

# **Non-renormalization Theorem and Cyclic Leibniz Rule in Lattice Supersymmetry**

M. Sakamoto (Kobe University)

in collaboration with M. Kato and H. So  
based on JHEP 1305(2013)089; arXiv:1311.4962;  
and in progress

# Supersymmetry on lattice



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Our results suggest that the answer is possibly ***negative***.

# Leibniz rule and SUSY algebra



We want to find lattice SUSY transf.  $\delta_Q, \delta_{Q'}$  such that

$$\overbrace{\delta_Q S[\phi, \chi, F]}^{\text{lattice SUSY transf.}} = \overbrace{\delta_{Q'} S'[\phi, \chi, F]}^{\text{lattice action}} = 0$$

with the SUSY algebra

$$\{ \delta_Q, \delta_{Q'} \} = \overbrace{\delta_P}^{\text{“translation” on lattice}}$$

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One might replace  $\delta_P$  by a difference operator  $\nabla$ .

Then, we need to find  $\nabla$  which satisfies the **Leibniz rule**.

$$\delta_P(\phi\psi) = (\delta_P\phi)\psi + \phi(\delta_P\psi)$$

$$\xrightarrow{\delta_P \rightarrow \nabla} \boxed{\nabla(\phi\psi) = (\nabla\psi)\psi + \phi(\nabla\psi)} \quad \text{Leibniz rule}$$

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However, we can show that it is hard to realize the Leibniz rule on lattice!!

To answer the question whether the Leibniz rule can be realized on lattice or not, let us consider general forms of difference operators and field products such as

**difference operator:**  $(\nabla \phi)_n \equiv \sum_m \nabla_{nm} \phi_m$

**field product:**  $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm} \phi_l \psi_m$

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For example,

$$(\nabla\phi)_n = \phi_{n+1} - \phi_n \implies \nabla_{nm} = \delta_{n+1,m} - \delta_{n,m}$$

$$(\phi * \psi)_n = \phi_n\psi_n \implies M_{nlm} = \delta_{n,l}\delta_{n,m}$$

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## No-Go Theorem

*M.Kato, M.S. & H.So, JHEP 05(2008)057*

There is no difference operator  $\nabla$  satisfying the following three properties:

- i) translation invariance
- ii) locality
- iii) Leibniz rule  $\nabla(\phi * \psi) = (\nabla \phi) * \psi + \phi * (\nabla \psi)$

# Our approach to construct lattice SUSY models

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The No-Go theorem tells us that we cannot realize SUSY algebras with  $\nabla$  equipped with the Leibniz rule.

Our strategy to construct lattice SUSY models is

full SUSY algebra  $\longrightarrow$

$$\text{Nilpotent SUSY algebra} \\ (\delta_Q)^2 = (\delta_{Q'})^2 = \{\delta_Q, \delta_{Q'}\} = 0$$

Leibniz rule  $\longrightarrow$

Cyclic Leibniz rule

# Complex SUSY quantum mechanics on lattice



## Lattice action

$$\begin{aligned} S = & (\nabla\phi_-, \nabla\phi_+) - (F_-, F_+) - i(\chi_-, \nabla\bar{\chi}_+) + i(\nabla\bar{\chi}_-, \chi_+) \\ & - \lambda_+(F_+, \phi_+ * \phi_+) + 2\lambda_+(\chi_+, \bar{\chi}_+ * \phi_+) \\ & - \lambda_-(F_-, \phi_- * \phi_-) - 2\lambda_-(\chi_-, \bar{\chi}_- * \phi_-) \end{aligned}$$

difference operator:  $(\nabla\phi)_n \equiv \sum_m \nabla_{nm} \phi_m$

field product:  $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm} \phi_l \psi_m$

inner product:  $(\phi, \psi) \equiv \sum_n \phi_n \psi_n$

- To make our discussions simple, we here put  $m=0$ .
- We can add mass terms as well as **supersymmetric Wilson terms** to prevent the doubling.

