5D SCFTs, Enhanced Symmetry & Nekrasov Partition Functions

Masato Taki RIKEN, iTHES group

2014.7/22
@ YITPws

references

[MT, arXiv:1401.7200]
[MT, arXiv:1310.7509]
[L.Bao-V.Mitev-E.Pomoni-MT-F.Yagi, arXiv:1310.3841]
[Hayashi-Kim-Nishinaka, arXiv:1310.3854]
[Bergman-Gomez-Zafrir, arXiv:1310.2150]
[Iqbal-Vafa, arXiv:1210.3605]
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etc

originated in an epoch-making paper ([Seiberg, 1996])

hep-th/9608111
RU-96-69

Five Dimensional SUSY Field Theories, Non-trivial Fixed Points and String Dynamics

Nathan Seiberg

Department of Physics and Astronomy
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We study (non-renormalizable) five dimensional supersymmetric field theories. The theories are parametrized by quark masses and a gauge coupling. We derive the metric on the Coulomb branch exactly. We use stringy considerations to learn about new non-trivial interacting field theories with exceptional global symmetry E_n (E_8, E_7, E_6, $E_5 = Spin(10)$, $E_4 = SU(5)$, $E_3 = SU(3) \times SU(2)$, $E_2 = SU(2) \times U(1)$ and $E_1 = SU(2)$). Their Coulomb branch is $\mathbf{R}^+$ and their Higgs branch is isomorphic to the moduli space of E_n instantons. One of the relevant operators of these theories leads to a flow to $SU(2)$ gauge theories with $N_f = n - 1$ flavors. In terms of these $SU(2)$ IR theories this relevant parameter is the inverse gauge coupling constant. Other relevant operators (which become quark masses after flowing to the $SU(2)$ theories) lead to flows between them. Upon further compactifications to four and three dimensions we find new fixed points with exceptional symmetries.

August 1996

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1. **Five-dimensional SUSY field theories, nontrivial fixed points and string dynamics**
Nathan Seiberg (Rutgers U., Piscataway). Aug 1996. 13 pp.
Published in **Phys.Lett. B388 (1996) 753-760**
RU-96-69
DOI: 10.1016/S0370-2693(96)01215-4
e-Print: **hep-th/9608111** | **PDF**
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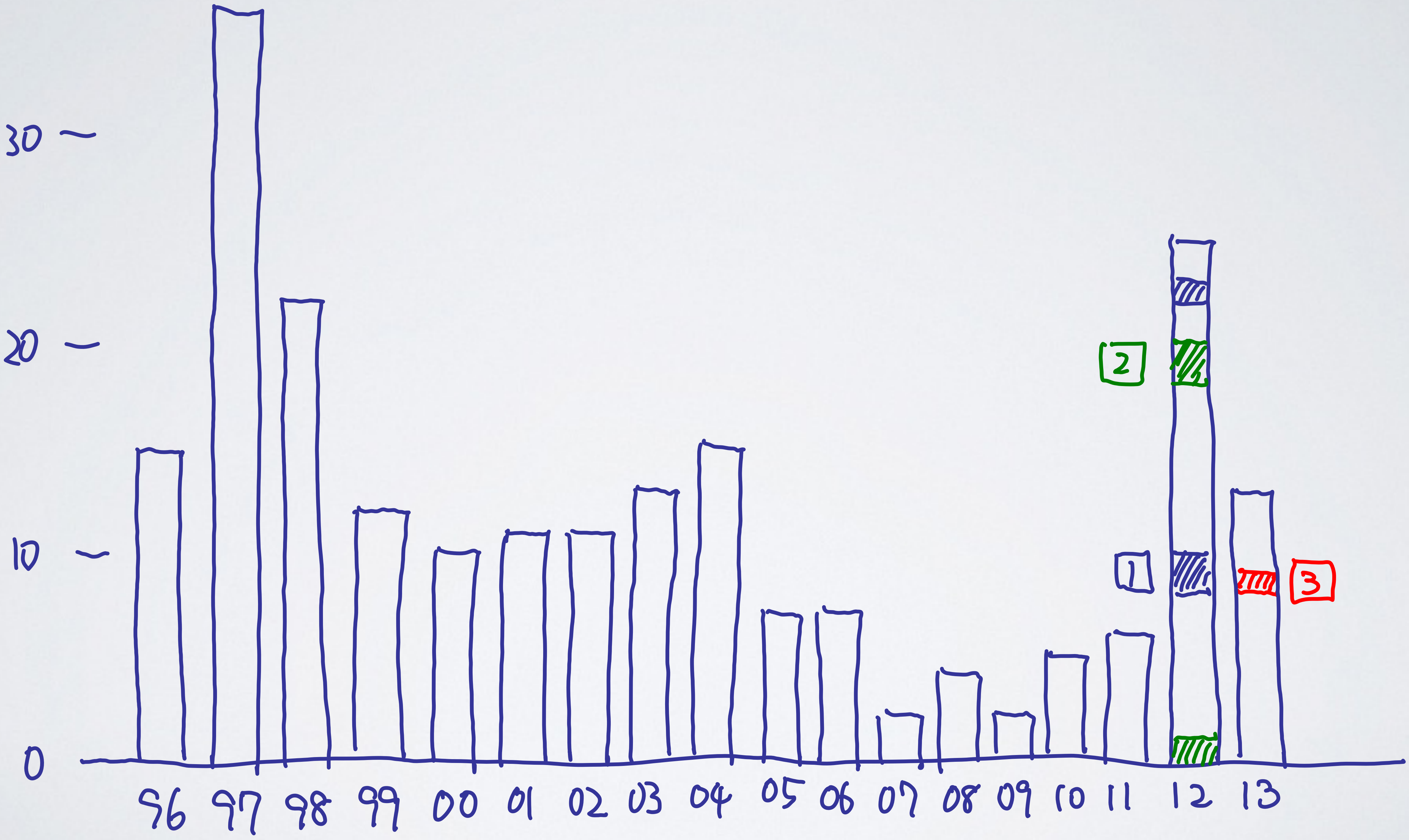
Seiberg, hep-th/9608111 の citation



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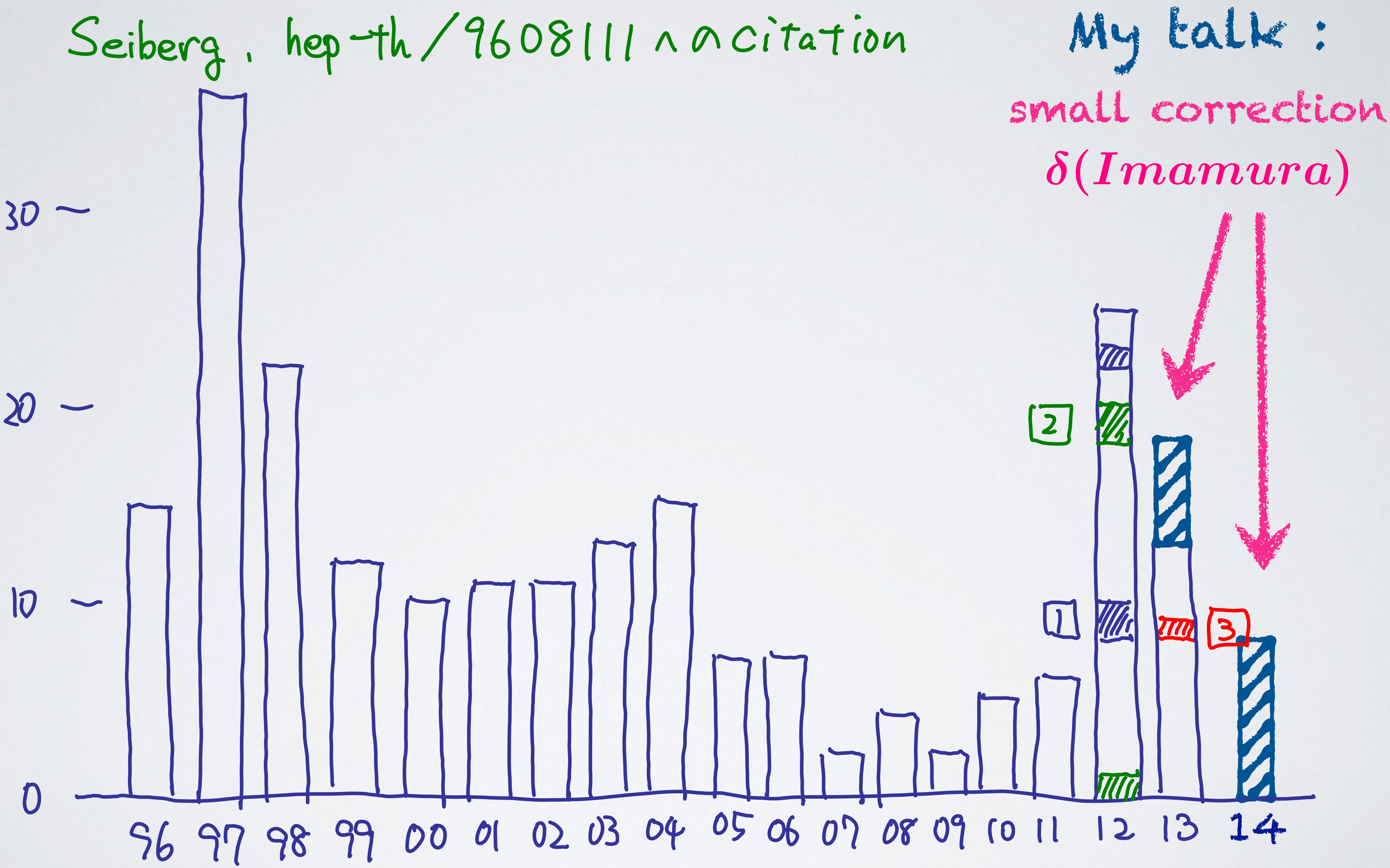

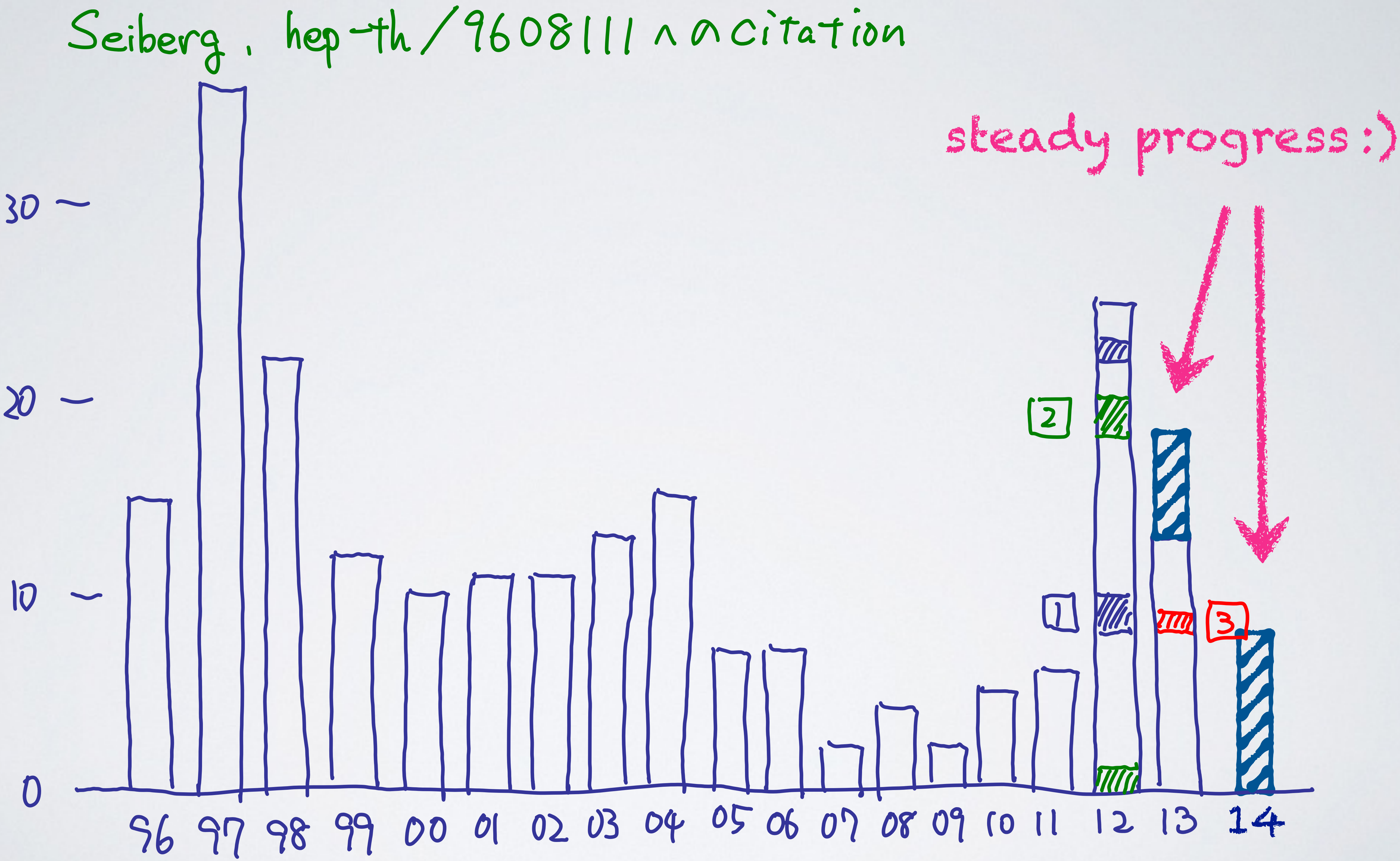
Seiberg, hep-th/9608111 ∧∩ citation
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Question:

Why 5d gauge theory?

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$$\frac{1}{g^2}\int d^d x\, F_{\mu\nu}F^{\mu\nu} \sim \frac{L^{d-4}}{g^2}$$

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$$\frac{1}{g^2}\int d^dx\, F_{\mu\nu}F^{\mu\nu} \sim \frac{L^{d-4}}{g^2}$$

$$\longrightarrow \quad [g] = [L^{\frac{d-4}{2}}]$$

Question:

Why 5d gauge theory?

 non-renormalizable & trivial

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 **only cut-off theory?**

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 only cut-off theory?

 some susy theories are well-defined via UV fixed points [Seiberg, ’96]

Question:

Why 5d gauge theory?

this strongly-coupled physics reflects string duality hiding behind

some susy theories are well-defined via UV fixed points [Seiberg, '96]

Plan

1. 5d SCFTs & Global Symmetry (1996-1998)
2. Localization & Partition Function (2002-2012)
3. Global Symmetry via SC Index (2012-2013)
4. Conjectures on Nekrasov Partition Functions (2013-2014)
5. Summary

1. 5D SCFTs
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August 1996

5d SUSY gauge theory and UV fixed point

* [Seiberg, ('96)] dealt with

  5D minimal SUSY gauge theories

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  - $\mathcal{N}$=2 in 4D.

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fundamental rep.

··· flavors

5d SUSY gauge theory and UV fixed point

[Seiberg, '96]

SU(2) gauge theory with 0, 1, 2, ... , 7 flavors has a UV fixed point.

5d SUSY gauge theory and UV fixed point

[Seiberg, '96]

UV

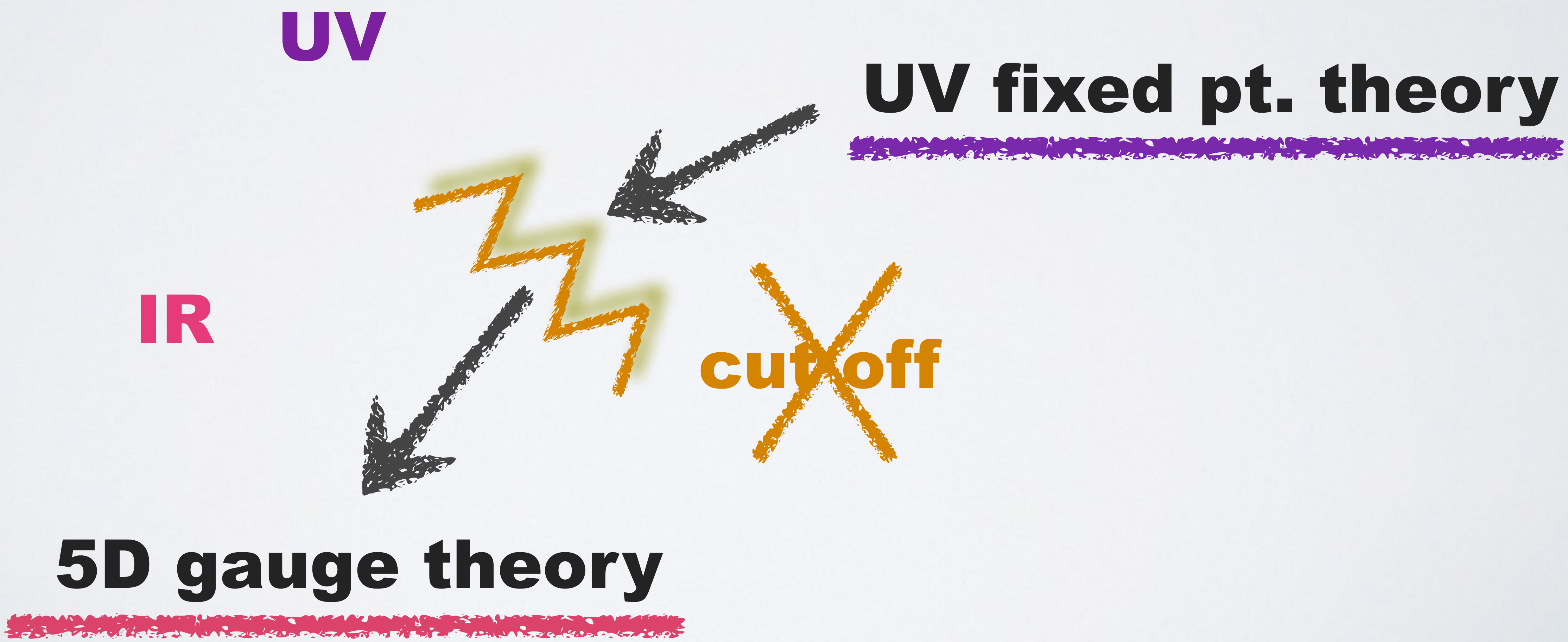
breakdown of Lagrangian

IR

cut off

5D gauge theory

5d SUSY gauge theory and UV fixed point
[Seiberg, ’96]



