $S = S_P + S_L + S_{PL}$

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$\rightarrow$ kinetic terms for Lorentz NG modes $\xi_{\widehat{\mu}}$ $(\widehat{\mu} = t, x, y)$

- S_{PL} : breaks both of diffs & local Lorentz

$$S_{PL} = \int d^4x \sqrt{-g} m^2 (e_3^{\mu} n_{\mu} - 1) \quad \text{with } n_{\mu} = \frac{\delta_{\mu}^{z}}{\sqrt{g^{zz}}}$$

vector domain-wall

decompose action schematically as $S = S_P + S_L + S_{PL}$

- S_L : breaks the local Lorentz

$$S_L = \int d^4x \left[\alpha_1 \left(\partial^{\widehat{\mu}} \xi_{\widehat{\mu}} \right)^2 + \alpha_2 \left(\partial_{\widehat{\mu}} \xi_{\widehat{\nu}} - \partial_{\widehat{\nu}} \xi_{\widehat{\mu}} \right)^2 + (2\alpha_2 + \alpha_3) \left(\partial_z \xi_{\widehat{\mu}} \right)^2 \right]$$

$\rightarrow$ kinetic terms for Lorentz NG modes $\xi_{\widehat{\mu}}$ $(\widehat{\mu} = t, x, y)$

- S_{PL} : breaks both of diffs & local Lorentz

$$S_{PL} = \int d^4x \sqrt{-g} m^2 (e_3^{\mu} n_{\mu} - 1) \rightarrow \int d^4x \left[-\frac{m^2}{2} \left(\xi_{\widehat{\mu}} - \partial_{\widehat{\mu}} \pi \right)^2 + \dots \right]$$

※ ξ becomes massive

※ at the energy scale E << m,

we obtain effective scalar interaction $\alpha_i \left(\partial_{\parallel}^2 \pi \right)^2$

cf. inverse Higgs integrates out the ξ field also

vector domain-wall

decompose action schematically as $S = S_P + S_L + S_{PL}$

- S_L : breaks the local Lorentz

applying a similar discussion in nonrelativistic systems,

obtain effective action for smectic A phase of liquid crystal

NG mode dispersion relations: $\omega^2 \sim 0 \cdot k_{\parallel}^2 + k_{\parallel}^4 + k_{\perp}^2$

※ ξ becomes massive

※ at the energy scale E << m,

we obtain effective scalar interaction $\alpha_i \left(\partial_{\parallel}^2 \pi\right)^2$

cf. inverse Higgs integrates out the ξ field also

5. Summary and prospects

summary

- EFT approach for spacetime symmetry breaking
  - from local symmetry picture
  - spacetime symmetry $\in$ diffeo x local Lorentz x (an)isotropic Weyl
  - effective action from gauge symmetry breaking

- in this talk, I discussed domain walls of scalar and vector
  - discussions on boundary linear term

    ex. application to inhomogeneous chiral condensation
  - vector domain walls $\rightarrow$ massive Lorentz NG modes

    ex. liquid crystal in smectic A phase at zero temperature
  - classification of physical meaning of inverse Higgs constraints

    ※ beyond gapless modes cf. cosmological application

other results and prospects

- other symmetry breaking patterns
  - relativistic $\to$ nonrelativistic (global Lorentz symmetry breaking)
  - extension to gravitational systems on cosmological background
  - effective action from gauge symmetry breaking
- extension to gravitational systems on cosmological background
  - application to inflation
- more on nonrelativistic case
  - finite temperatures, finite densities, ...
- inclusion of SUSY, ...

Thank you!