# N=2 nilpotent SUSYs



N=2 Nilpotent SUSYs:  $(\delta_+)^2 = (\delta_-)^2 = \{\delta_+, \delta_-\} = 0$

$$\left\{ \begin{array}{l} \delta_+ \phi_+ = \bar{\chi}_+ \\ \delta_+ \chi_+ = F_+ \\ \delta_+ \chi_- = -i\nabla \phi_- \\ \delta_+ F_- = -i\nabla \bar{\chi}_- \\ \text{others} = 0 \end{array} \right. \quad \left\{ \begin{array}{l} \delta_- \chi_+ = i\nabla \phi_+ \\ \delta_- F_+ = -i\nabla \bar{\chi}_+ \\ \delta_- \phi_- = -\bar{\chi}_- \\ \delta_- \chi_- = F_- \\ \text{others} = 0 \end{array} \right.$$

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$$\delta_{\pm} S = 0$$



$$(\nabla \bar{\chi}_{\pm}, \phi_{\pm} * \phi_{\pm}) + (\nabla \phi_{\pm}, \phi_{\pm} * \bar{\chi}_{\pm}) + (\nabla \phi_{\pm}, \bar{\chi}_{\pm} * \phi_{\pm}) = 0$$

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We call this **Cyclic Leibniz rule**.

# Cyclic Leibniz rule vs. Leibniz rule



We have found that the **Cyclic Leibniz Rule** guarantees the N=2 nilpotent SUSYs.

*Cyclic Leibniz Rule (CLR)*

$$(\nabla A, B * C) + (\nabla B, C * A) + (\nabla C, A * B) = 0$$

vs.

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$$(\nabla A, B * C) + (A, \nabla B * C) + (A, B * \nabla C) \not= 0$$

↑  
No-Go theorem



The cyclic Leibniz rule ensures **a lattice analog of vanishing surface terms!**

$$\underbrace{(\nabla \phi, \phi * \phi)}_{\text{on lattice}} \stackrel{\text{CLR}}{=} 0 \longleftarrow \int dx \partial_x (\phi(x))^3 \stackrel{\text{in continuum}}{=} 0$$

# An example of CLR



An explicit example of the **Cyclic Leibniz Rule** :

$$(\nabla\phi)_n = \frac{1}{2}(\phi_{n+1} - \phi_{n-1})$$

$$(\phi * \psi)_n = \frac{1}{6}(2\phi_{n+1}\psi_{n+1} + 2\phi_{n-1}\psi_{n-1} + \phi_{n+1}\psi_{n-1} + \phi_{n-1}\psi_{n+1})$$

lattice spacing  
 $a=1$

*M.Kato, M.S. & H.So, JHEP 05(2013)089*

which satisfy i) *translation invariance*, ii) *locality* and iii) *Cyclic Leibniz Rule*.



*The field product  $(\phi * \psi)_n$  should be non-trivial!*

# Advantages of CLR

Advantages of our lattice model with **CLR** are given by

	CLR	no CLR
nilpotent SUSYs		
Nicolai maps		
“surface” terms		
non-renormalization theorem		
cohomology		

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# Non-renormalization theorem in continuum



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## □ 4d N=1 Wess-Zumino model in continuum

$$S = \int d^4x \left\{ \int d^2\theta d^2\bar{\theta} \overbrace{\Phi^\dagger(\bar{\theta})\Phi(\theta)}^{\text{chiral superfield}} + \int d^2\theta \overbrace{W(\Phi)}^{\text{superpotential}} + \text{c.c.} \right\}$$

D term (kinetic terms)                      F term (potential terms)

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### Non-renormalization Theorem

There is *no quantum correction to the F-terms* in any order of perturbation theory.

# Essence of non-renormalization theorem

$$S = \int d^4x \left\{ \int d^2\theta d^2\bar{\theta} \Phi^\dagger(\bar{\theta})\Phi(\theta) + \int d^2\theta W(\Phi) + c.c. \right\}$$

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**Holomorphy** plays an important role in the non-renormalization theorem.

