Holographic Holes in Higher Dimensions

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1. Introduction

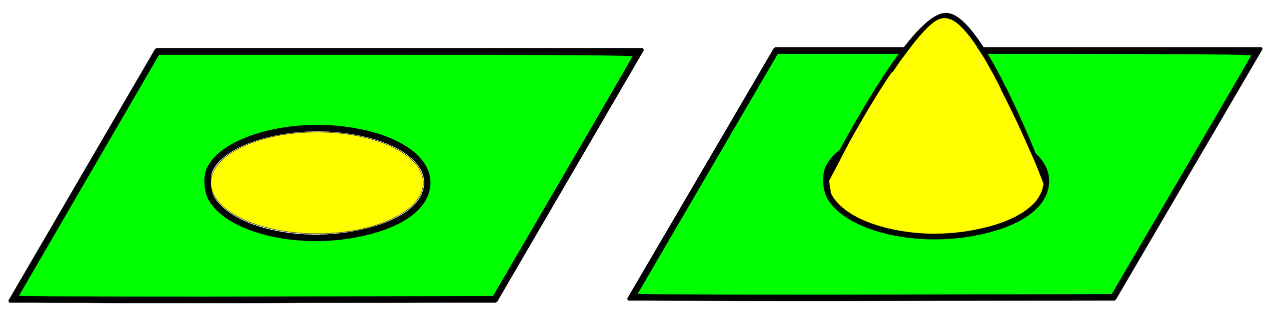
➢ **Bekenstein-Hawking entropy**

Entropy of a black hole is expressed by its horizon area. $S = \dfrac{A}{4\,G_N}$

➢ **Holographic entanglement entropy** Ryu & Takayanagi (2006)

In the context of the AdS/CFT correspondence, it is conjectured that

EE in the bdry theory	=	the BH entropy for the extremal surface in the bulk



◆ We generalize this correspondence to more general surfaces in the bulk.

- motivation We want to reconstruct geometrical quantities in the bulk from the field theoretical quantities.

Recently, it was shown that the area of a general hole in AdS_3 can be obtained by a combination of EE of 2-dim CFT using HEE formula. Balasubramanian, Chowdhury, Czech, de Boer & Heller (2013)

We extend this construction to higher dimensional background.