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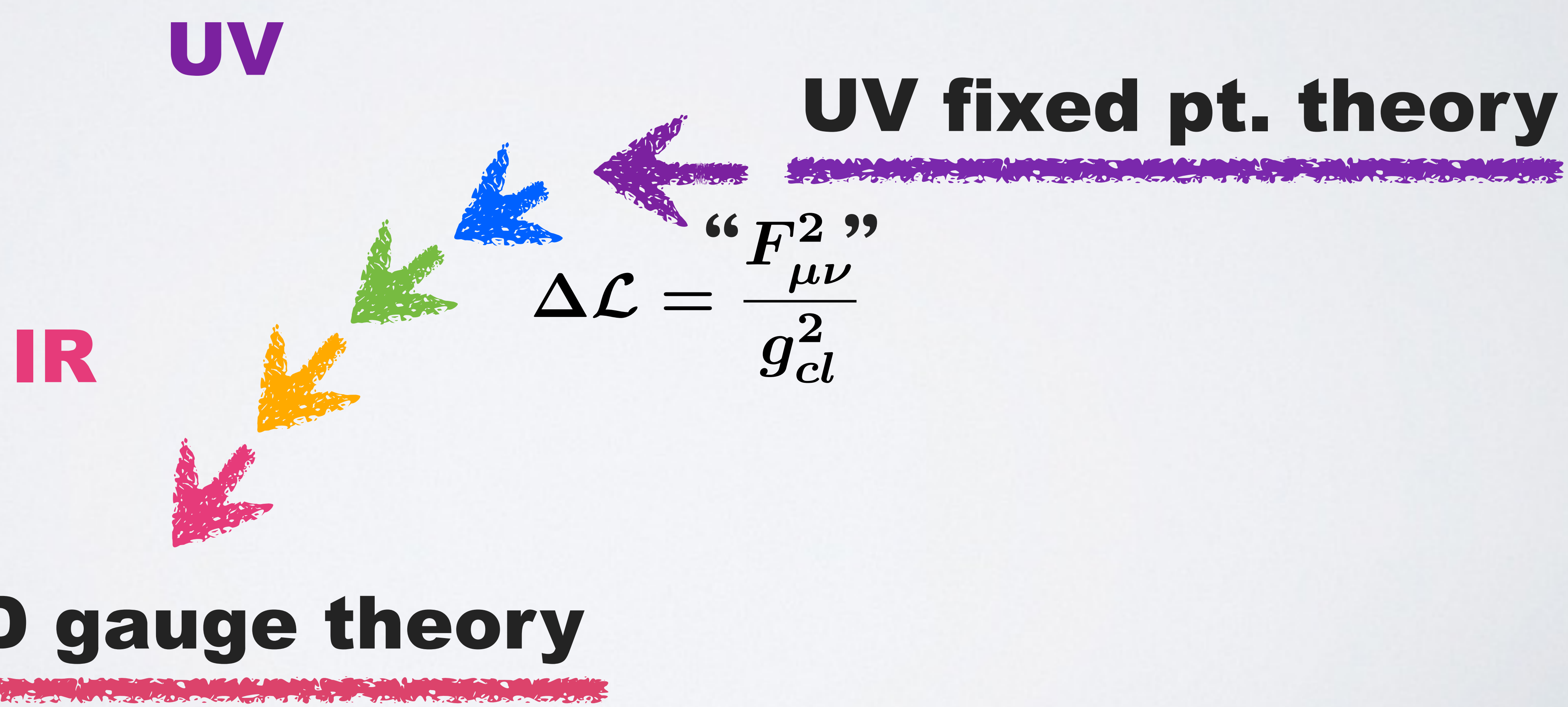
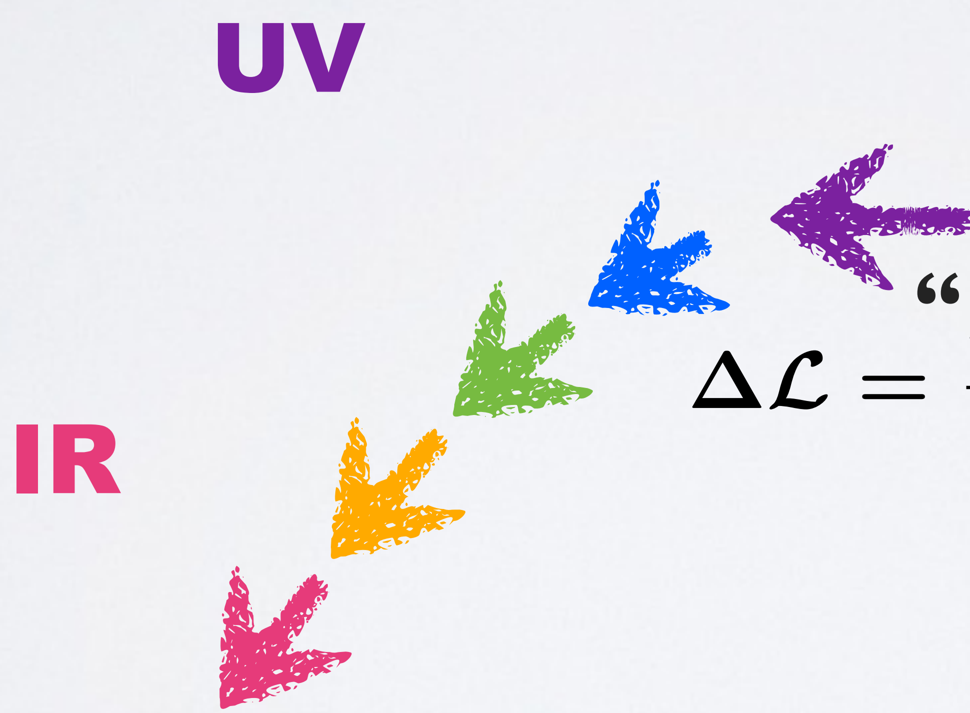


$$\Delta\mathcal{L} = \frac{“F_{\mu\nu}^2”}{g_{cl}^2}$$

5d SUSY gauge theory and UV fixed point
[Seiberg, ’96]

> **SU(2) gauge theory with 0, 1, 2, … , 7 flavors has a UV fixed point.**



UV fixed pt. theory

$$\Delta\mathcal{L} = \frac{\text{“}F_{\mu\nu}^2\text{”}}{g_{cl}^2}$$

strongly-coupled SCFT

5D gauge theory

Flavor symmetry of UV SCFT [Seiberg, '96]

flavor symmetry of IR gauge theory

$$SO(2N_f) \times U(1)$$

$q_1,\ \tilde{q}_1,\ \cdots q_{N_f},\ \tilde{q}_{N_f}$

instantons

this symmetry mixes them

Flavor symmetry of UV SCFT [Seiberg, '96]

flavor symmetry of IR gauge theory

$$SO(2N_f) \times U(1)$$

$$\uparrow$$

subgroup of the global symmetry of UV SCFT

$\leftarrow$ **deformation** $\Delta\mathcal{L} = \frac{F_{\mu\nu}^2}{g_{cl}^2}$ **and masses breaks it.**

Flavor symmetry of UV SCFT [Seiberg, '96]

$$SO(2N_f) \times U(1) \longrightarrow$$

$$N_f = 0 \qquad E_1 = SU(2)$$

$$N_f = 1 \qquad E_2 = SU(2) \times U(1)$$

$$N_f = 2 \qquad E_3 = SU(3) \times SU(2)$$

$$N_f = 3 \qquad E_4 = SU(5)$$

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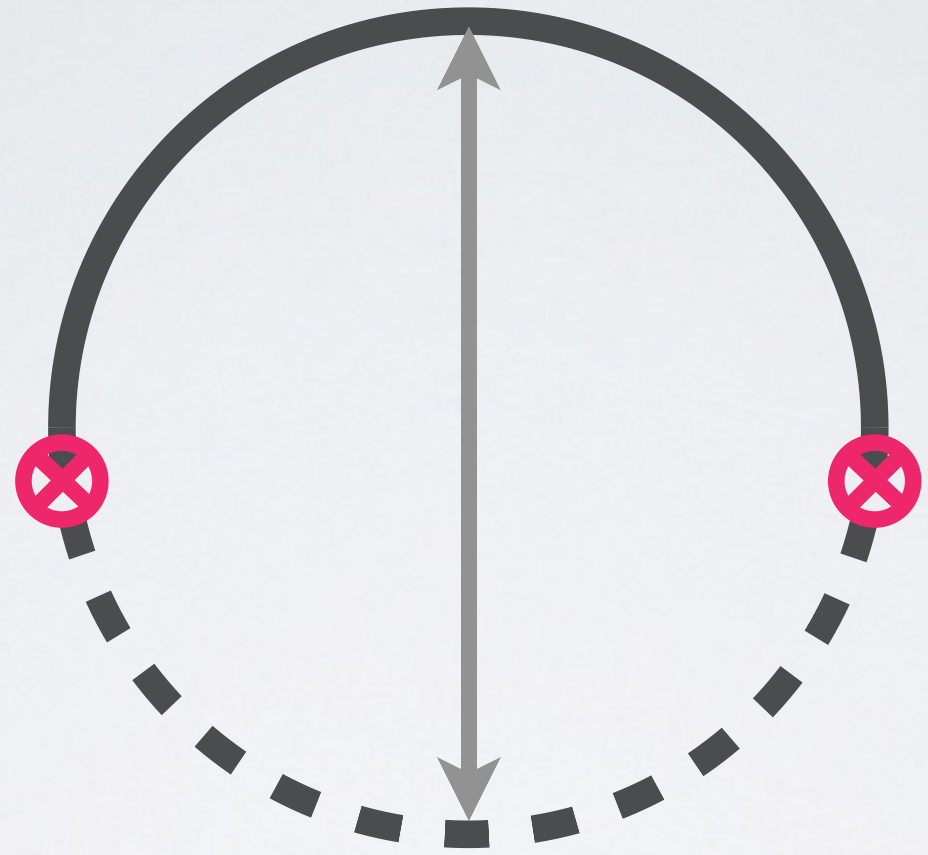
$$N_f = 5 \qquad E_6$$

$$N_f = 6 \qquad E_7$$

$$N_f = 7 \qquad E_8$$

$\mathbf{E_n}$ via TypeII on orbifold [Seiberg, '96]

(TypeI' = T-dual to TypeI)



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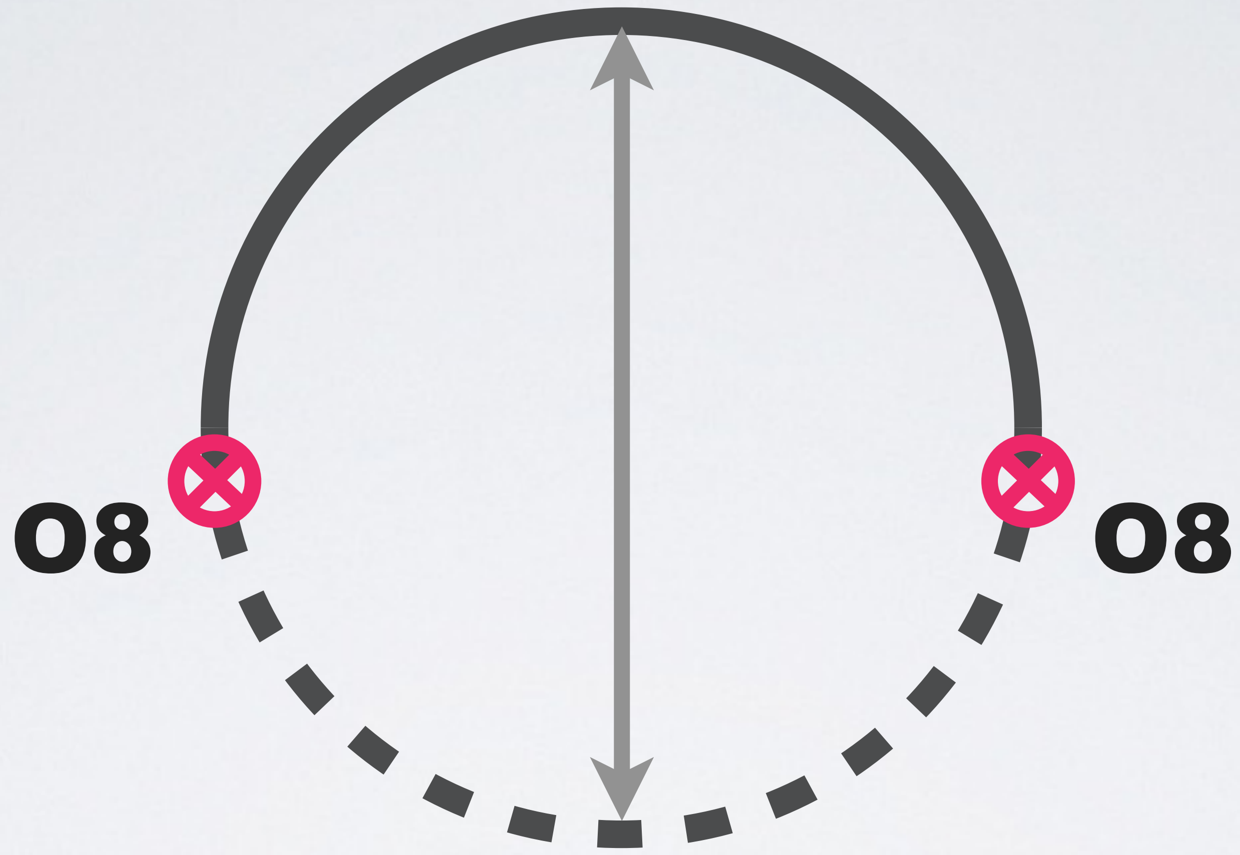

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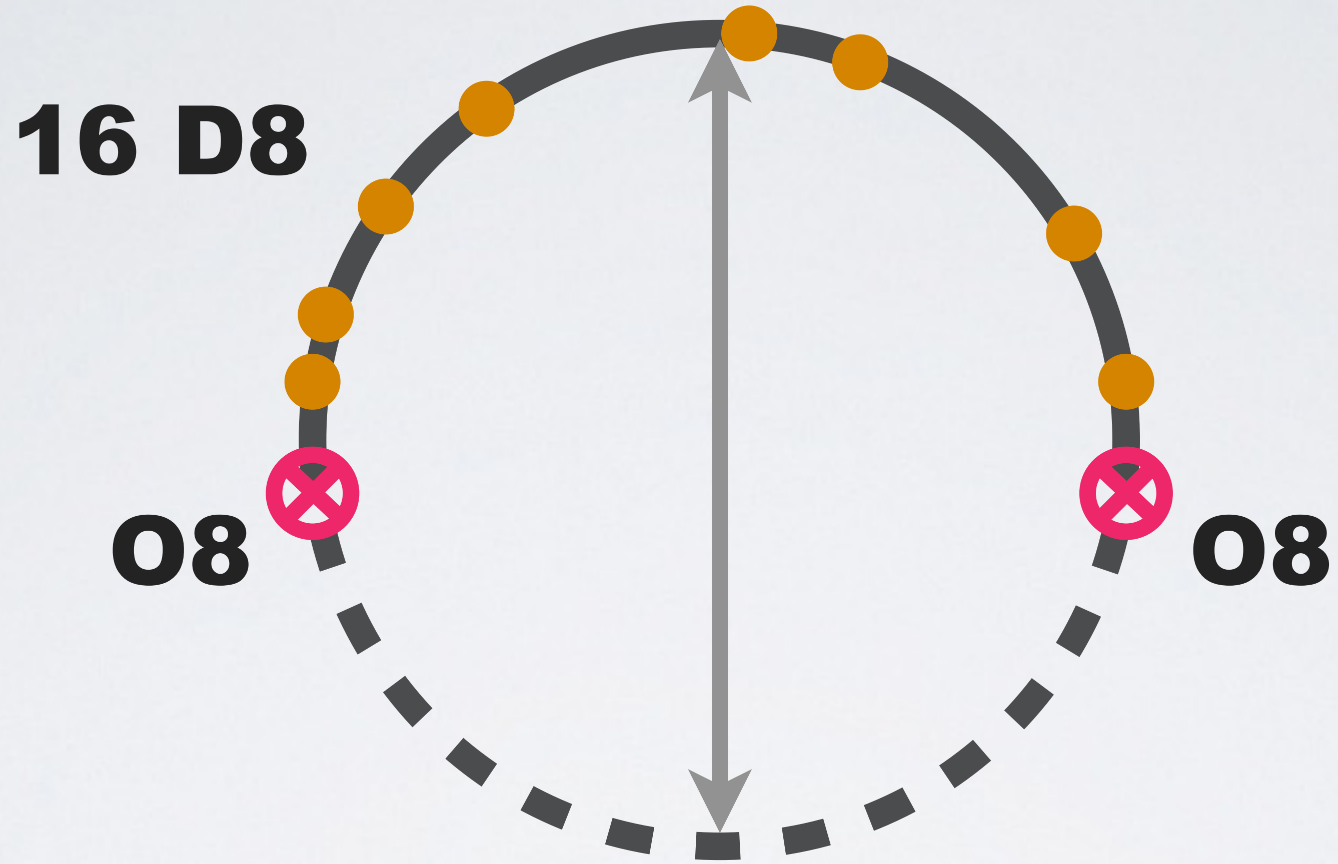

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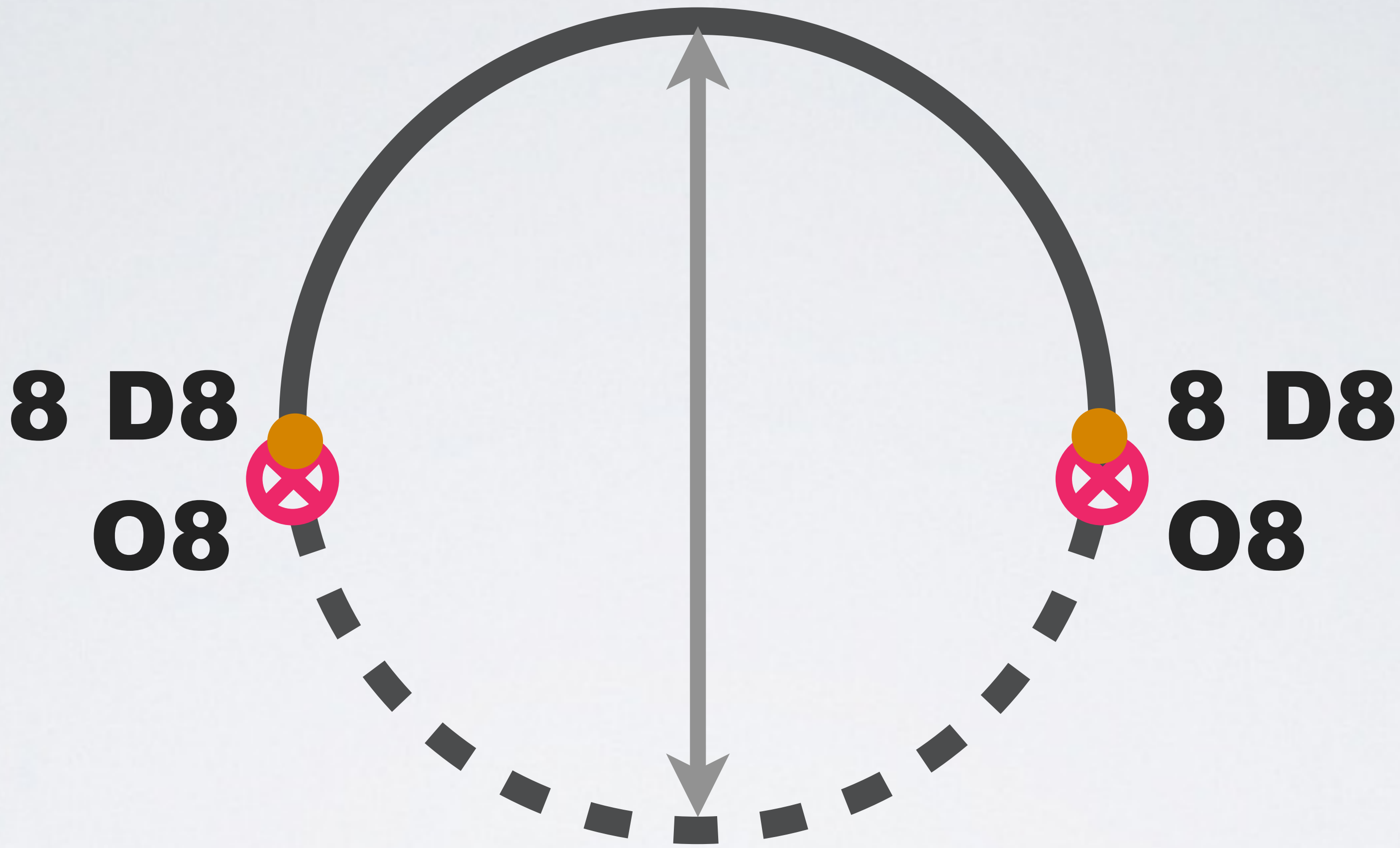