*tree superpotential* ————

$$\int d^2\theta \mathcal{W}_{\text{tree}}(\Phi, \lambda) + \int d^2\bar{\theta} \bar{\mathcal{W}}_{\text{tree}}(\Phi^\dagger, \lambda^*)$$

└── *chiral superfield*  
└── *coupling constant*  
└── *anti-chiral superfield*

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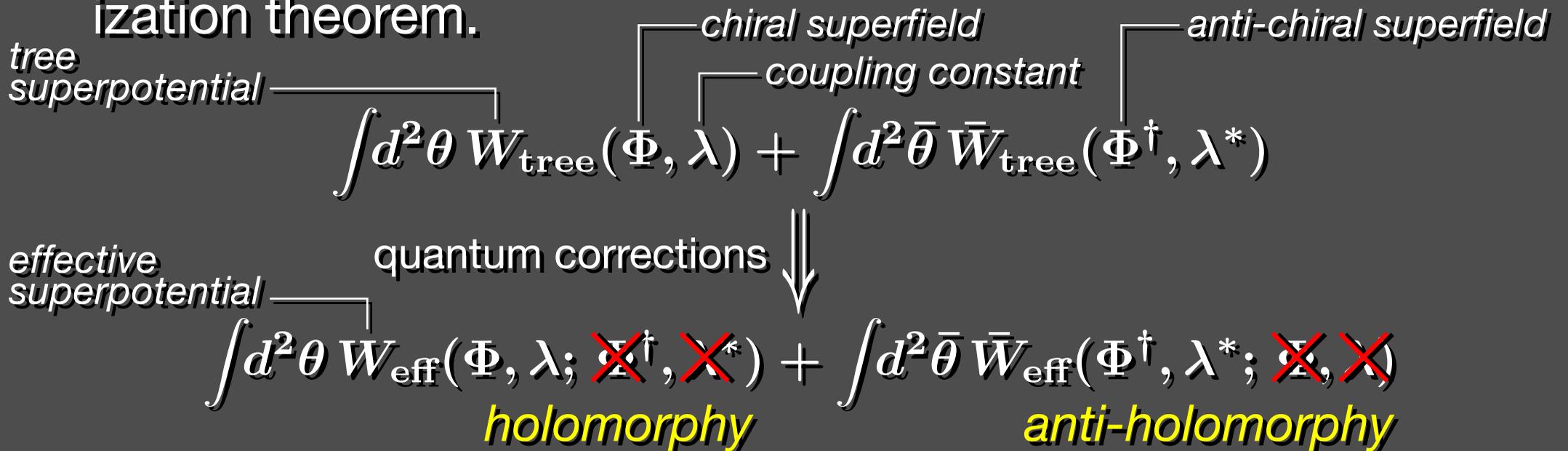
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$$\begin{array}{c}
 \text{tree superpotential} \quad \left[ \begin{array}{l} \text{chiral superfield} \\ \text{coupling constant} \\ \text{anti-chiral superfield} \end{array} \right. \\
 \int d^2\theta \mathcal{W}_{\text{tree}}(\Phi, \lambda) + \int d^2\bar{\theta} \bar{\mathcal{W}}_{\text{tree}}(\Phi^\dagger, \lambda^*) \\
 \text{effective superpotential} \quad \left[ \begin{array}{l} \text{quantum corrections} \\ \Downarrow \end{array} \right. \\
 \int d^2\theta \mathcal{W}_{\text{eff}}(\Phi, \lambda; \Phi^\dagger, \lambda^*) + \int d^2\bar{\theta} \bar{\mathcal{W}}_{\text{eff}}(\Phi^\dagger, \lambda^*; \Phi, \lambda)
 \end{array}$$