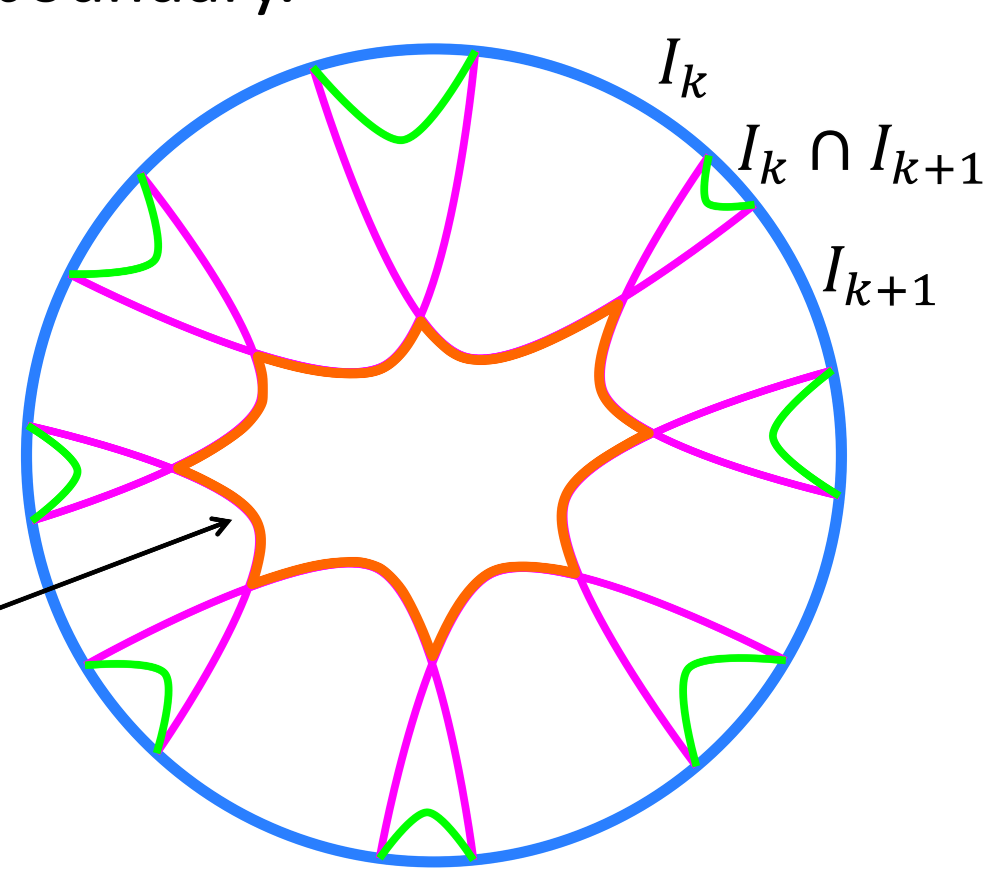
2. Holes in AdS_3

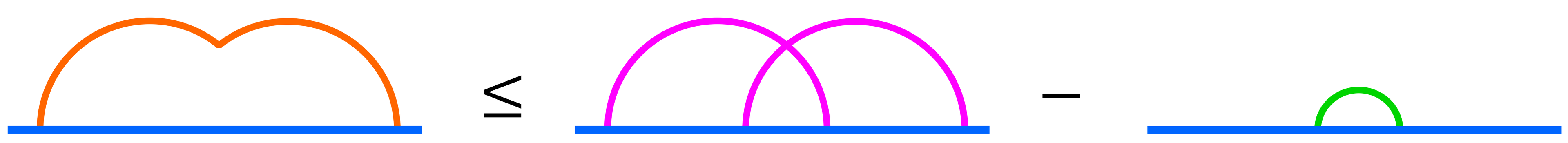
◆ Consider the overlapping intervals I_k which cover a time slice in the boundary.

◆ Define the outer envelope as the bdry of the union of the bulk regions enclosed by the minimal curves determining EE, $S(I_k)$.

$$\Rightarrow \quad \boxed{\hat{S}(\{I_k\}) \leq \sum_k [S(I_k) - S(I_k \cap I_{k+1})]}$$

(length of outer envelope)/$4G_N$





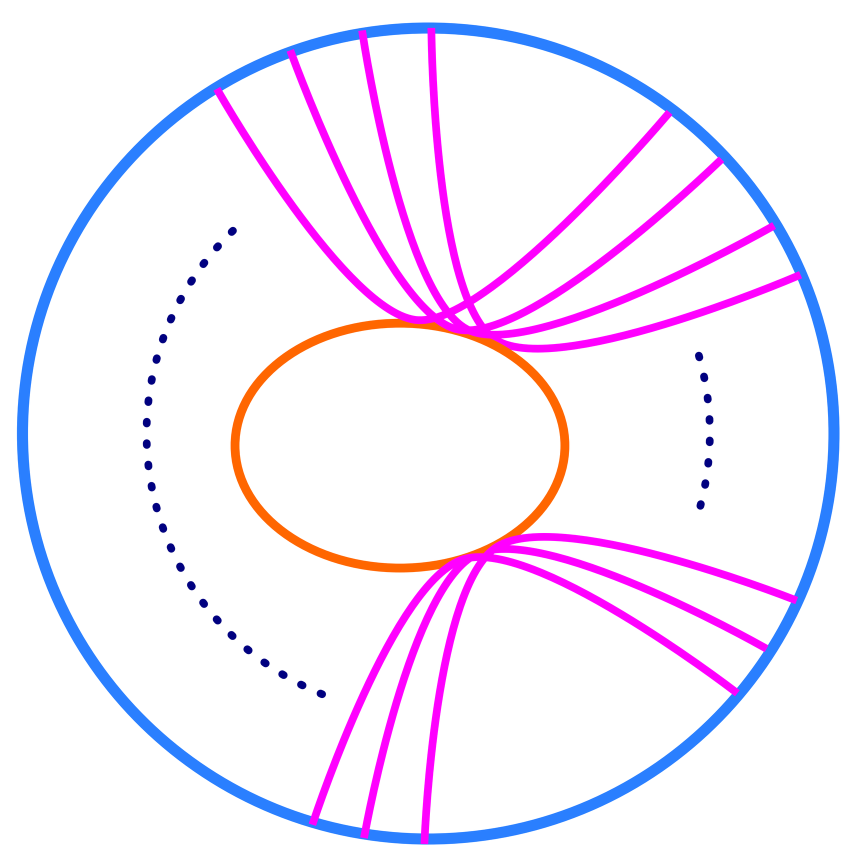
- Note that this inequality is stronger than strong subadditivity: $S(I_1 \cup I_2) \leq \hat{S}(I_1, I_2) \leq S(I_1) + S(I_2) - S(I_1 \cap I_2)$