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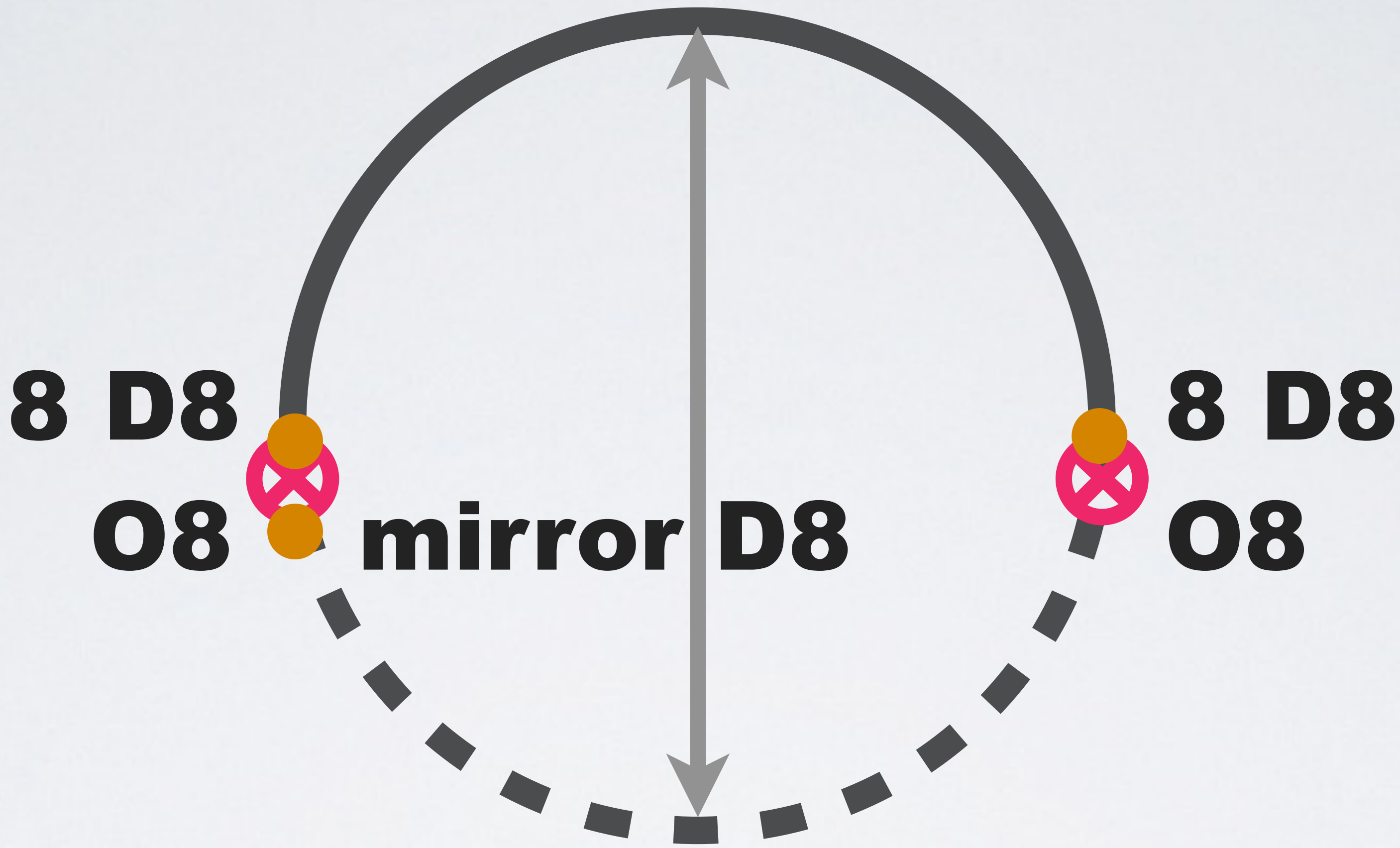

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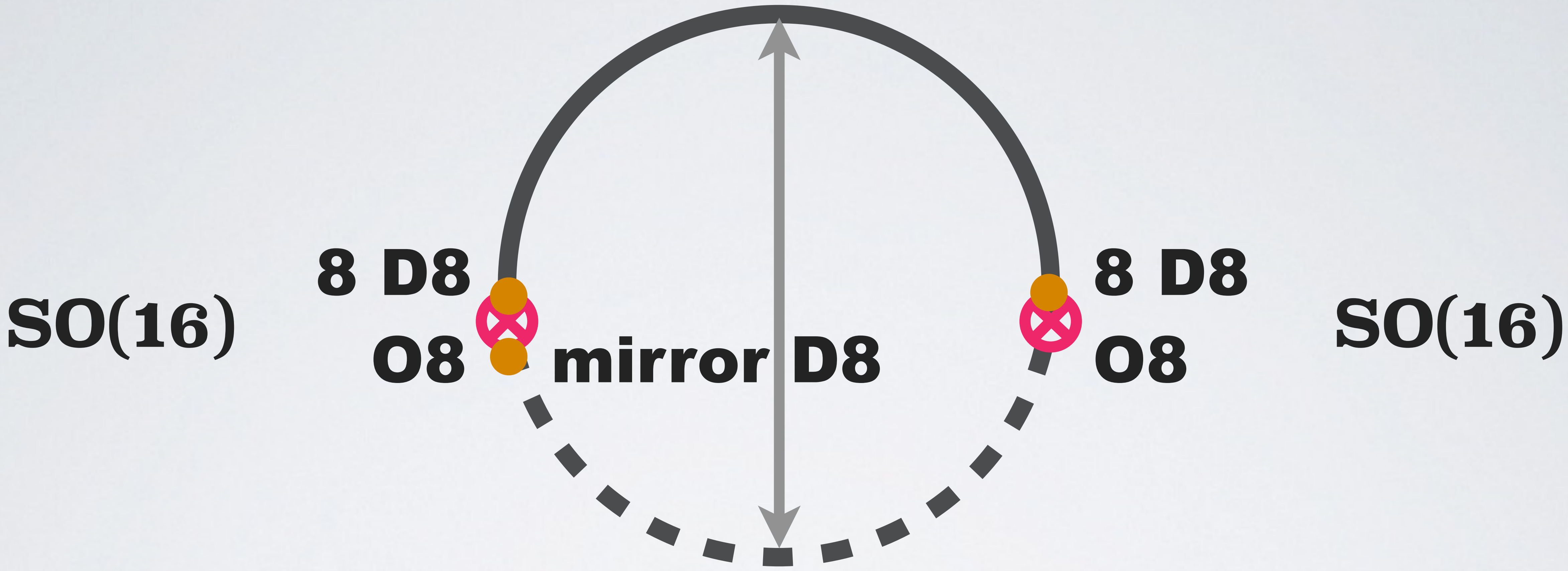
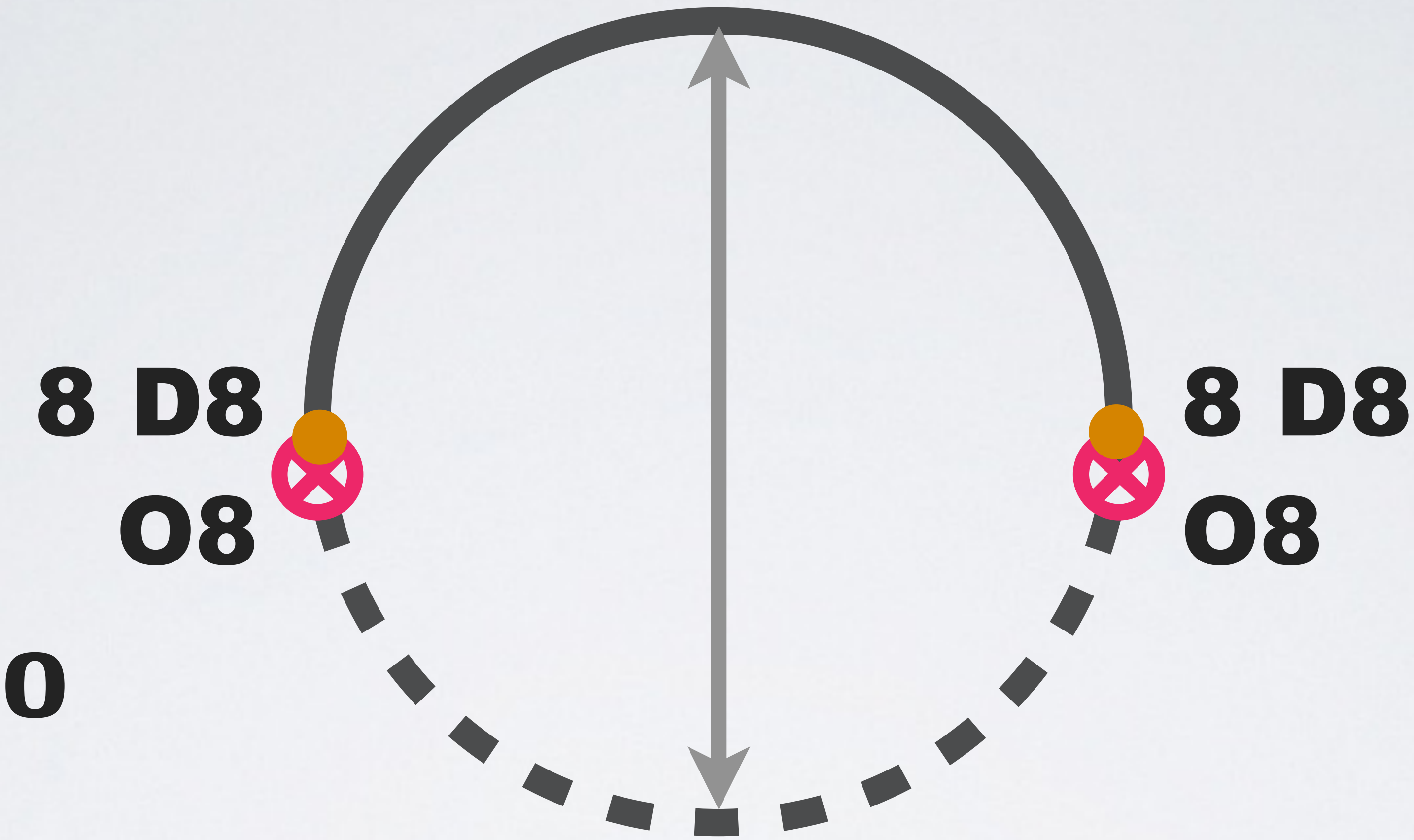


E_n via TypeII on orbifold [Seiberg, '96]

SO(16)

adj = 120



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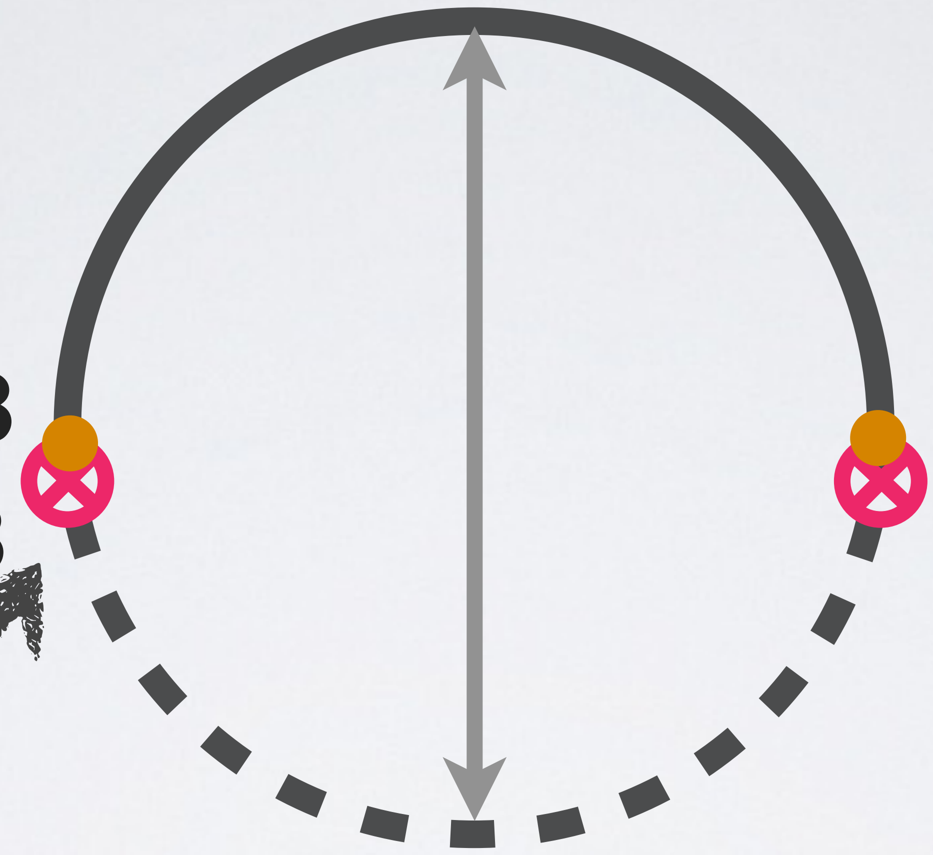

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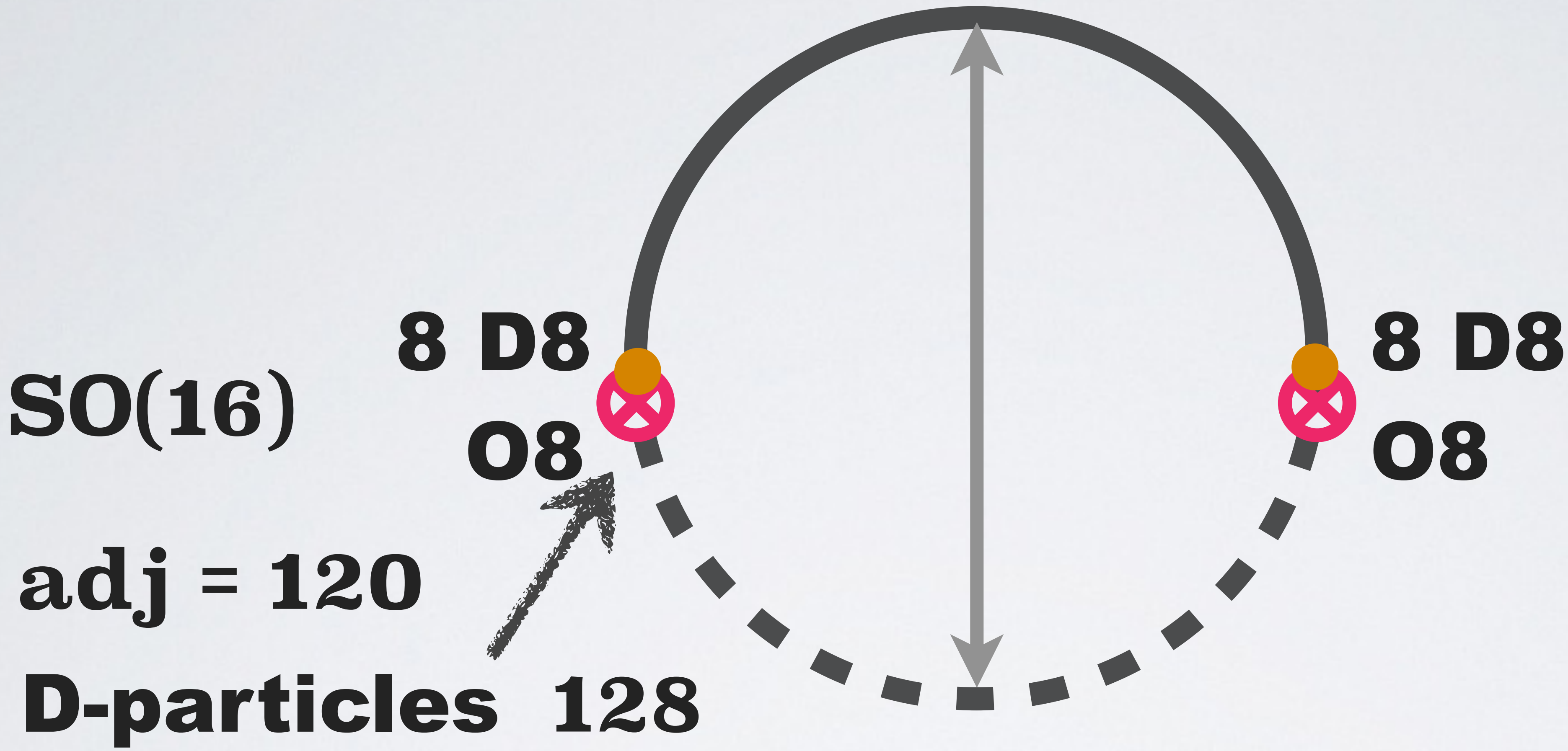