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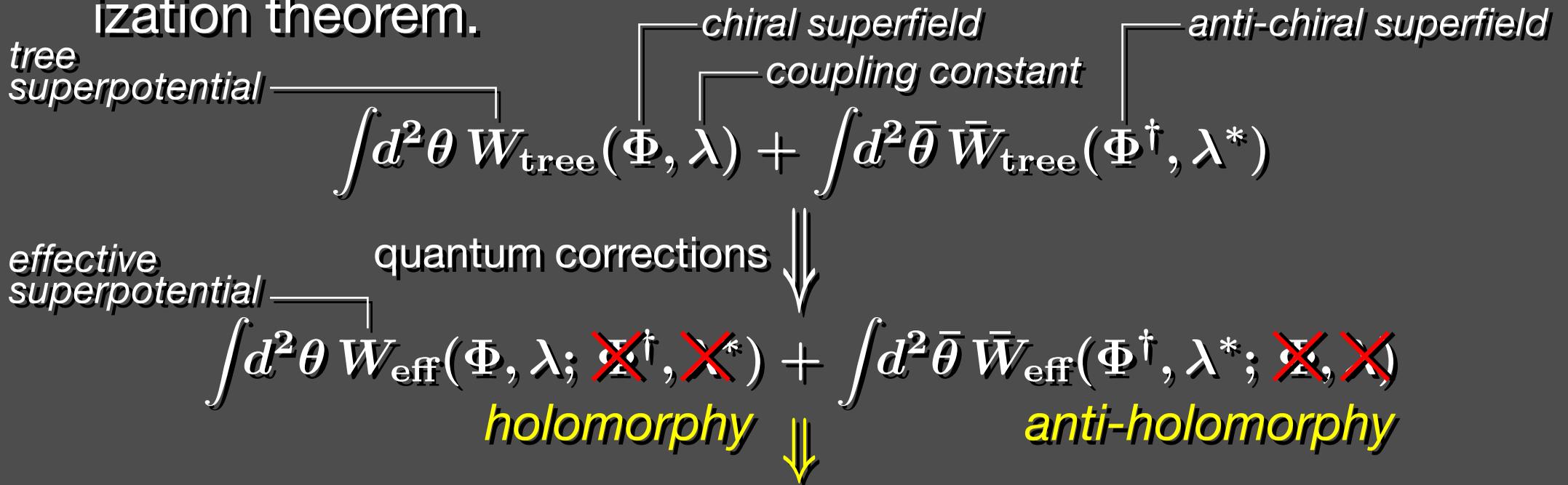
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**No quantum correction!!**

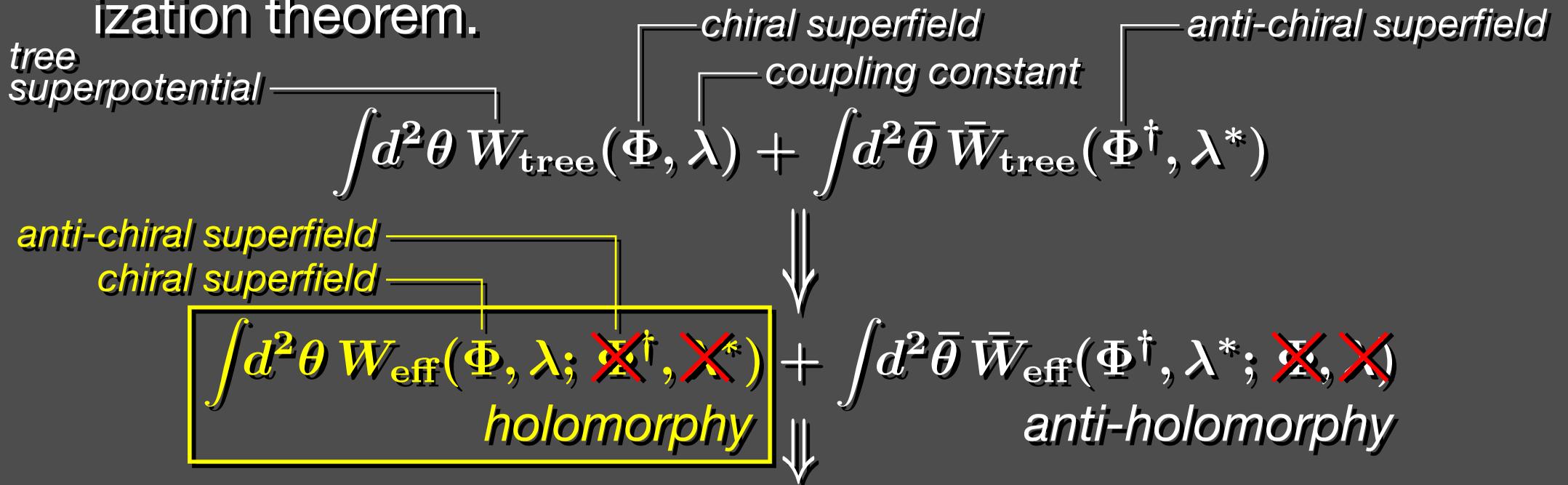
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*N. Seiberg, Phys. Lett. B318 (1993) 469*

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$$W_{\text{eff}} = W_{\text{tree}}$$

N.Seiberg, Phys. Lett. B318 (1993) 469

# Difficulty in defining chiral superfield on lattice 13

The holomorphy requires that the F term  $W(\Phi)$  depends only on the **chiral** superfield  $\Phi(x, \theta)$ , which is defined by

$$\bar{D}\Phi(x, \theta) \equiv \left( \frac{\partial}{\partial \bar{\theta}} - i\theta\sigma_{\mu}\partial_{\mu} \right) \Phi(x, \theta) = 0 \quad \text{in continuum}$$