◆ Take a "continuum limit", i.e. take the number of intervals to infinity.

Then, the outer envelope becomes a smooth curve.
Furthermore, the above inequality is saturated.

$$\boxed{\frac{(\text{length})}{4\,G_N} = \sum_k^{\infty} [S(I_k) - S(I_k \cap I_{k+1})]}$$

We call this combination of EE the "differential entropy".



3. Holes in more general backgrounds

◆ Consider the following metric in (d+1)-dim background

$$ds^2 = -g_0(z)dt^2 + \sum_{i=1}^{d-1} g_i(z)\left(dx^i\right)^2 + g_1(z)f(z)dz^2$$

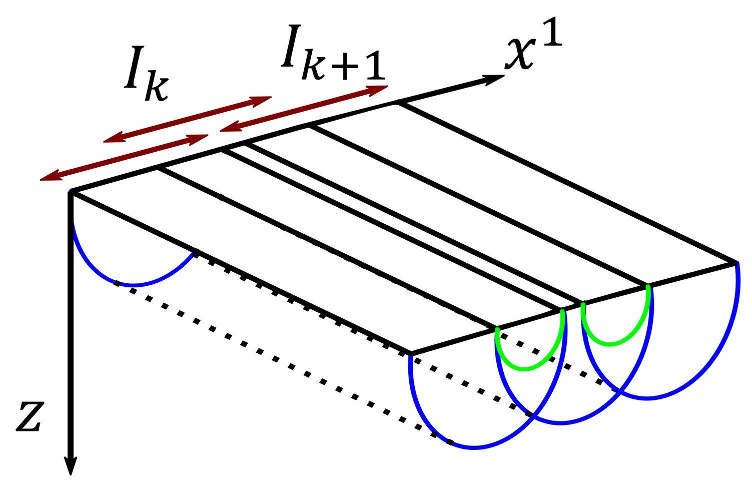
- boundary: $z = 0$
- x^i-directions are periodic with periods ℓ_i

- e.g. planar AdS black hole $g_0(z) = \frac{L^2}{z^2}\left(1 - \frac{z^d}{z_h^d}\right), g_i(z) = \frac{L^2}{z^2}, f(z) = \left(1 - \frac{z^d}{z_h^d}\right)^{-1}$

◆ We show that the Bekenstein-Hawking entropy of bulk surfaces which have translational symmetry in x^j-directions (j $= 2, \cdots, d-1$) can be evaluated by the differential entropy.

- bulk surface: $z(x^1)$
- assume that there is the extremal surface which is tangent to the given profile $z(x^1)$ at each point x^1.