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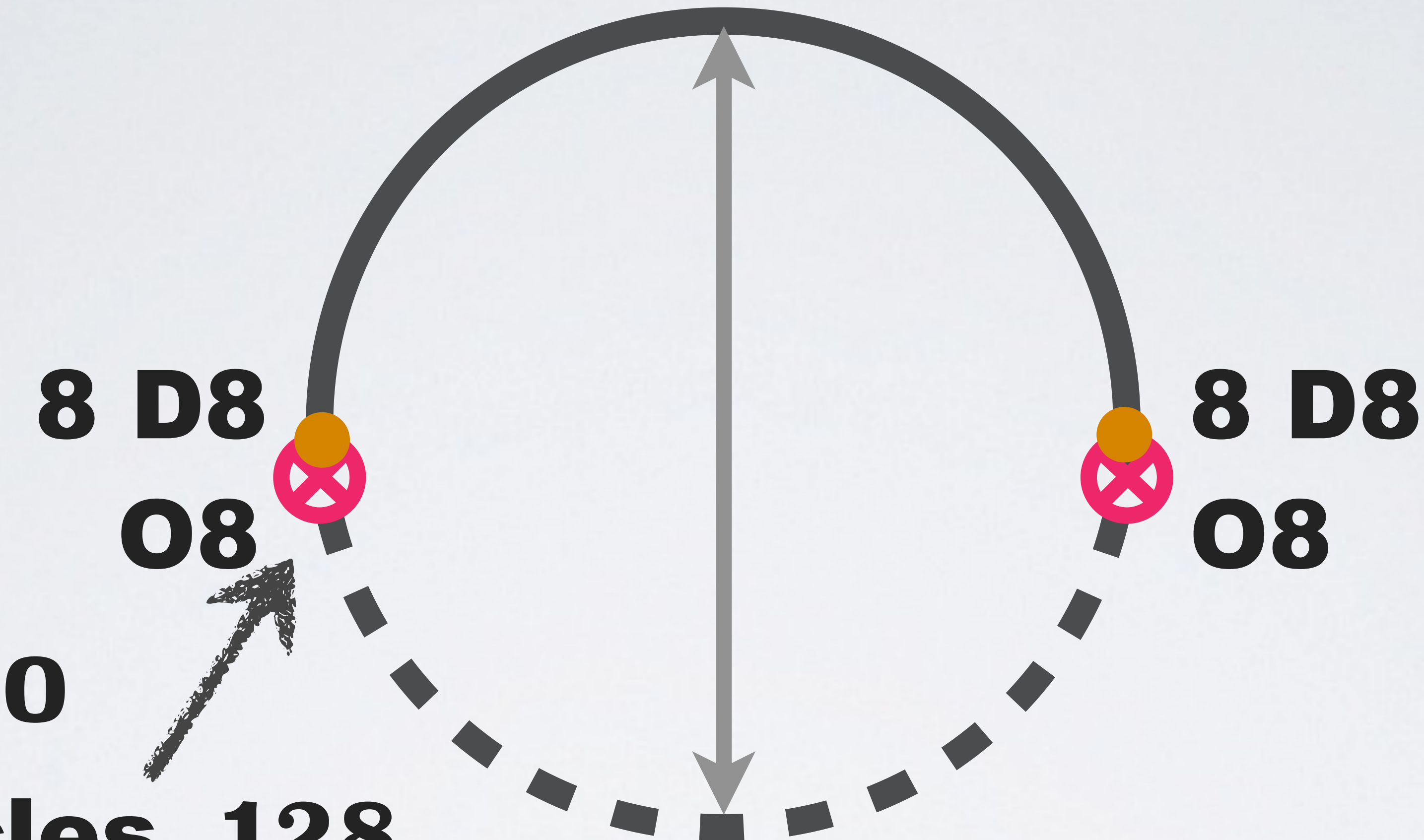
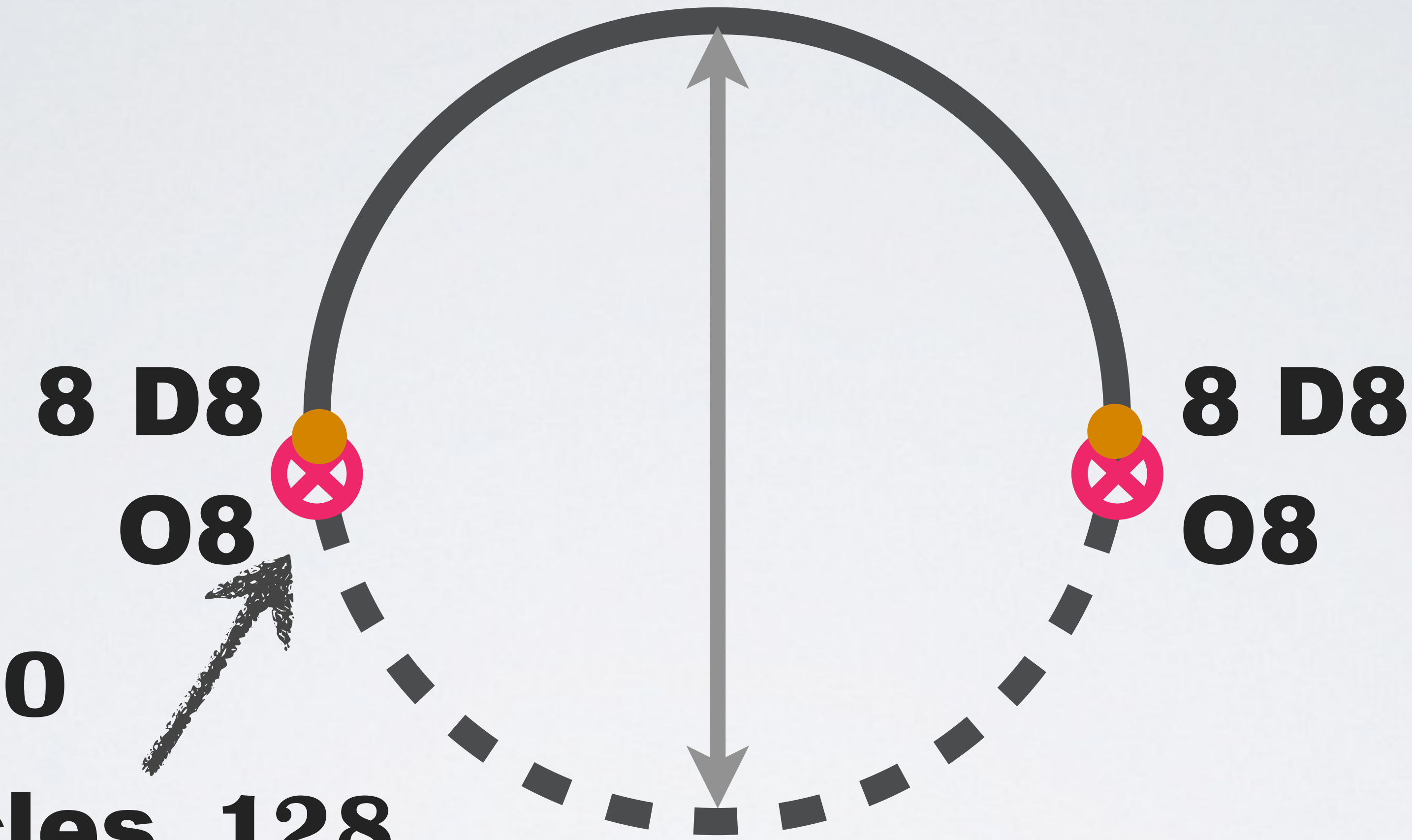


SO(16)

- adj = 120
- D-particles 128

$\rightarrow$ 248

E_n via TypeII on orbifold [Seiberg, '96]

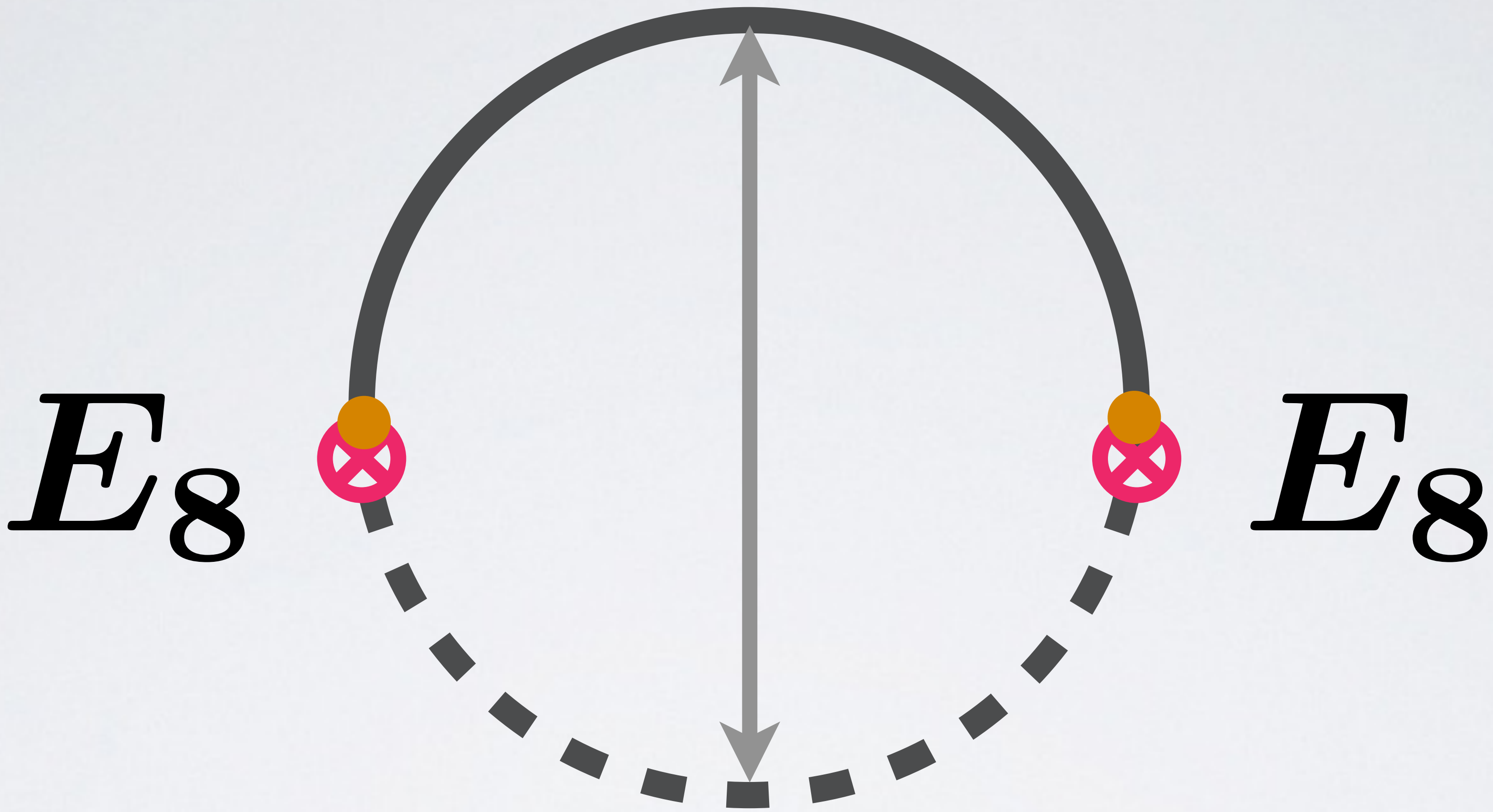


SO(16)

$\left\{ \begin{array}{l} \text{adj} = 120 \\ \text{D-particles } 128 \end{array} \right.$

$\longrightarrow$ 248 = E_8's adj !

$\mathbf{E_n}$ via TypeII on orbifold [Seiberg, '96]



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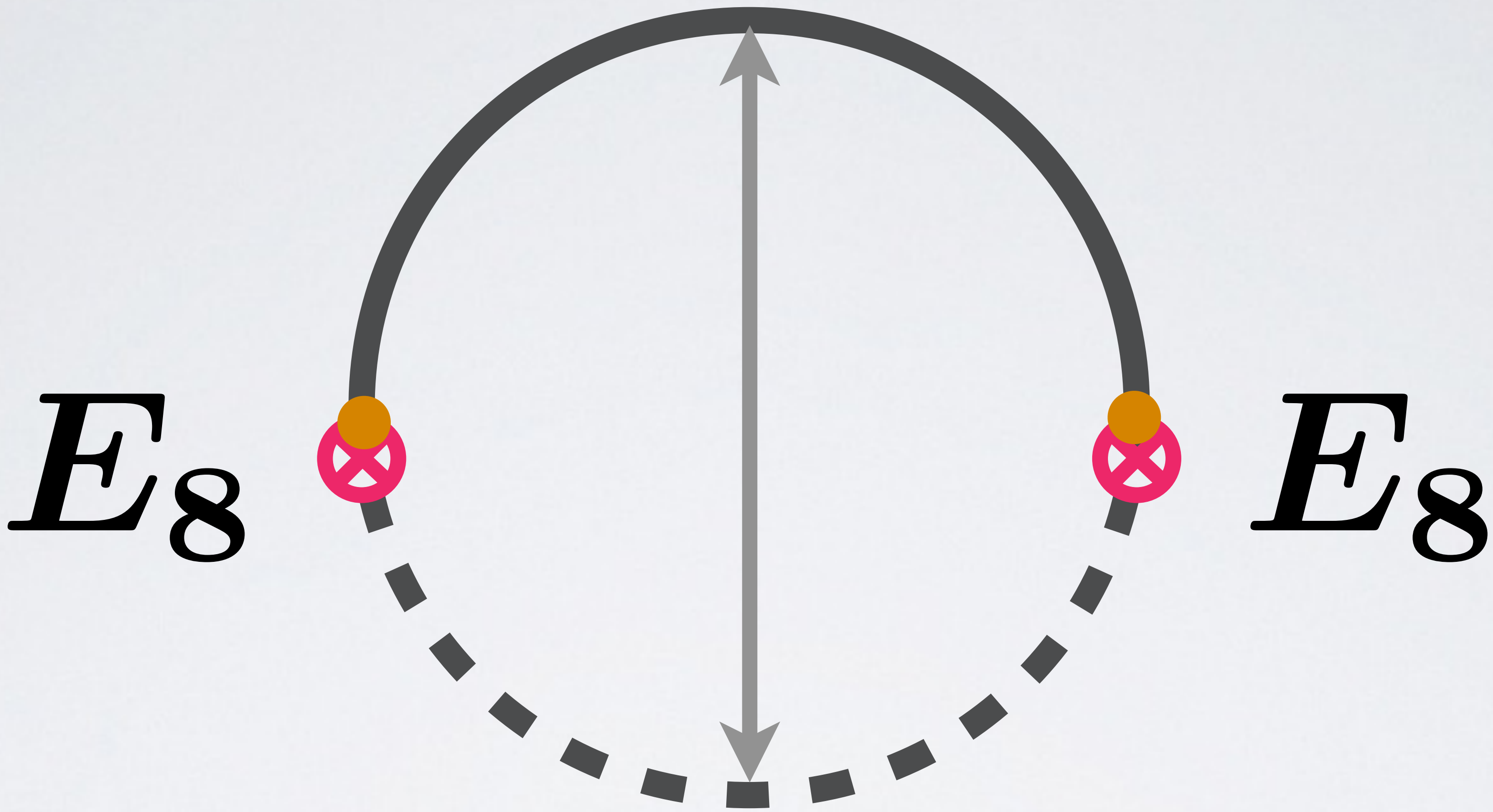
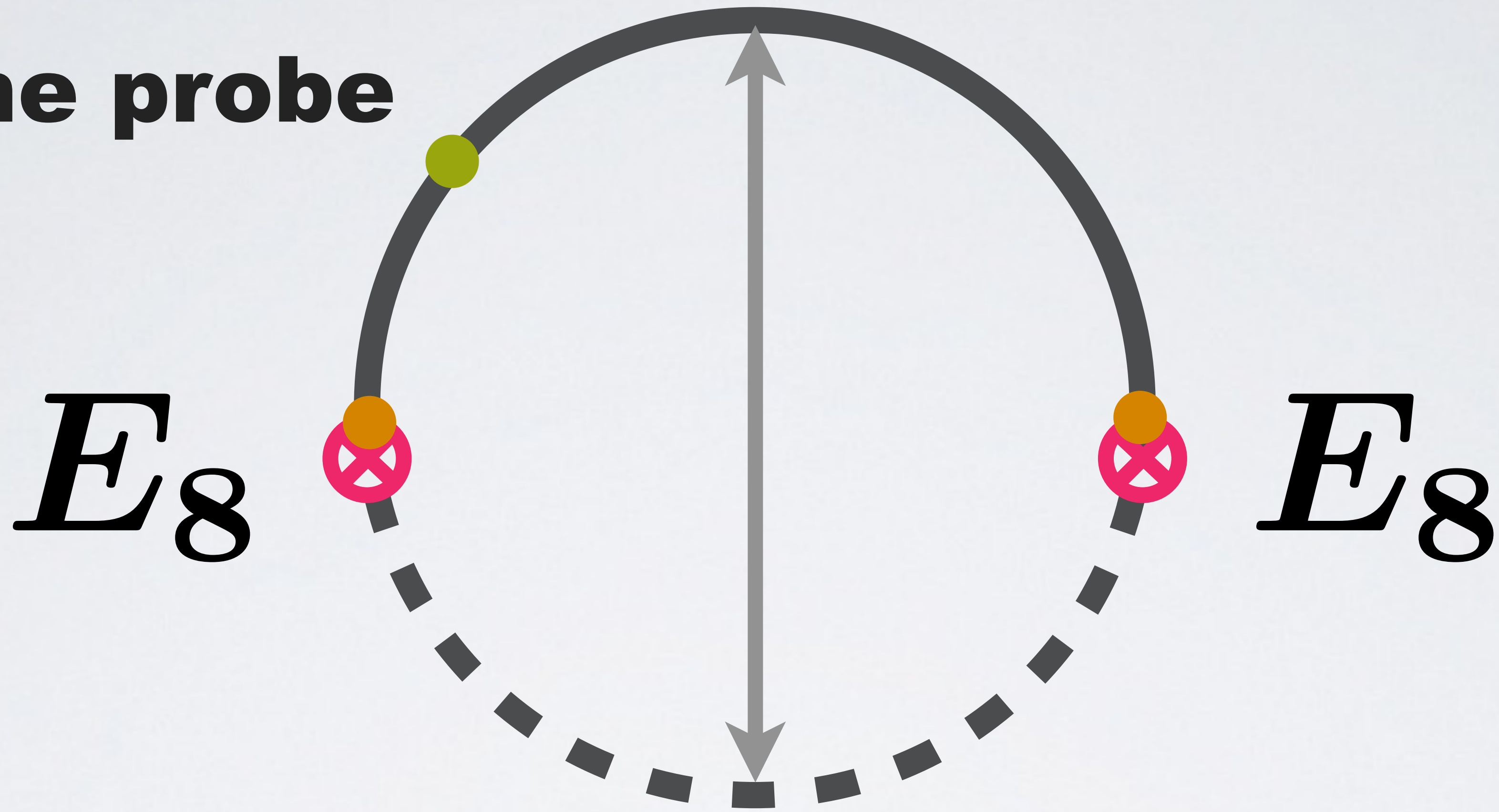


this E_8 x E_8 comes from Heterotic/Type I duality

(M-theory lift of it is strongly-coupled Heterotic string)

$\mathbf{E}_n$ via TypeII on orbifold [Seiberg, '96]