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However, the above definition of the chiral superfield is **ill-defined** because any products of chiral superfields are not chiral due to the **breakdown of LR on lattice!**

$$\bar{D}\Phi_1 = \bar{D}\Phi_2 = 0 \quad \Longrightarrow \quad \bar{D}(\Phi_1 \Phi_2) \neq 0$$

↑  
*the breakdown of the  
Leibniz rule on lattice*

# Superfield formulation in our lattice model

## □ Lattice superfields

$$\Psi_{\pm}(\theta_{+}, \theta_{-}) \equiv \chi_{\pm} + \theta_{\pm} F_{\pm} + \theta_{\mp} i \nabla \phi_{\pm} + \theta_{\pm} \theta_{\mp} i \nabla \bar{\chi}_{\pm}$$

$$\Lambda_{\pm}(\theta_{\pm}) \equiv \phi_{\pm} + \theta_{\pm} \chi_{\pm}$$

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## □ Lattice action in superspace $S = S_{\text{type I}} + S_{\text{type II}}$

$$S_{\text{type I}} = \int d\theta_{+} d\theta_{-} \Psi_{-} \Psi_{+} \implies \text{kinetic terms (D-term)}$$

$$S_{\text{type II}} = \int d\theta_{+} d\theta_{-} \left\{ \theta_{-} \lambda_{+}(\Psi_{+}, \Lambda_{+} * \Lambda_{+}) + \theta_{+} \lambda_{-}(\Psi_{-}, \Lambda_{-} * \Lambda_{-}) \right\} \\ \implies \text{potential terms (F-term)}$$

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$S_{\text{type II}}$  is SUSY-invariant **if and only if**  $W(\Psi_{+}, \Lambda_{+})$  depends only on  $\Psi_{+}, \Lambda_{+}$  and is written into the form

$$W(\Psi_{+}, \Lambda_{+}) = \sum_n \lambda_{+}^{(n)}(\Psi_{+}, \overbrace{\Lambda_{+} * \Lambda_{+} * \cdots * \Lambda_{+}}^{n-1})$$

and  $(\Psi_{+}, \Lambda_{+} * \Lambda_{+} * \cdots * \Lambda_{+})$  has to obey **CLR**.

M.Kato, M.S., H.So, in preparation

M. SAKAMOTO, talk at Strings and Fields @ YITP, July 24, 2014

# Non-renormalization theorem in our lattice model 15

$$\int d\theta_+ d\theta_- \theta_- W_{\text{tree}}(\Psi_+, \Lambda_+, \lambda_+)$$

$\downarrow$  quantum corrections

$$W_{\text{tree}} \equiv \lambda_+(\Psi_+, \Lambda_+ * \Lambda_+)$$

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SUSY-invariance  
with **CLR** forbids them!

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$$\int d\theta_+ d\theta_- \theta_- W_{\text{eff}}(\Psi_+, \Lambda_+, \lambda_+; \underbrace{\cancel{\Psi_-}, \cancel{\Lambda_-}, \cancel{\lambda_-}})$$

SUSY-invariance  
with **CLR** forbids them!

The **holomorphic property**  
is realized in our lattice model.

# Non-renormalization theorem in our lattice model 15

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SUSY-invariance  
with **CLR** forbids them!

The **holomorphic property**  
is realized in our lattice model.

no quantum corrections:  $W_{\text{eff}} \equiv W_{\text{tree}}$

**The non-renormalization theorem holds  
even for a finite lattice spacing in our lattice model.**

$$S = \overbrace{\delta_+ \delta_- K(\phi_{\pm}, F_{\pm}, \chi_{\pm}, \bar{\chi}_{\pm})}^{S_{\text{type I}}} + \overbrace{\delta_+ W(\phi_+, \chi_+) + \delta_- \bar{W}(\phi_-, \chi_-)}^{S_{\text{type II}}}$$

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invariant under  $\delta_{\pm}$   
because of the **nilpotency**:  
 $(\delta_+)^2 = (\delta_-)^2 = \{\delta_+, \delta_-\} = 0$