⇒ a time slice in the boundary is partitioned by a set of strips as the following fig.

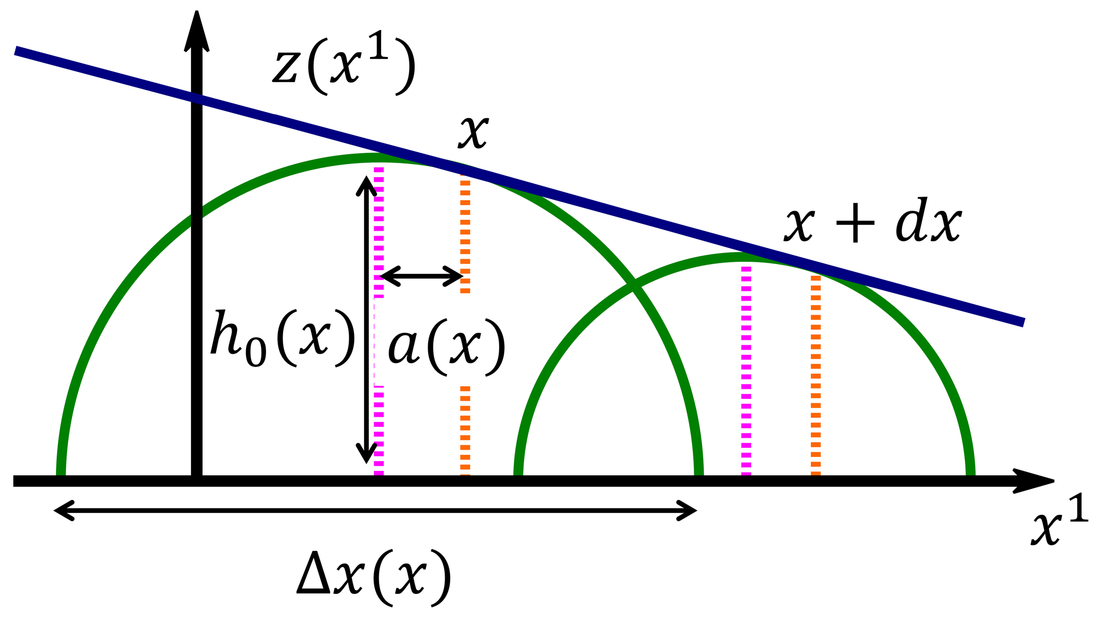


□ the BH entropy of the given surface

$$\frac{A}{4\,G_N} = \frac{\ell_2 \cdots \ell_{d-1}}{4\,G_N}\int_0^{\ell_1} dx \sqrt{G(z)(1 + f(z)z'^2)}$$

$$G(z) = g_1(z) \cdots g_{d-1}(z)$$

□ the differential entropy



$$E = \sum_k^{\infty}[S(I_k) - S(I_k \cap I_{k+1})] = \int_0^{\ell_1} dx \frac{dS}{d\Delta x}(1 + a')$$

$$\frac{dS}{d\Delta x} = \frac{\ell_2 \cdots \ell_{d-1}}{4\,G_N} G(h_0)$$

We can show that the difference of the integrands is a total derivative.

⇒ $$\boxed{\frac{A}{4\,G_N} = E}$$

4. Discussion

◆ Spacetime entanglement conjecture

Bianchi & Myers (2012)

- The leading contribution of EE in a theory of quantum gravity is given by the Bekenstein-Hawking formula.

◆ Interpretation of the differential entropy

- This might be related to entanglement between UV and IR degrees of freedom in the boundary theory.

5. Summary

◆ The Bekenstein-Hawking entropy of codimension-two surfaces with planar sym. in the bulk can be written in terms of the differential entropy in the boundary theory.

- See also Czech, Dong & Sully (arXiv:1406.4889)

◆ We can extend this construction to Lovelock gravity.

◆ Our results provide the relation between geometry and entanglement.