D4-brane probe



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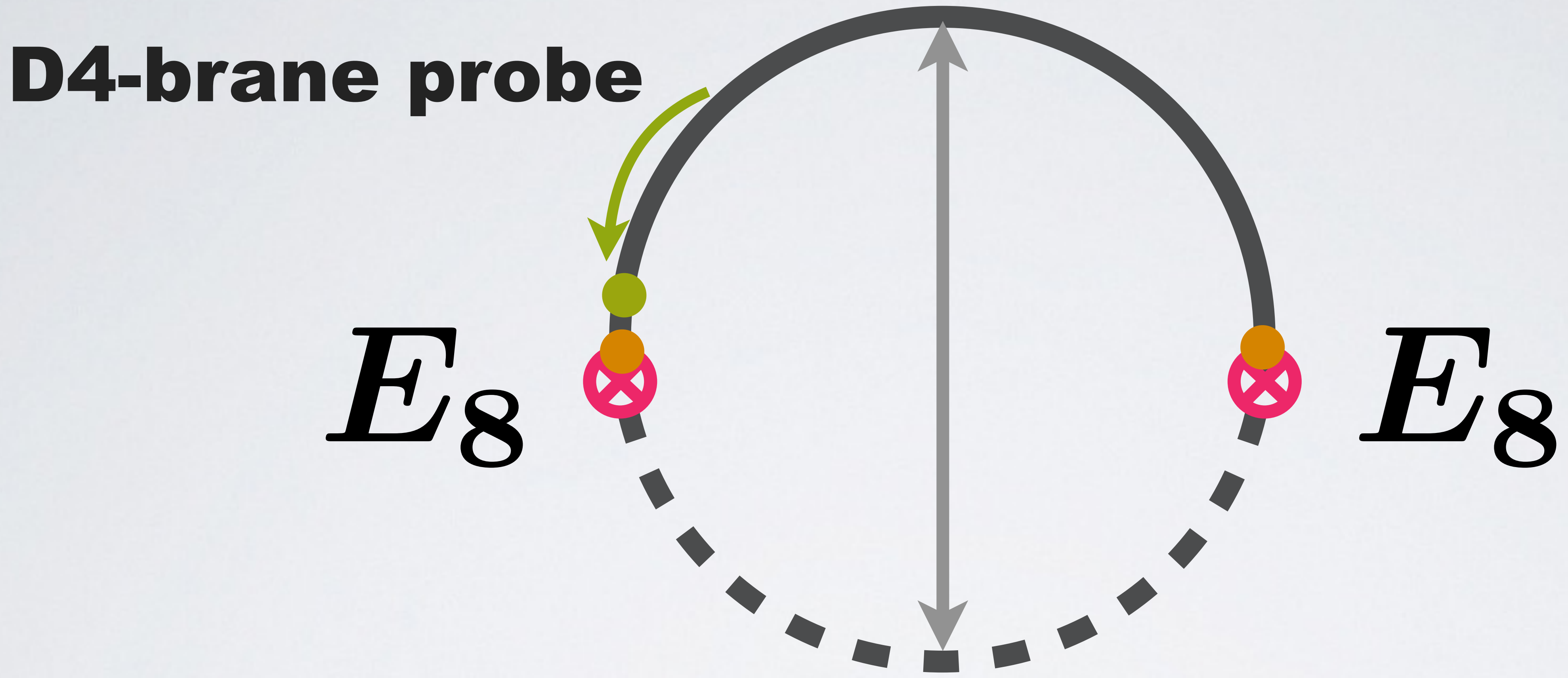

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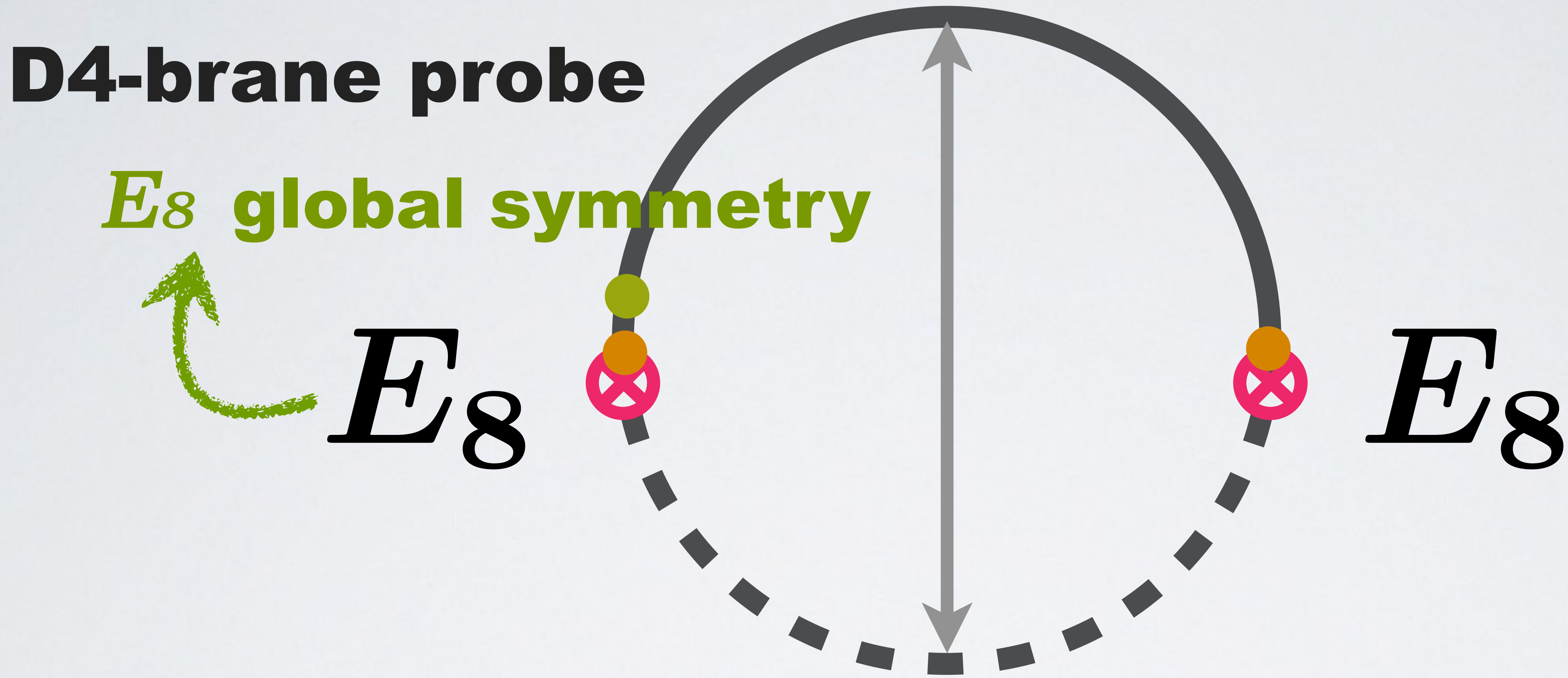

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2. Localization
& Partition Functions

Supersymmetric localization

susy exact integral

$$Z = \int \mathcal{D}\phi \mathcal{D}\psi \, e^{-t\, Q\cdot V}$$

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independent of t : we can evaluate at $t \to \infty$

Supersymmetric localization

susy exact integral

$$Z = \int \mathcal{D}\phi \mathcal{D}\psi \, e^{-t\, Q\cdot V}$$

independent of t **: we can evaluate at** $t \to \infty$

$$= \frac{\det D_\psi}{\sqrt{\det D_\phi}}$$

WKB is exact !

Various 5D geometries where localization works

$$S^1 \times \mathbb{R}^4_{\epsilon_1,\epsilon_2}$$

$$S^1 \times S^4$$

$$S^5$$

Various 5D geometries where localization works

$S^1 \times \mathbb{R}^4{}_{\epsilon_1, \epsilon_2}$: **Nekrasov partition function**

[Nekrasov, '02] [Nekrasov-Okounkov, '03]

$S^1 \times S^4$

S^5

Various 5D geometries where localization works

$S^1 \times \mathbb{R}^4_{\epsilon_1,\epsilon_2}$: Nekrasov partition function

[Nekrasov, '02] [Nekrasov-Okounkov, '03]

$S^1 \times S^4$: superconformal(spherical) index

[Bhattacharya-Bhattacharyya-Minwalla-Raju, '08]
[H.-C.Kim-S.-S.Kim-K.Lee, '12] [Iqbal-Vafa, '12] etc

S^5

Various 5D geometries where localization works

$S^1 \times \mathbb{R}^4_{\epsilon_1,\epsilon_2}$: **Nekrasov partition function**

[Nekrasov, '02] [Nekrasov-Okounkov, '03]

$S^1 \times S^4$: **superconformal(spherical) index**

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S^5 : **sphere partition function**

[Hosomichi-Seong-Terashima, '12] [Kallen-Qiu-Zabezine, '12] [Lockhart-Vafa, '12] [Imamura, '12] [H.-C.Kim-S.Kim, '12] etc

Superconformal Index

[Bhattacharya-Bhattacharyya-Minwalla-Raju, ’08]

$Q = Q^{\mathbf{1}}_{\mathbf{2}}$ **a super charge**

$S = S^{\mathbf{2}}_{\mathbf{1}}$ **a superconformal charge**

Superconformal Index

[Bhattacharya-Bhattacharyya-Minwalla-Raju, ’08]

$$\Delta = \{Q, S\} = \epsilon_0 - 2j_1 - 3R \geq 0$$

1/8 BPS with energy $\epsilon_0 = 2j_1 + 3R$

Superconformal Index

[Bhattacharya-Bhattacharyya-Minwalla-Raju, '08]

bosonic part of 5d N=1 SCA

$$SO(2,5) \times \underbrace{SU(2)_R}$$

$$\cup$$

$$SO(5) \simeq Sp(2) \supset \underbrace{SU(2)} \times SU(2)$$

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$$\Delta = \{Q, S\} = \epsilon_0 - 2j_1 - 3R \geq 0$$

1/8 BPS with energy $\epsilon_0 = 2j_1 + 3R$

Index counts these BPS states

Superconformal Index

[Bhattacharya-Bhattacharyya-Minwalla-Raju, '08]

$$\boldsymbol{I} = \mathrm{Tr}\,(-1)^{\boldsymbol{F}} = \boldsymbol{N}_{\mathrm{bosons}} - \boldsymbol{N}_{\mathrm{fermions}}$$

Superconformal Index

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$$\boldsymbol{I} = \mathrm{Tr}\,(-1)^{\boldsymbol{F}} = \boldsymbol{N}_{\mathrm{bosons}} - \boldsymbol{N}_{\mathrm{fermions}}$$

$$= \infty - \infty \; !$$

Superconformal Index

[Bhattacharya-Bhattacharyya-Minwalla-Raju, '08]

$$\boldsymbol{I} = \mathrm{Tr}\,(-1)^{\boldsymbol{F}} = \boldsymbol{N}_{\mathrm{bosons}} - \boldsymbol{N}_{\mathrm{fermions}}$$

$$= \infty - \infty \; !$$

we need to introduce regulators which correspond to the Cartan generators commuting with Q, S, and each other

$$\boldsymbol{\Delta}, \quad \boldsymbol{j_1 + R}, \quad \boldsymbol{j_2}, \quad \boldsymbol{H_i}, \quad \boldsymbol{J = *\mathrm{tr}(F \wedge F)}$$

Superconformal Index

[Bhattacharya-Bhattacharyya-Minwalla-Raju, '08]

$$\boldsymbol{I(x,y,m,u)} =$$

$$\mathrm{Tr}\,(-1)^{\boldsymbol{F}}\boldsymbol{e}^{-\boldsymbol{\beta}\{\boldsymbol{Q},\boldsymbol{S}\}}\boldsymbol{x}^{\mathbf{2}(\boldsymbol{j_1}+\boldsymbol{R})}\boldsymbol{y}^{\mathbf{2}\boldsymbol{j_2}}\boldsymbol{e}^{-\boldsymbol{i}\sum\boldsymbol{m_i}\boldsymbol{H_i}}\boldsymbol{u}^{\boldsymbol{k}}$$

Superconformal Index

[Bhattacharya-Bhattacharyya-Minwalla-Raju, ’08]

$$I(x,y,m,u) =$$

$$\mathrm{Tr}\,(-1)^F e^{-\beta\{Q,S\}} x^{2(j_1+R)} y^{2j_2} e^{-i\sum m_i H_i} u^k$$

5d SCFTs are strongly-coupled

→ **Localization works !!**

Superconformal Index [Kim-Kim-Lee, '12]

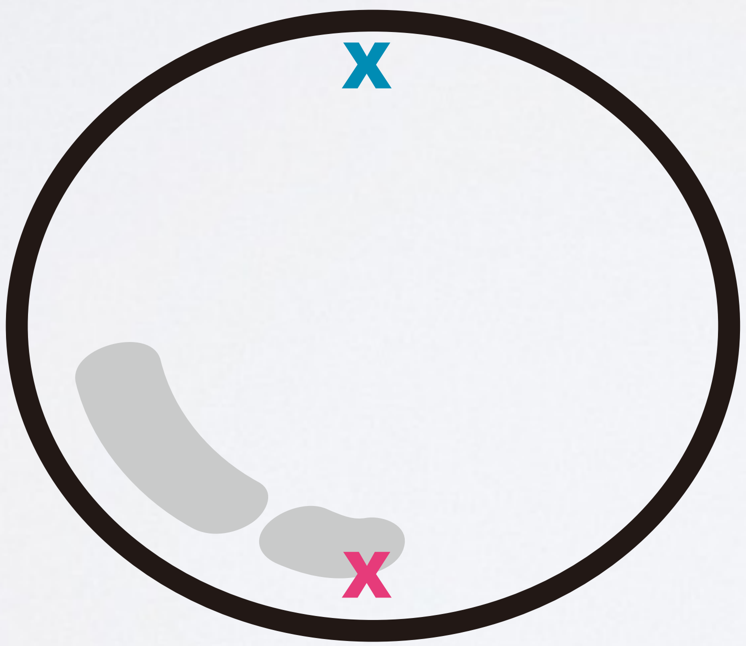
$I(x, y, m, u) =$ **a path integral expression on** $S^1 \times S^4$

This is the form where we can apply the localization method !

Superconformal Index [Kim-Kim-Lee, '12], [Iqbal-Vafa, '12]

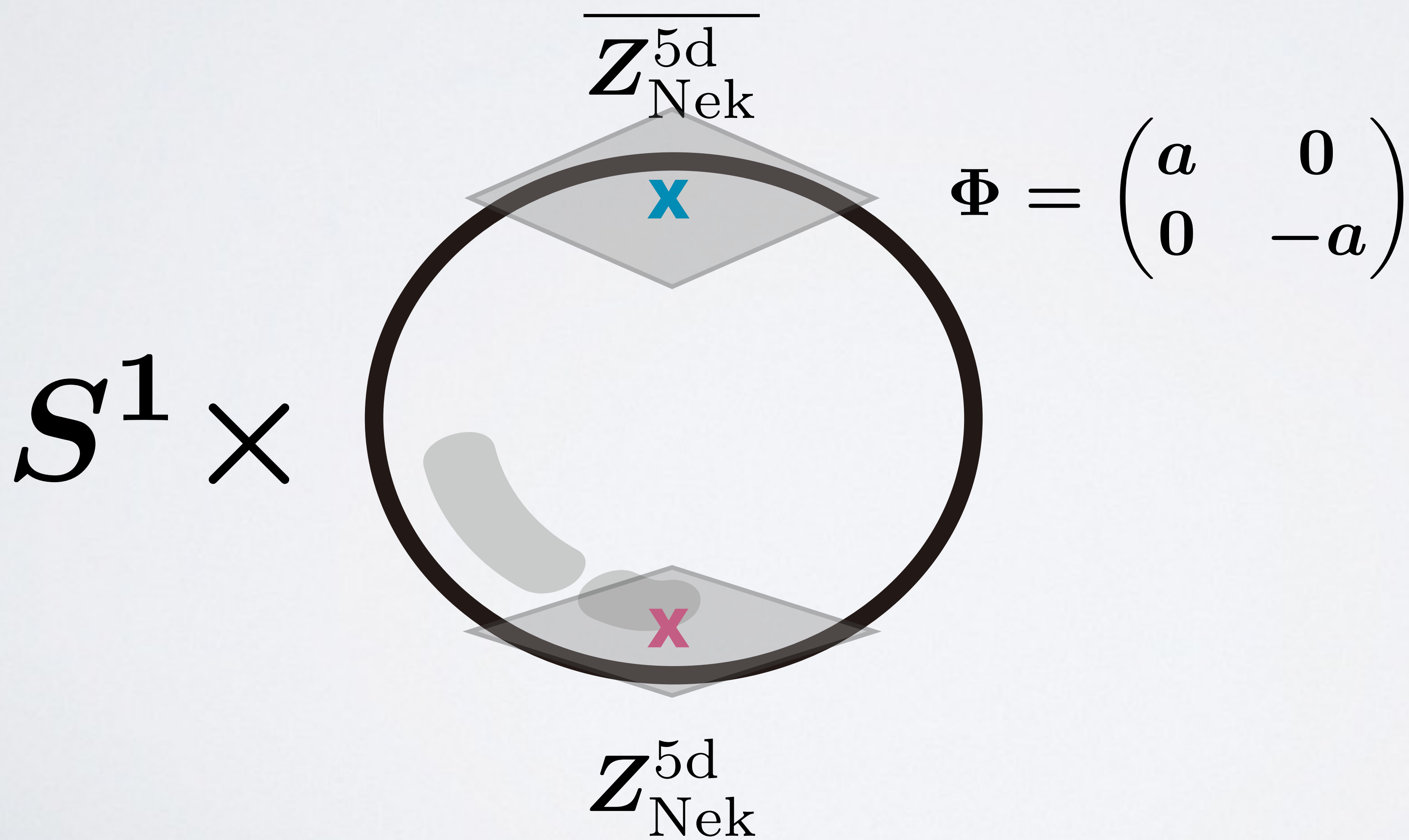
$$I(x,y,m,u) = \int [da] \left| Z^{\text{5d}}_{\text{Nek}}(t,q,m,u,a) \right|^2$$

$$S^1 \times$$



Superconformal Index [Kim-Kim-Lee, '12], [Iqbal-Vafa, '12]

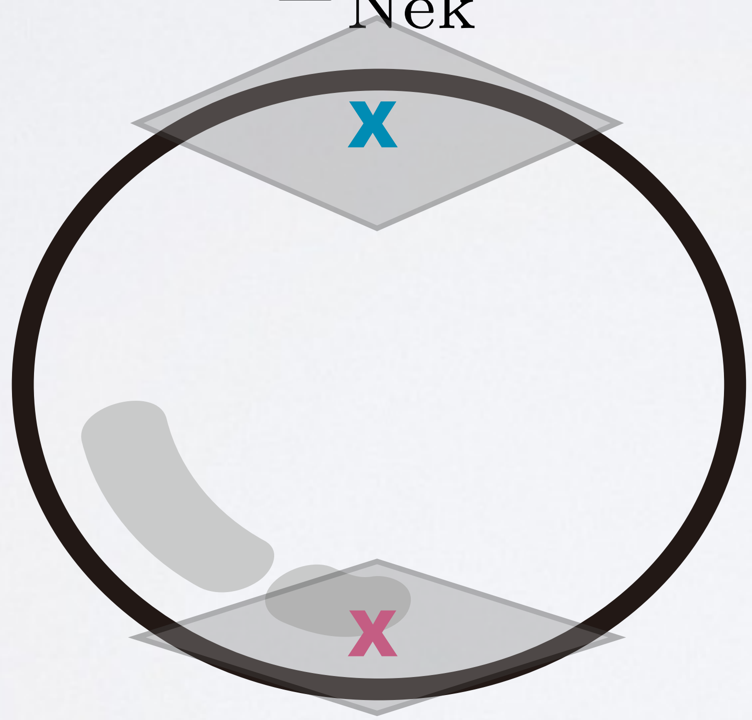
$$I(x,y,m,u)=\int [da]\,\left|Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a)\right|^2$$



Superconformal Index [Kim-Kim-Lee, ’12], [Iqbal-Vafa, ’12]

$$I(x,y,m,u) = \int [da] \left| Z^{\text{5d}}_{\text{Nek}}(t,q,m,u,a) \right|^2$$

$$S^1 \times$$



$$q = e^{R\epsilon_1},\ t = e^{-R\epsilon_2}$$

$$q = xy,\ t = \frac{y}{x}$$

3. Global Symmetry via SC Index

Superconformal Index [Kim-Kim-Lee, '12]

$$I(x,y,m,u) = \int [da] \left| Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a) \right|^2$$

[Kim-Kim-Lee] uses those for Sp(1) gauge theories.

Superconformal Index [Kim-Kim-Lee, '12]

$$I(x,y,m,u) = \int [da] \left| Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a) \right|^2$$

Sp(1)=SU(2)

[Kim-Kim-Lee] uses those for Sp(1) gauge theories.