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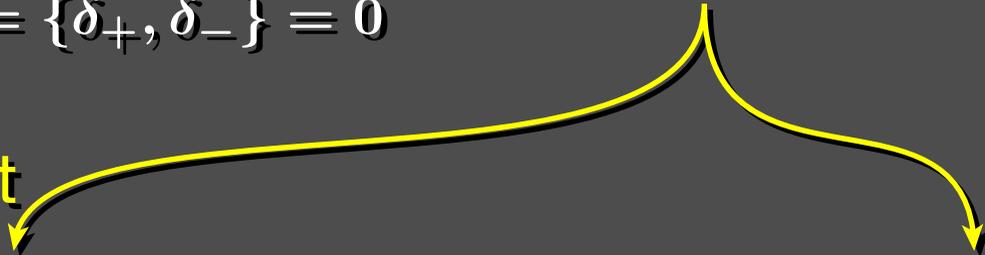
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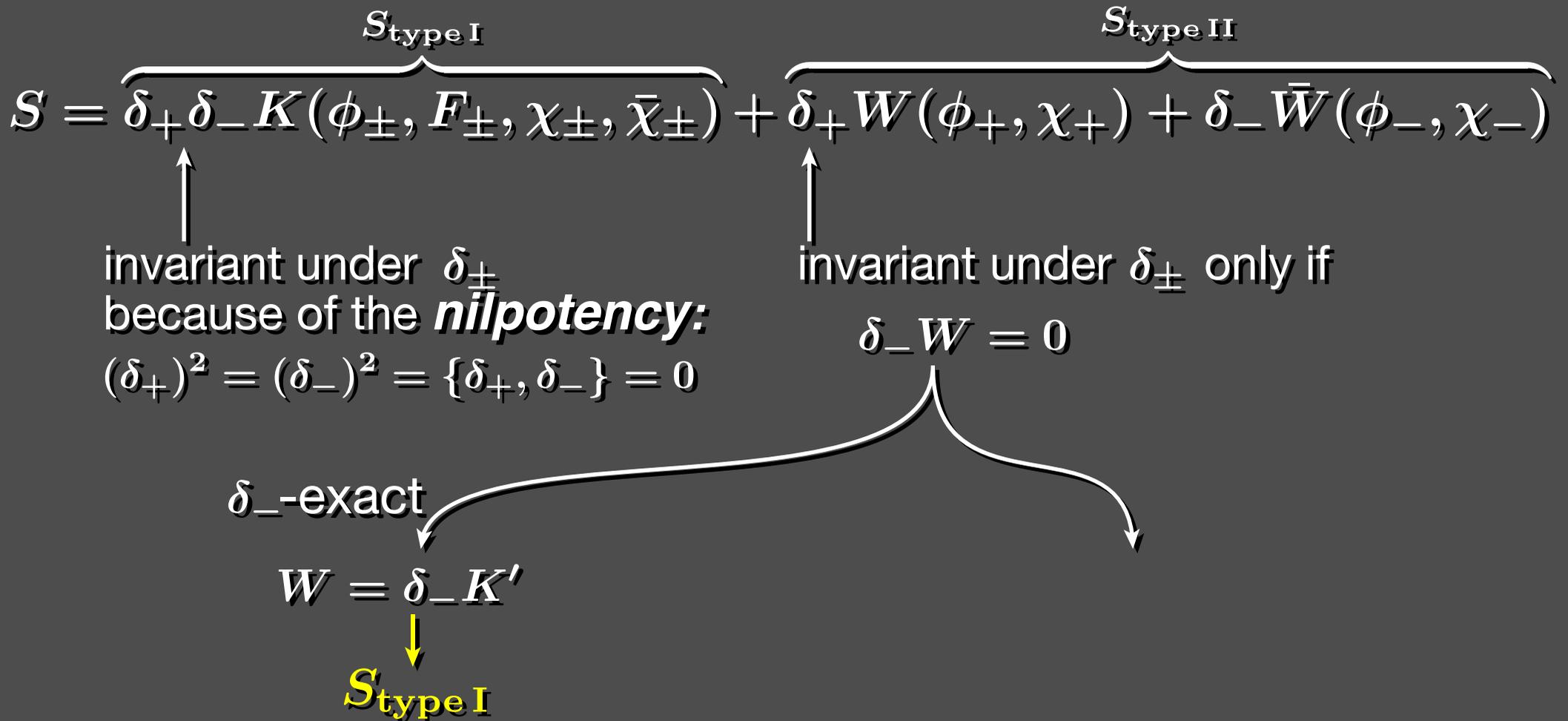
$\delta_-$ -exact
 