Superconformal Index [Kim-Kim-Lee, '12]

$$I(x,y,m,u) = \int [da] \left| Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a) \right|^2$$

[Kim-Kim-Lee] uses those for Sp(1) gauge theories.

ADHM works for $\mathrm{A_n}$, $\mathrm{B_n}$, $\mathrm{C_n}$ & $\mathrm{D_n}$ instantons

Superconformal Index [Kim-Kim-Lee, '12]

$$I(x,y,m,u) = \int [da] \left| Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a) \right|^{2}$$

[Kim-Kim-Lee] uses those for Sp(1) gauge theories.

ADHM works for A_n, B_n, C_n & D_n instantons

→ **Nekrasov partition function** [Nekrasov-Shadchin, '04]

Superconformal Index [Kim-Kim-Lee, '12]

$$I(x,y,m,u) = \int [da] \left| Z^{\mathrm{5d}}_{\mathrm{Nek}}(t,q,m,u,a) \right|^{\mathbf{2}}$$

[Kim-Kim-Lee] found N_f=0, ..., 6 enjoys E_n symmetry

$$I_{N_f} = 1 + \chi^{E_{N_f+1}}_{\mathrm{adj}}(m,u)\, x^2 + \cdots$$

quantitative verification of E_n enhancement

Superconformal Index [Iqbal-Vafa, '12]

$$I(x,y,m,u) = \int [da]\left|Z^{\mathrm{5d}}_{\mathrm{Nek}}(t,q,m,u,a)\right|^2$$

[Iqbal-Vafa] uses those for SU(2) gauge theories.

Superconformal Index [Iqbal-Vafa, '12]

$$I(x, y, m, u) = \int [da] \left| Z^{\text{5d}}_{\text{Nek}}(t, q, m, u, a) \right|^{2}$$

Sp(1)=SU(2)

[Iqbal-Vafa] uses those for SU(2) gauge theories.

- N_f=0, 1, 2 enjoys E_n symmetry (the same as Sp(1))

Superconformal Index

[Bao-Mitev-Pomoni-MT-Yagi, '13]
[Hayashi-Kim-Nishinaka, '13]

$$I(x,y,m,u) = \int [da] \left| Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a) \right|^{2}$$

Sp(1)=SU(2)

[Iqbal-Vafa] uses those for SU(2) gauge theories.

- N_f=0, 1, 2 enjoys E_n symmetry
- $N_f > 2$ does not show enhancement

[Bao-Mitev-Pomoni-MT-Yagi, '13]
[Hayashi-Kim-Nishinaka, '13]

Superconformal Index [Bao-Mitev-Pomoni-MT-Yagi, '13] [Hayashi-Kim-Nishinaka, '13]

$$I(x,y,m,u) = \int [da] \left| Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a) \right|^{2}$$

Do we physicists need to accept the mathematical nonsense? (as we sometimes do)

$$Sp(1) \neq SU(2)$$

Superconformal Index [Bao-Mitev-Pomoni-MT-Yagi, '13] [Hayashi-Kim-Nishinaka, '13]

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Do we physicists need to accept the mathematical nonsense? (as we sometimes do)

$$Sp(1) \neq SU(2)$$

We found solution to this problem

[Bao-Mitev-Pomoni-MT-Yagi, '13]
[Hayashi-Kim-Nishinaka, '13]

4. Conjectures on Nekrasov Partition Functions

5d theories via 5-branes

5d theory via (p,q) 5-brane web

[Aharony-Hanany, ’97]

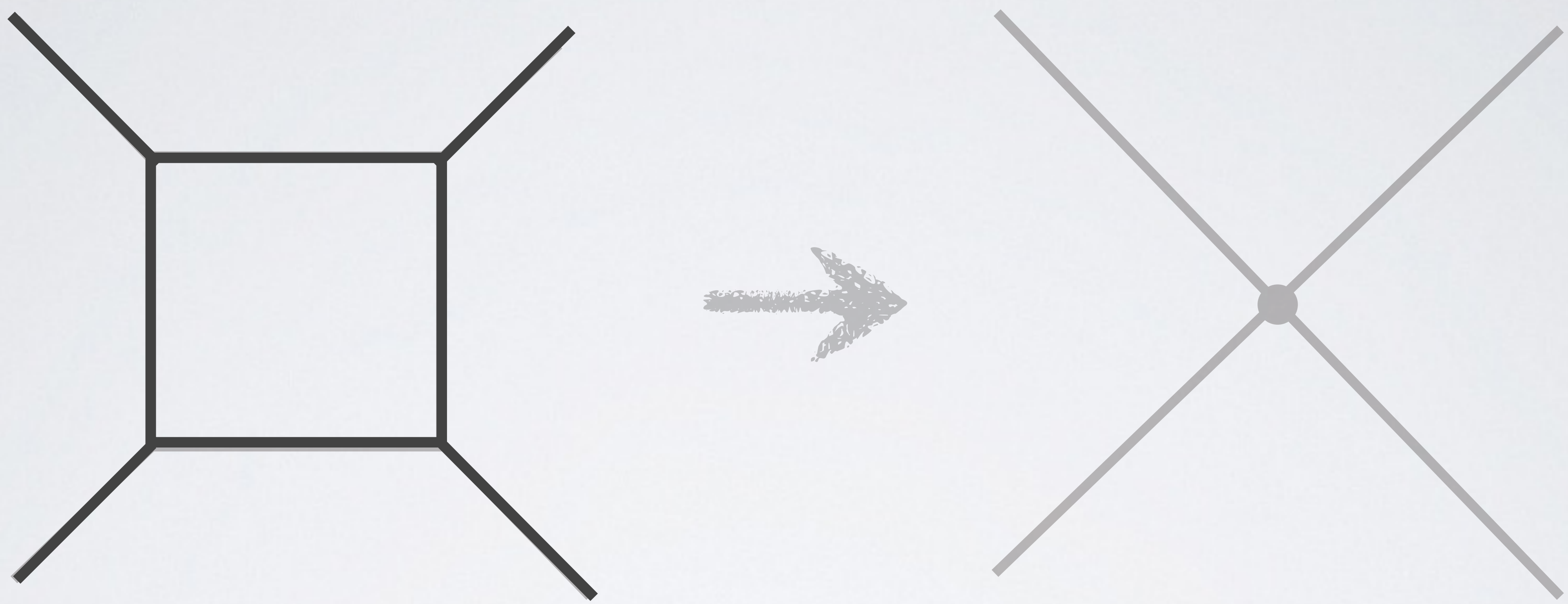
We can construct 5d theories by using (p,q) 5-brane web in Type IIB string

(1,0) 5-brane : D5-brane

(0,1) 5-brane : NS5-brane

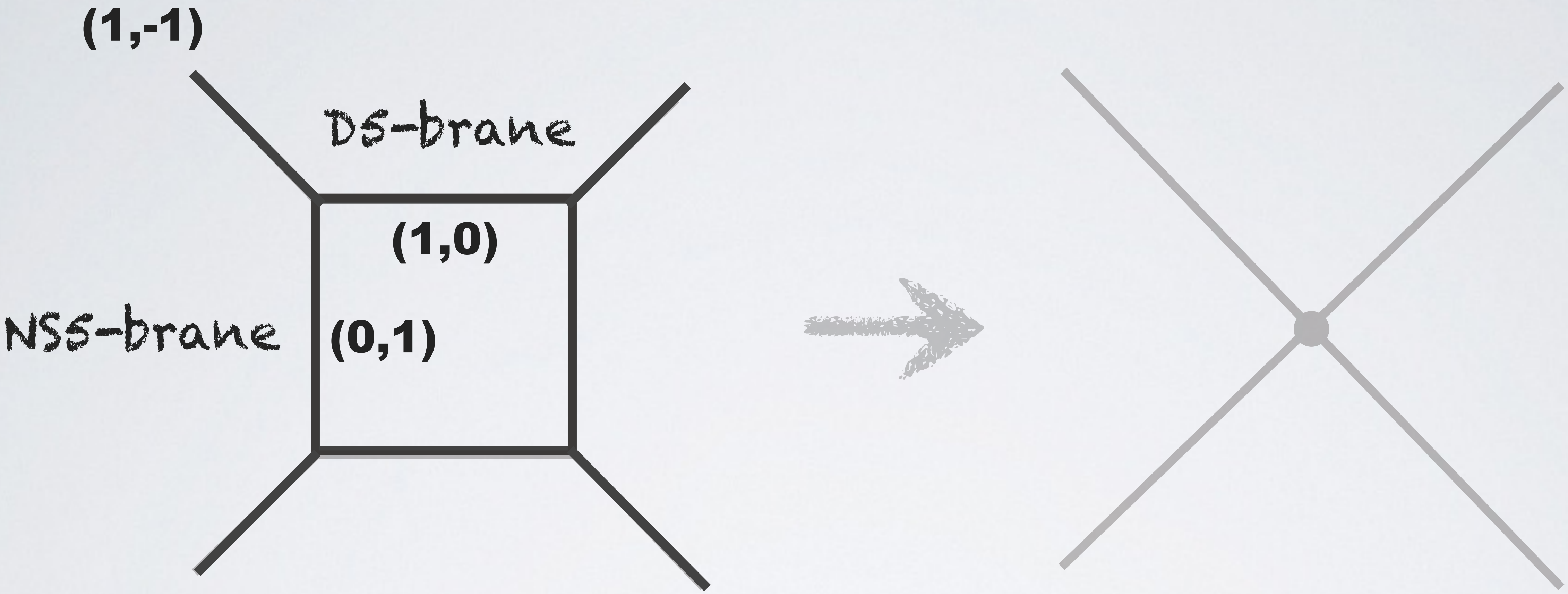
Pure SU(2) YM [Aharony-Hanany, '97]

x^5- x^6 plane



	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
5-brane	○	○	○	○	○	1D	confg.			

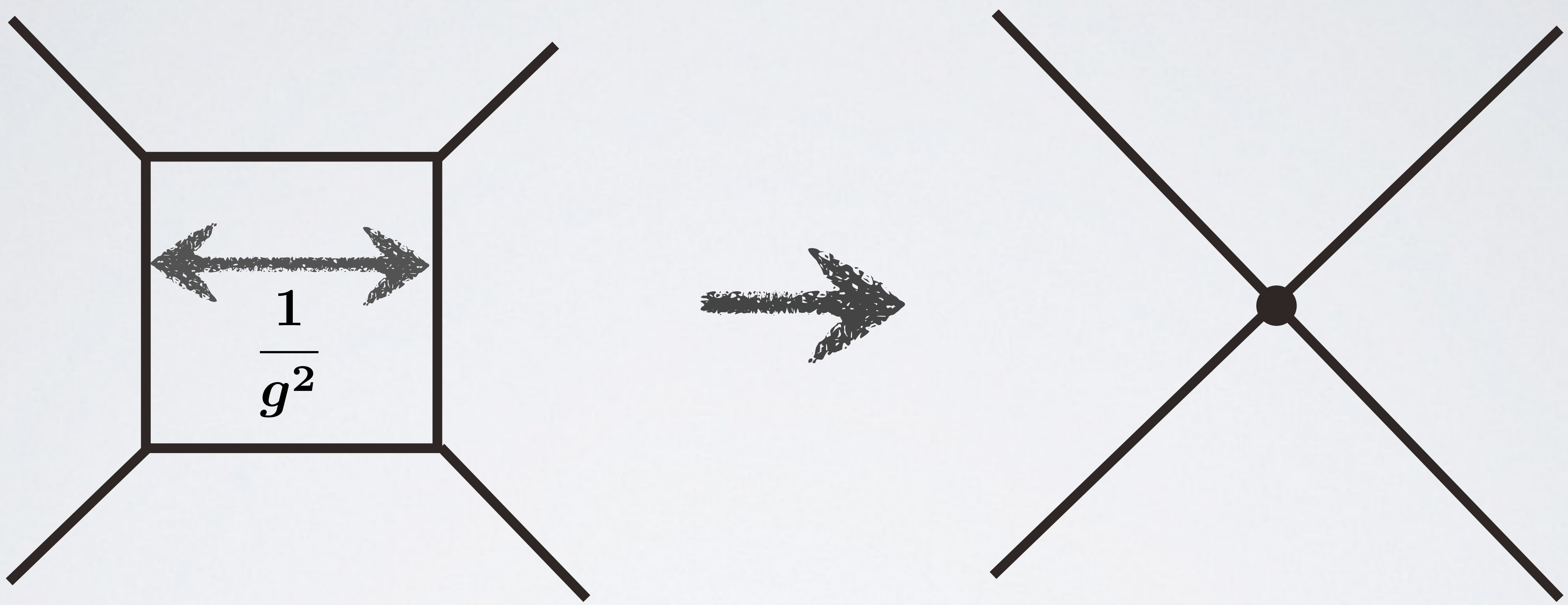
Pure SU(2) YM [Aharony-Hanany, ’97]



	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
5-brane	○	○	○	○	○	1D confg.				

Pure SU(2) YM [Aharony-Hanany, ’97]

IR **UV**



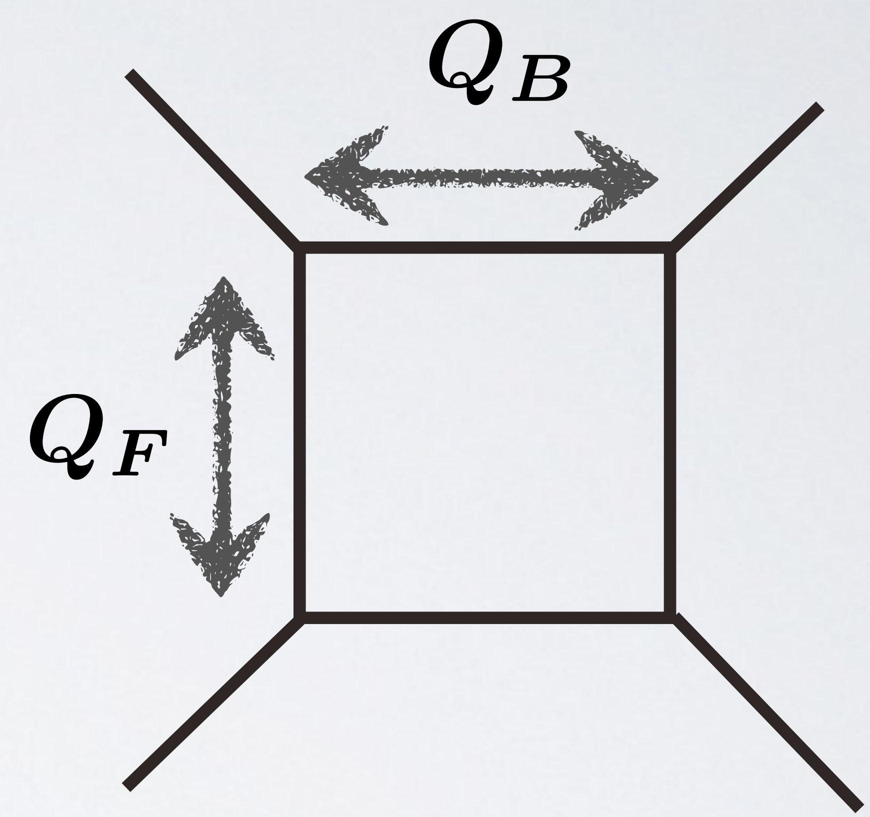
strongly-coupled

E_1 SCFT

Pure SU(2) YM [Aharony-Hanany, '97]

$$Q_F = e^{2Ra}$$

$$Q_B = u\, e^{2Ra}$$

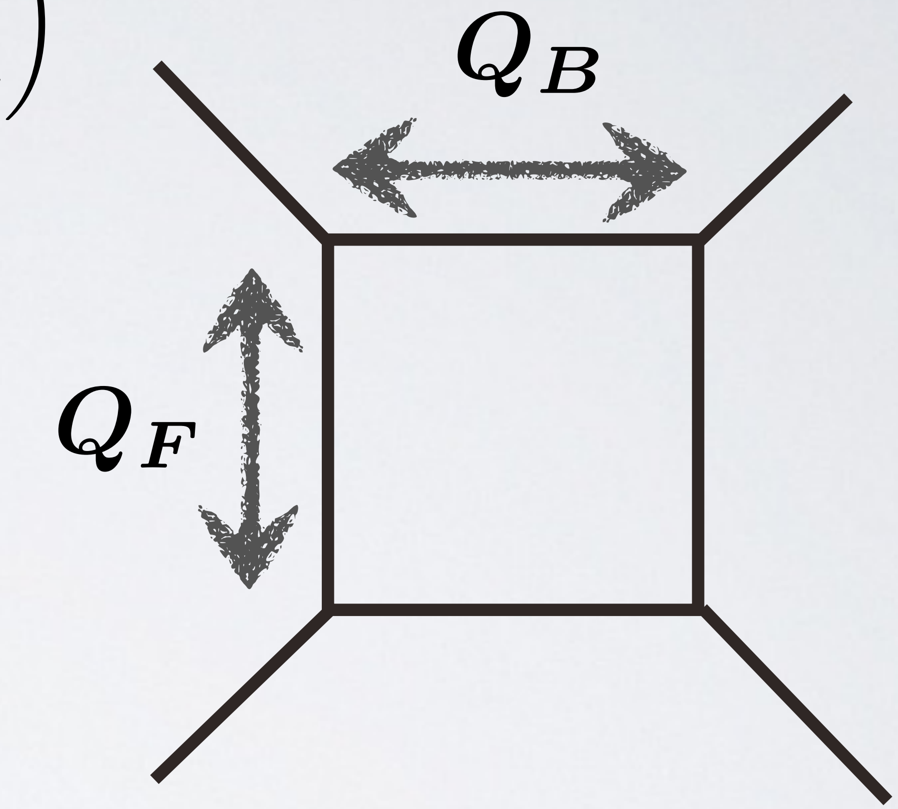


Pure SU(2) YM [Aharony-Hanany, '97]

Coulomb branch $\Phi = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}$

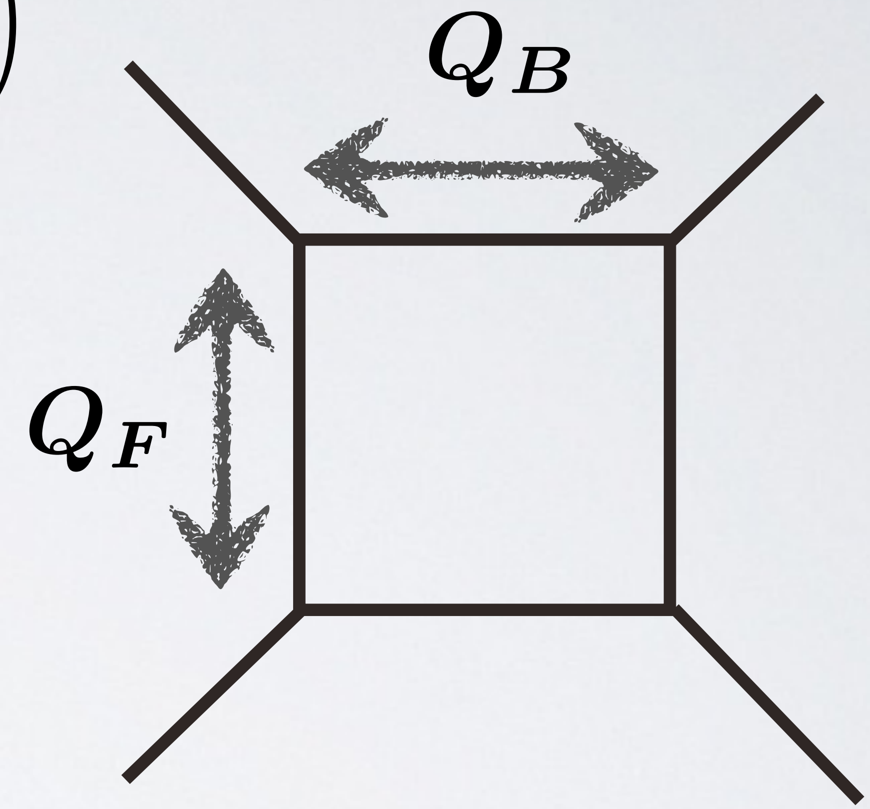
$$Q_F = e^{2Ra}$$

$$Q_B = u\, e^{2Ra}$$



Pure SU(2) YM [Aharony-Hanany, ’97]