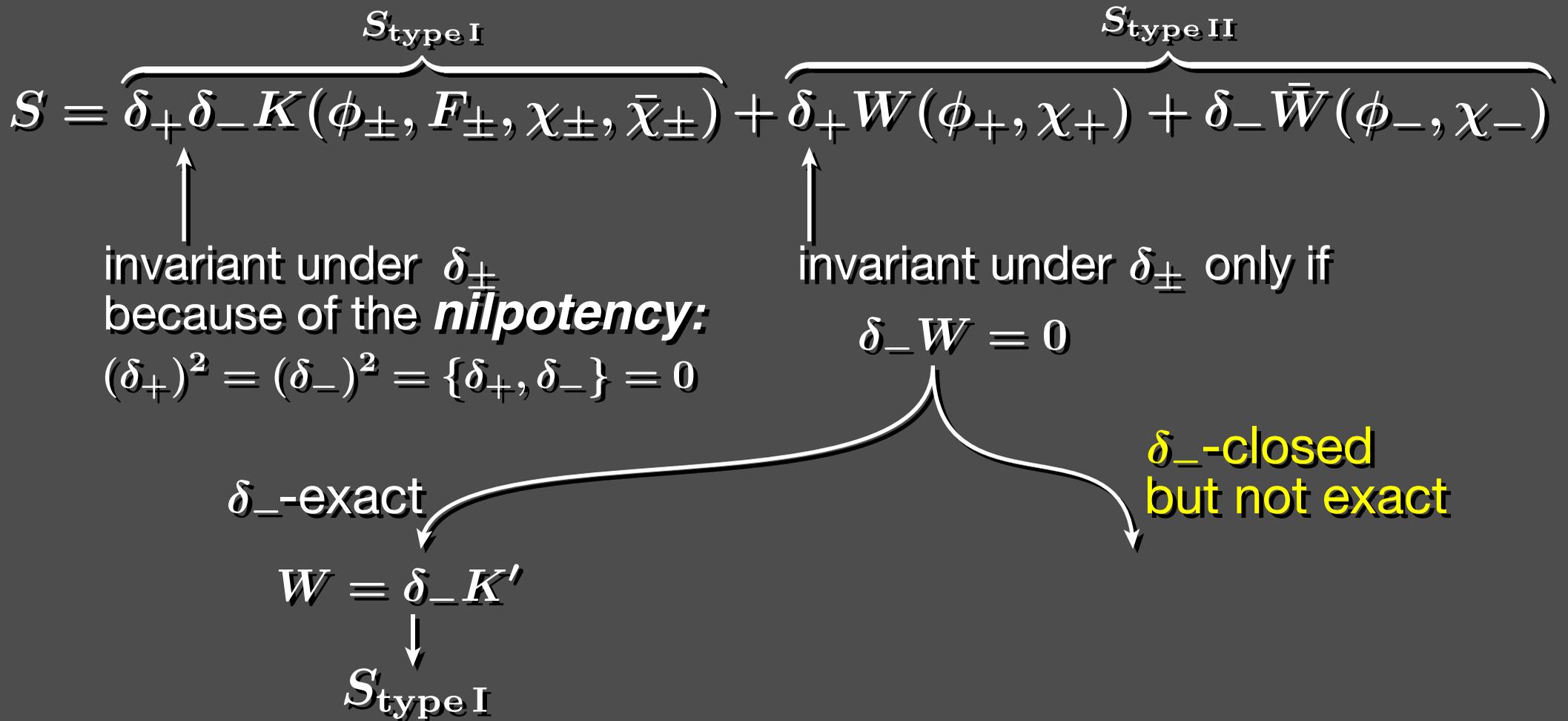
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$$W = \delta_- K'$$

$S_{\text{type I}}$

$\delta_-$ -closed  
but not exact

$$W \sim (\chi_+, \phi_+ * \phi_+ * \dots * \phi_+)$$

which has the properties:

$$W \neq \delta_- K'$$

$$\delta_- W \sim (\nabla \phi_+, \phi_+ * \phi_+ * \dots * \phi_+) \stackrel{CLR}{=} 0$$

M.Kato, M.S., H.So in preparation

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The type II terms are **cohomologically non-trivial!**

- ❑ We have proved the ***No-Go theorem*** that the Leibniz rule cannot be realized on lattice under reasonable assumptions.
- ❑ We proposed a lattice SUSY model equipped with the ***cyclic Leibniz rule*** as a modified Leibniz rule.
- ❑ A striking feature of our lattice SUSY model is that the ***non-renormalization theorem*** holds for a finite lattice spacing.
- ❑ Our results suggest that ***the cyclic Leibniz rule grasps important properties of SUSY.***

- Extension to higher dimensions

*We have to extend our analysis to higher dimensions. Especially, we need to find solutions to CLR in more than one dimensions.*

- inclusion of gauge fields

- Nilpotent SUSYs with CLR  $\overset{?}{\longleftrightarrow}$  full SUSYs

*Are nilpotent SUSYs extended by CLR enough to guarantee full SUSYs ?*

# Appendix

# SUSY transformations of superfields

$$\Psi_{\pm}(\theta_{+}, \theta_{-}) \equiv \chi_{\pm} + \theta_{\pm} F_{\pm} + \theta_{\mp} i \nabla \phi_{\pm} + \theta_{\pm} \theta_{\mp} i \nabla \bar{\chi}_{\pm}$$

$$\Lambda_{\pm}(\theta_{\pm}) \equiv \phi_{\pm} + \theta_{\pm} \chi_{\pm}$$

transform under SUSY transformations  $\delta_{\pm}$  as

$$\delta_{\pm} \mathcal{O}(\theta_{\pm}) = \frac{\partial}{\partial \theta_{\pm}} \mathcal{O}(\theta_{\pm})$$

Two Nicolai maps:

$$\xi_{\pm} \equiv \nabla \phi_{\mp} \pm \phi_{\pm} * \phi_{\pm}$$

$$\bar{\xi}_{\pm} \equiv \nabla \phi_{\pm} \pm \phi_{\mp} * \phi_{\mp}$$

Action:  $S = S_B + S_F$

$$S_B = (\bar{\xi}_{+}, \xi_{+}) = (\bar{\xi}_{-}, \xi_{-})$$

$$(\nabla \phi_{\pm}, \phi_{\pm} * \phi_{\pm}) = 0$$

↑  
CLR

# Proof of No-Go Theorem

difference operator:  $(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$

field product:  $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm}\phi_l\psi_m$

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## i) translation invariance

$$\nabla_{nm} \equiv \nabla(n - m)$$

$$M_{nlm} \equiv M(l - n, m - n)$$

difference operator:  $(\nabla \phi)_n \equiv \sum_m \nabla_{nm} \phi_m$

field product:  $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm} \phi_l \psi_m$

## ii) locality

$$\nabla(m) \xrightarrow{|m| \rightarrow \infty} 0 \text{ (exponentially)}$$

$$M(l, m) \xrightarrow{|l|, |m| \rightarrow \infty} 0 \text{ (exponentially)}$$

## holomorphic representation

$$\tilde{\nabla}(z) \equiv \sum_m \nabla(m) z^m \quad \text{on } 1 - \varepsilon < |z|, |w| < 1 + \varepsilon$$

$$\tilde{M}(z, w) \equiv \sum_{lm} M(l, m) z^l w^m$$

$\tilde{\nabla}(z), \tilde{M}(z, w)$  have to be holomorphic on  $1 - \varepsilon < |z|, |w| < 1 + \varepsilon$

difference operator:  $(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$

field product:  $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm}\phi_l\psi_m$

### iii) Leibniz rule

$$\nabla(\phi * \psi) = (\nabla\phi) * \psi + \phi * (\nabla\psi)$$

$$\implies M(z, w) (\nabla(zw) - \nabla(z) - \nabla(w)) = 0$$

$$\implies \nabla(zw) - \nabla(z) - \nabla(w) = 0$$

$$\implies \nabla(z) \propto \log z$$

$$\implies \log z \text{ is non-holomorphic on } 1 - \epsilon < |z| < 1 + \epsilon.$$

$$\implies \text{The Leibniz rule cannot be realized on lattice!}$$