Coulomb branch $\Phi = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}$



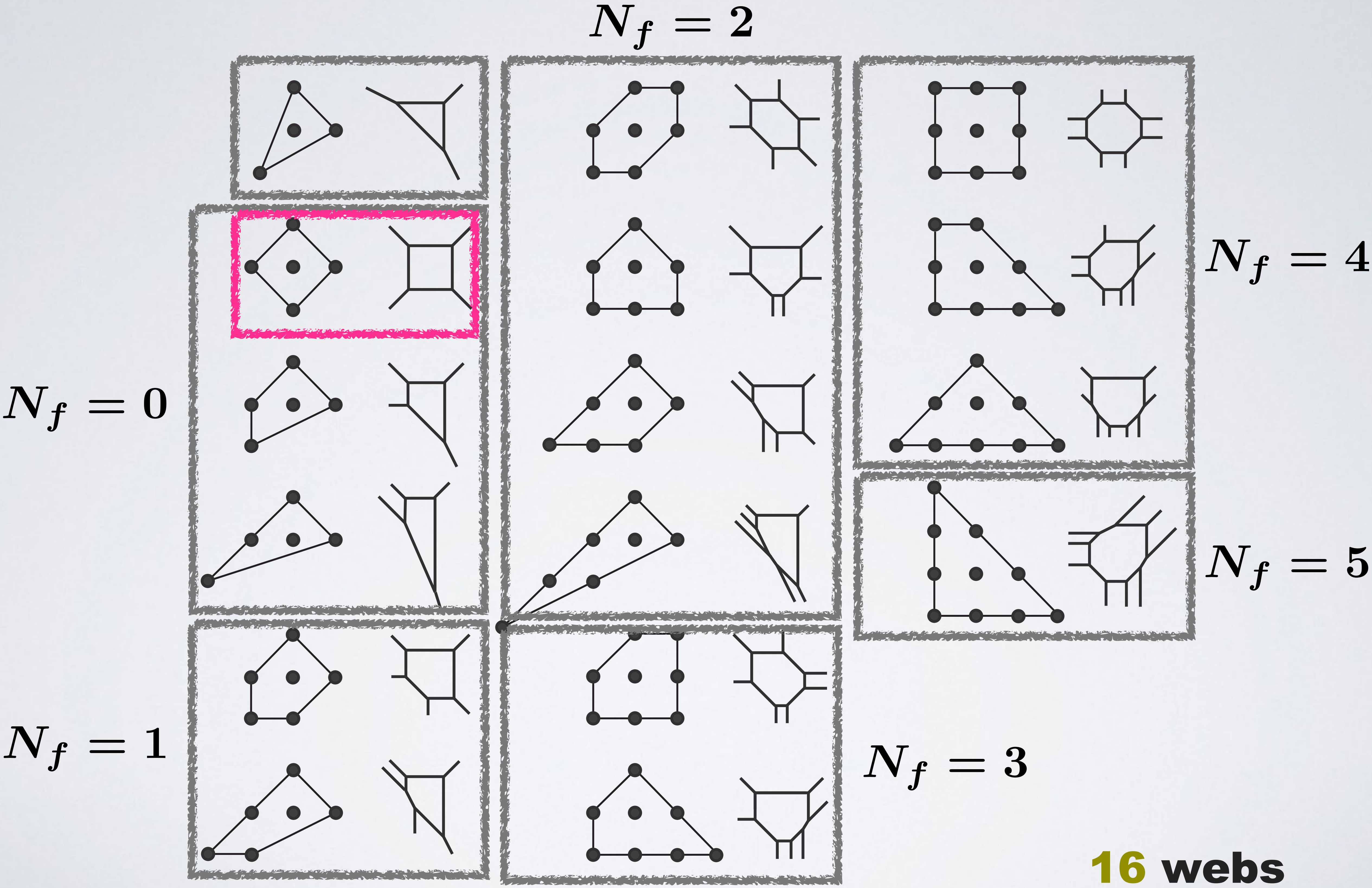
$$Q_F = e^{2Ra}$$

$$Q_B = u\, e^{2Ra}$$

instanton factor

$$(R\mu)^4 e^{-\frac{8\pi^2}{g(\mu)^2} + i\theta(\mu)}$$

Possible SU(2) theories (grids & webs)



16 webs

Refined topological strings

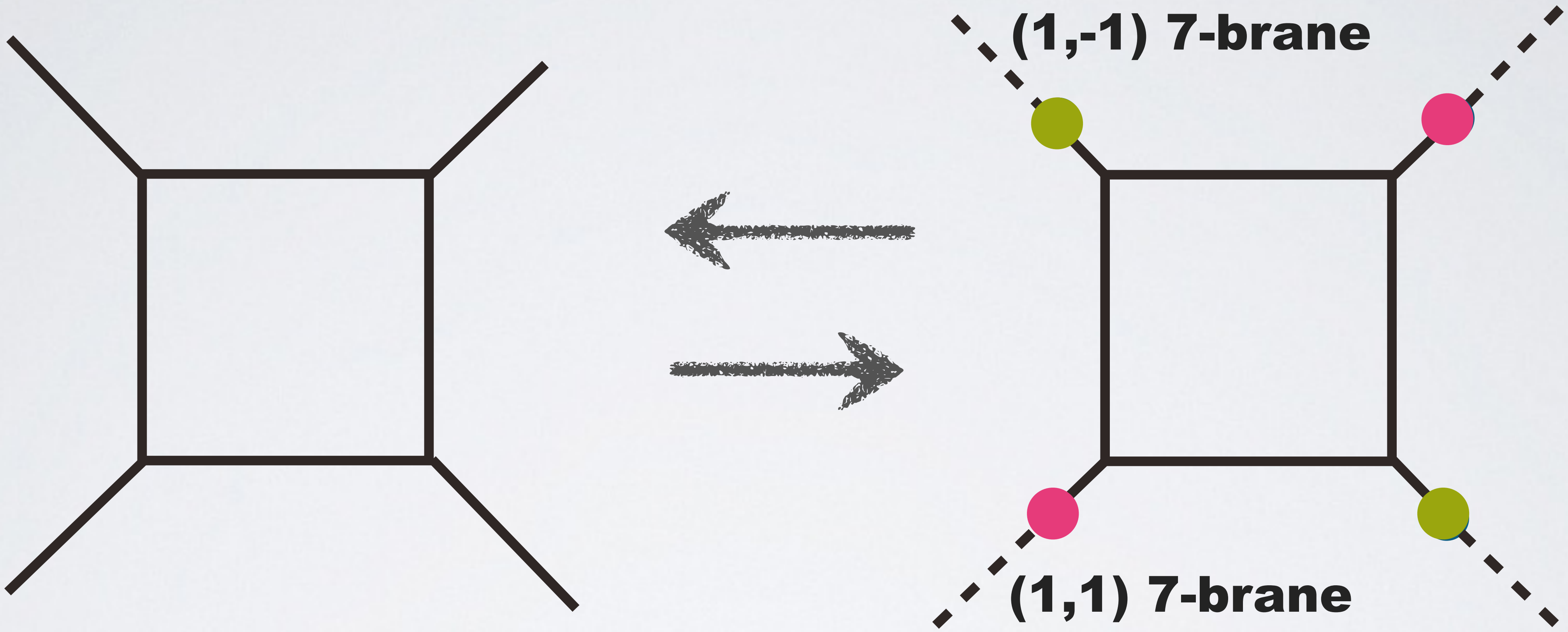
For 5-brane webs, the refined topological string exactly computes the partition function of the world-vol. theory

[Aganagic-Klemm-Marino-Vafa, '03]

[Awata-Kanno] [Iqbal-Kozcaz-Vafa]

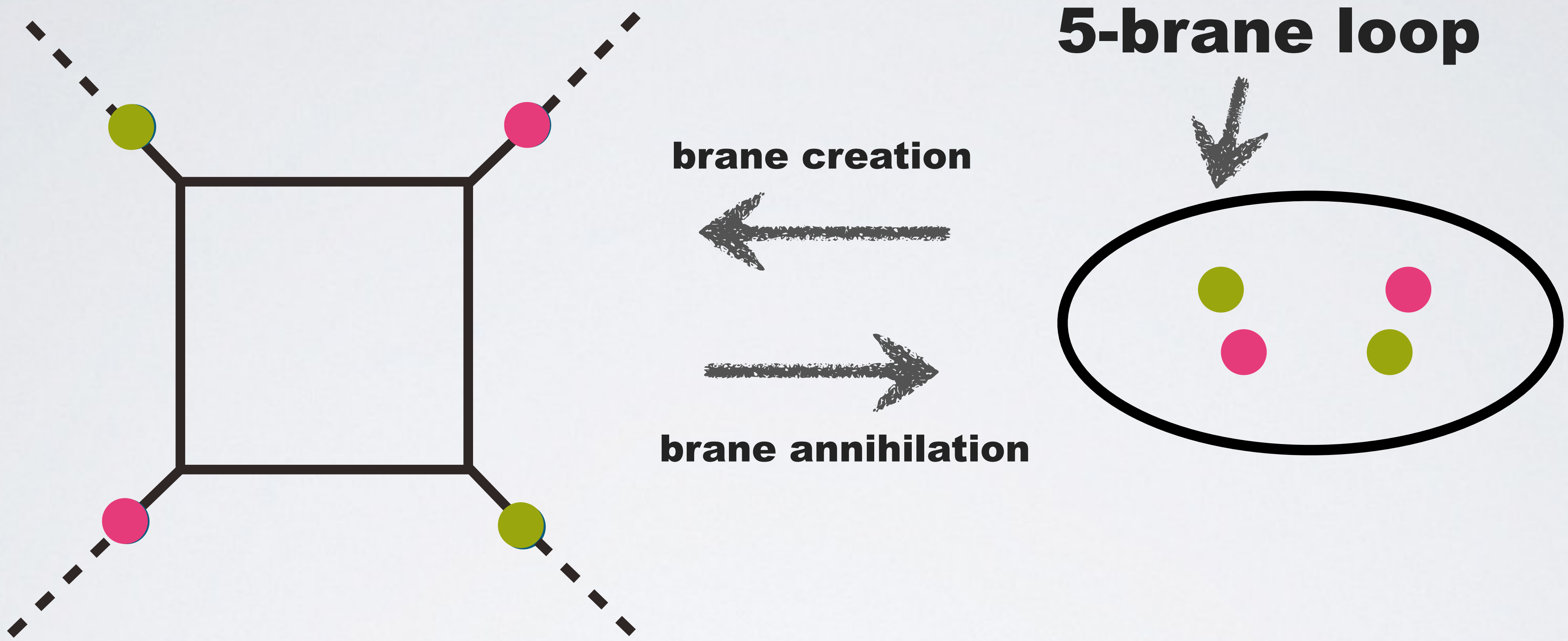
$$Z_{web} = Z^{5d}_{Nek}$$

Pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, '99]

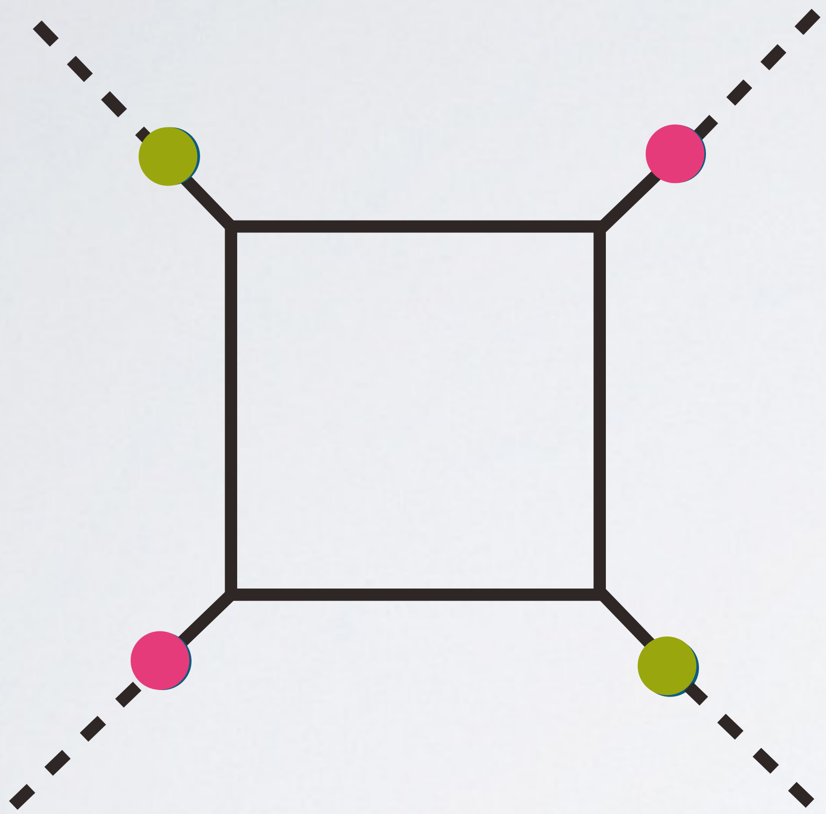


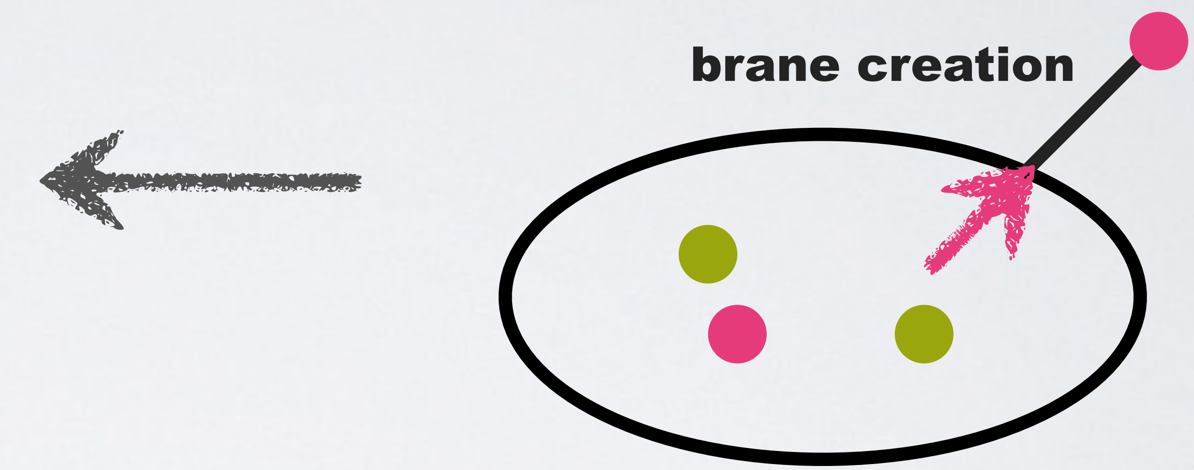
	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
5-brane	○	○	○	○	○	1D confg.				
7-brane	○	○	○	○	○	pt.		○	○	○

Pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, '99]

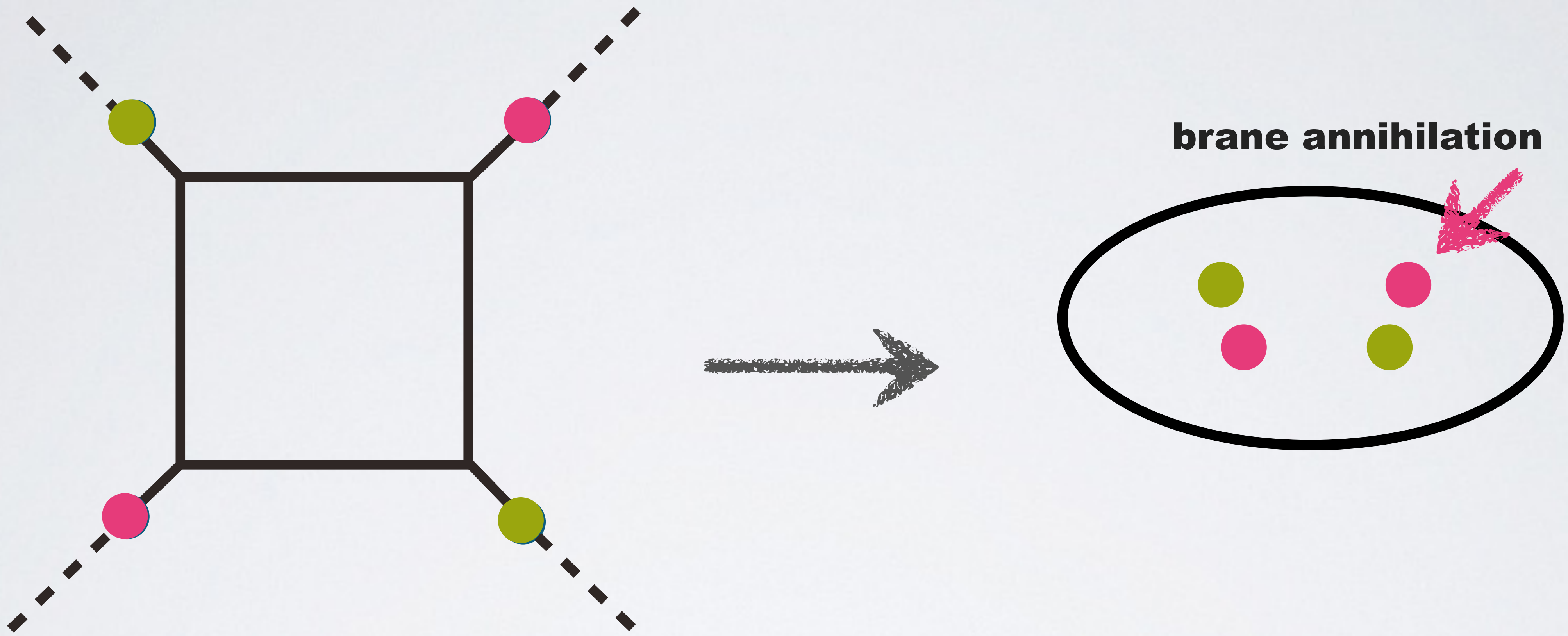


Pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, ’99]

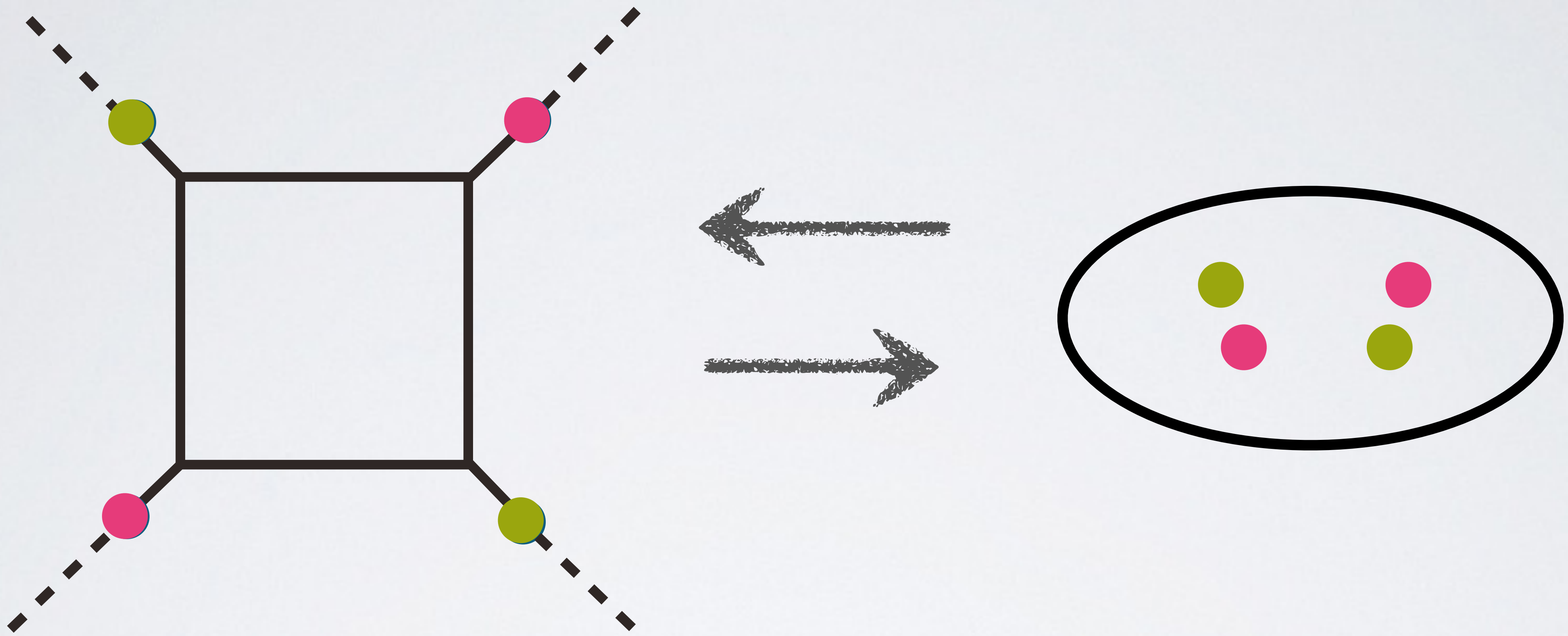




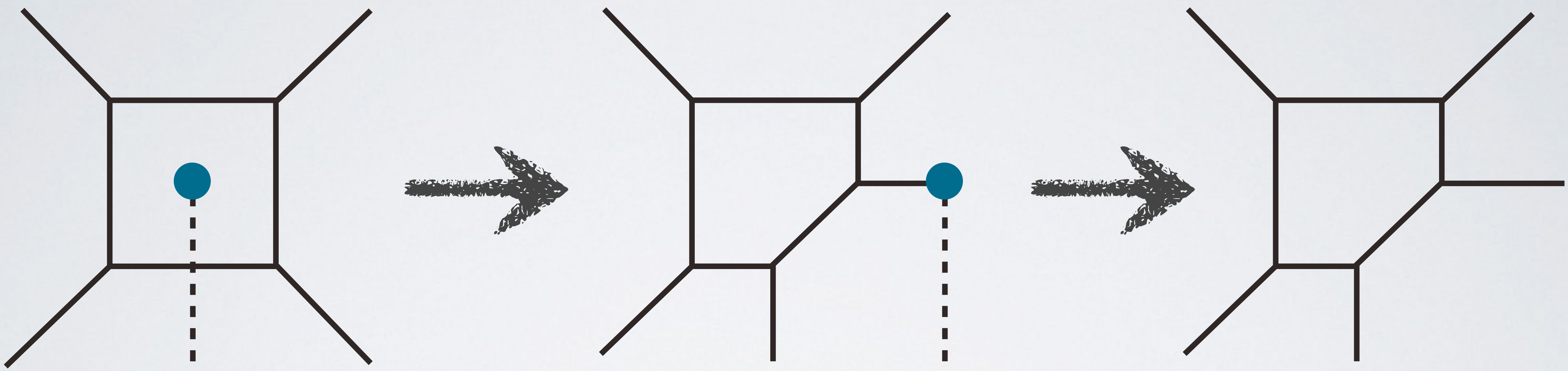
Pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, ’99]



Pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, ’99]

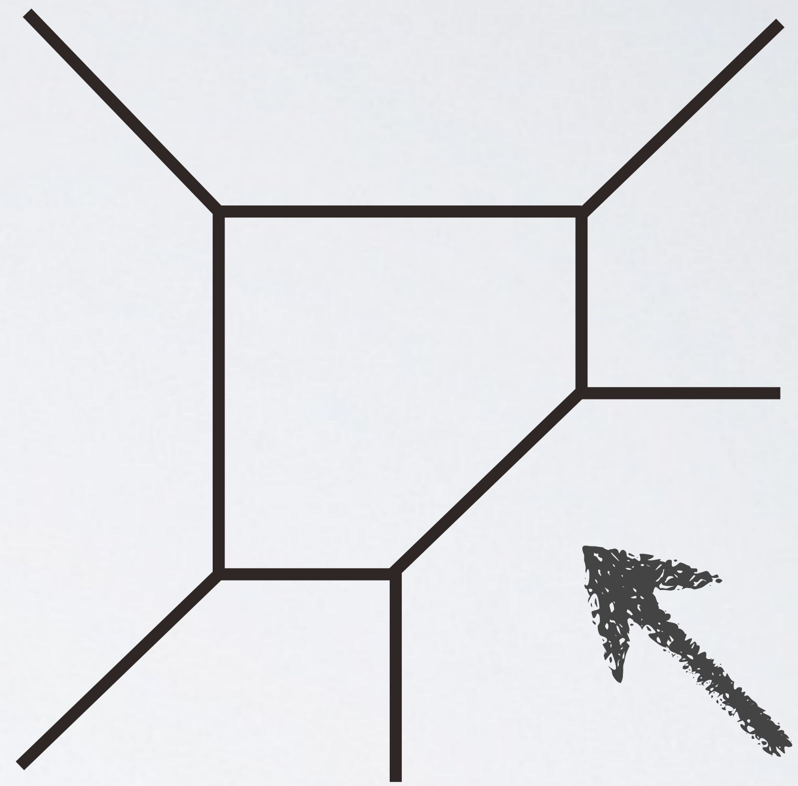


N_f=1 SU(2) SQCD [DeWolfe-Hanany-Iqbal-Katz, ’99]



N_f=1 SU(2) SQCD

$$Q_m = e^{-t_E} = e^{-R(a+m)}$$



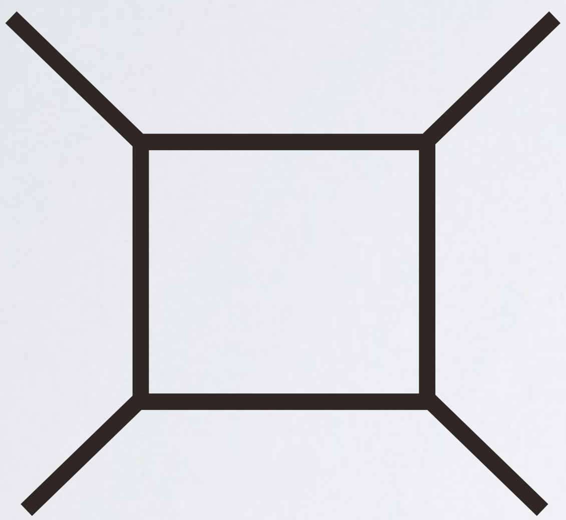
blow up

What happens in problematic SU(2) theories ?

[MT, ’13]

Rank-one theories without flavor

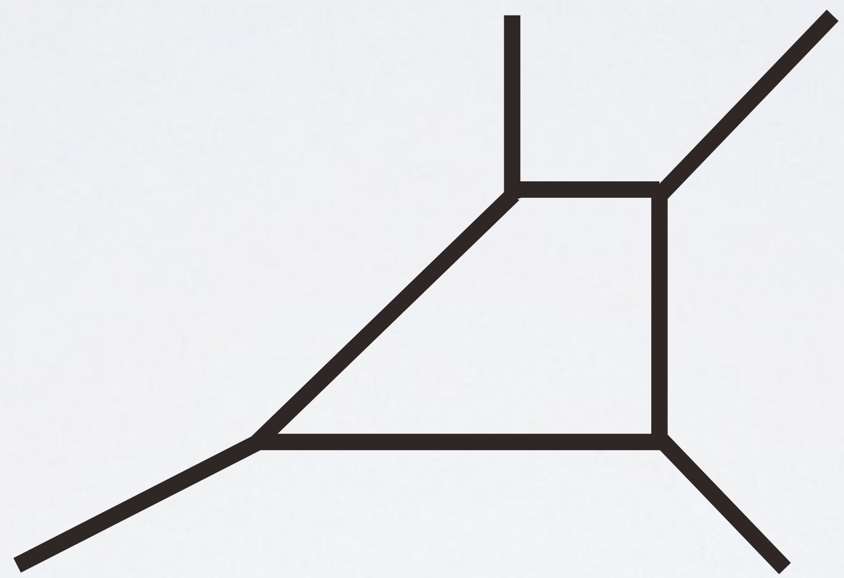
discussion so far : [Douglas-Katz-Vafa,'97]
[Aharony-Hanany,'97]
[Aharony-Hanany-Kol,'97] ...



$\mathbb{F}_0$

$\mathbf{E}_1$

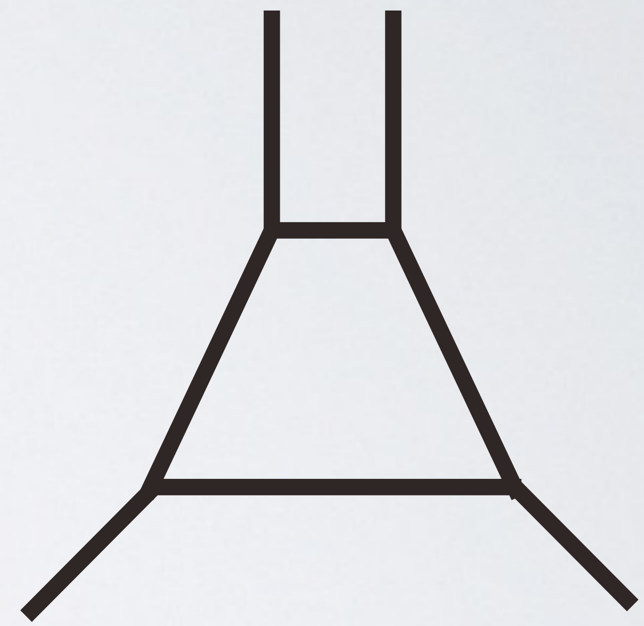
SU(2) YM $\theta=0$



$\mathbb{F}_1$

$\tilde{\mathbf{E}}_1$

SU(2) YM $\theta=\pi$: discrete theta angle



$\mathbb{F}_2$

??

Rank-one theories

$$\boxed{\boldsymbol{\pi_4(SU(2)) = \mathbb{Z}_2}}$$

Only two gauge theories !

Rank-one theories

$$\boxed{\pi_4(SU(2)) = \mathbb{Z}_2}$$

$$\longrightarrow \quad Z^{\theta=0,\pi} = Z_{[0]} \pm Z_{[1]}$$

Rank-one theories

$$\boxed{\pi_4(SU(2)) = \mathbb{Z}_2}$$

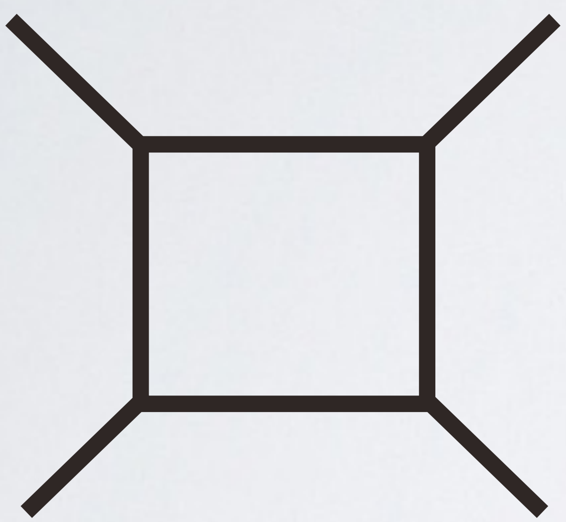
$$\longrightarrow \quad Z^{\theta=0,\pi} = Z_{[0]} \pm Z_{[1]}$$

discrete theta angle

$$e^{in\theta}$$

Rank-one theories [MT, arXiv:1310.7509]

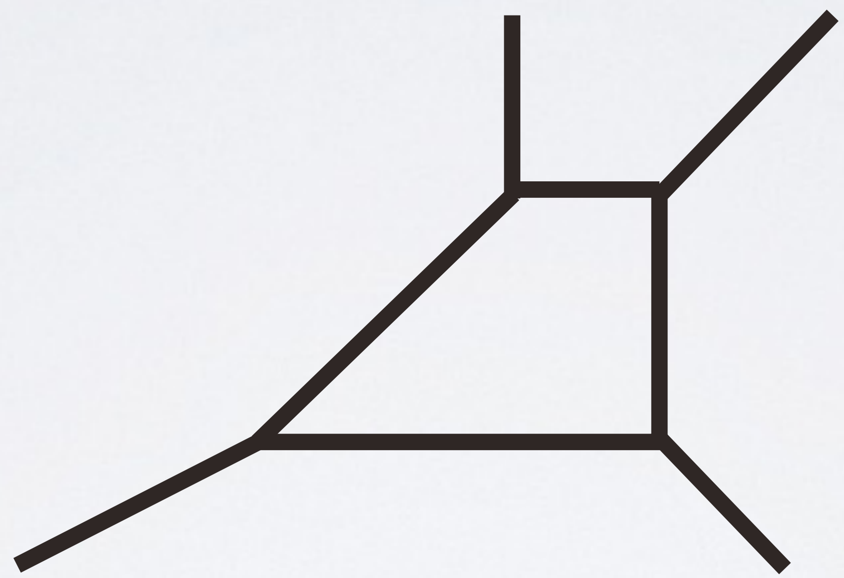
7-brane pict. works



$\mathbb{F}_0$

$\mathbf{E}_1$

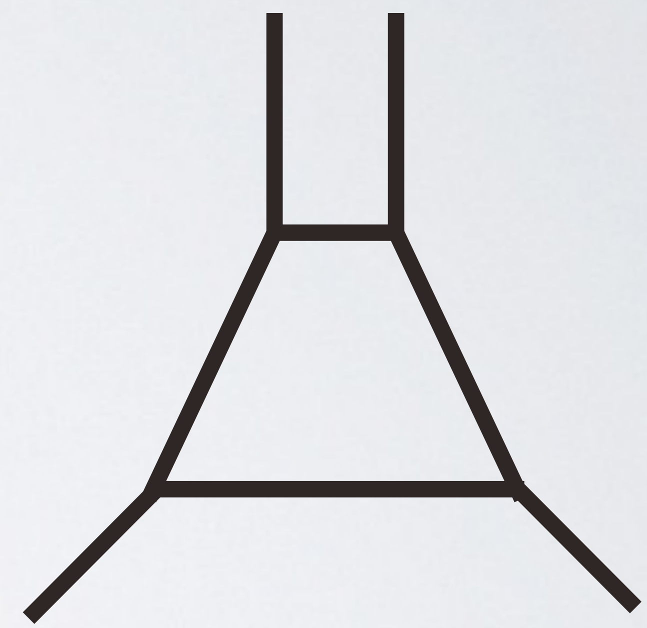
SU(2) YM θ=0



$\mathbb{F}_1$

$\tilde{\mathbf{E}}_1$

SU(2) YM θ=π

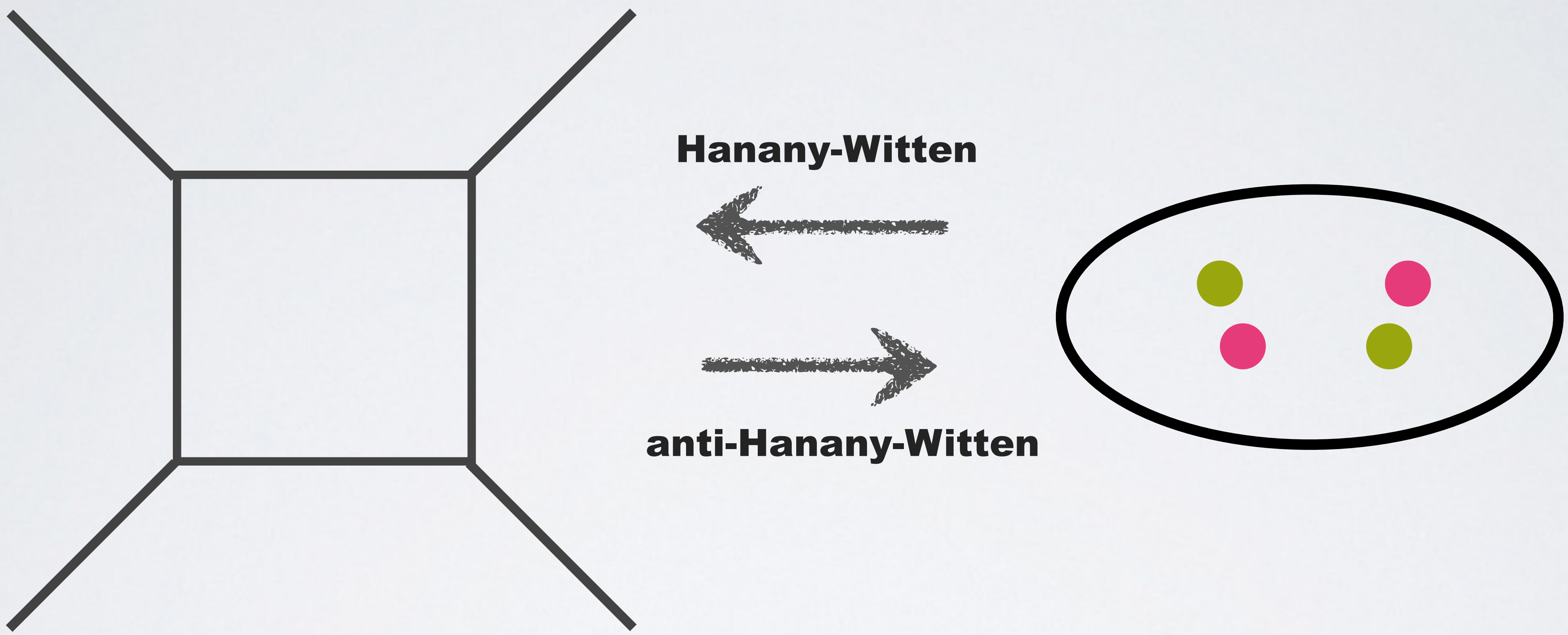


$\mathbb{F}_2$

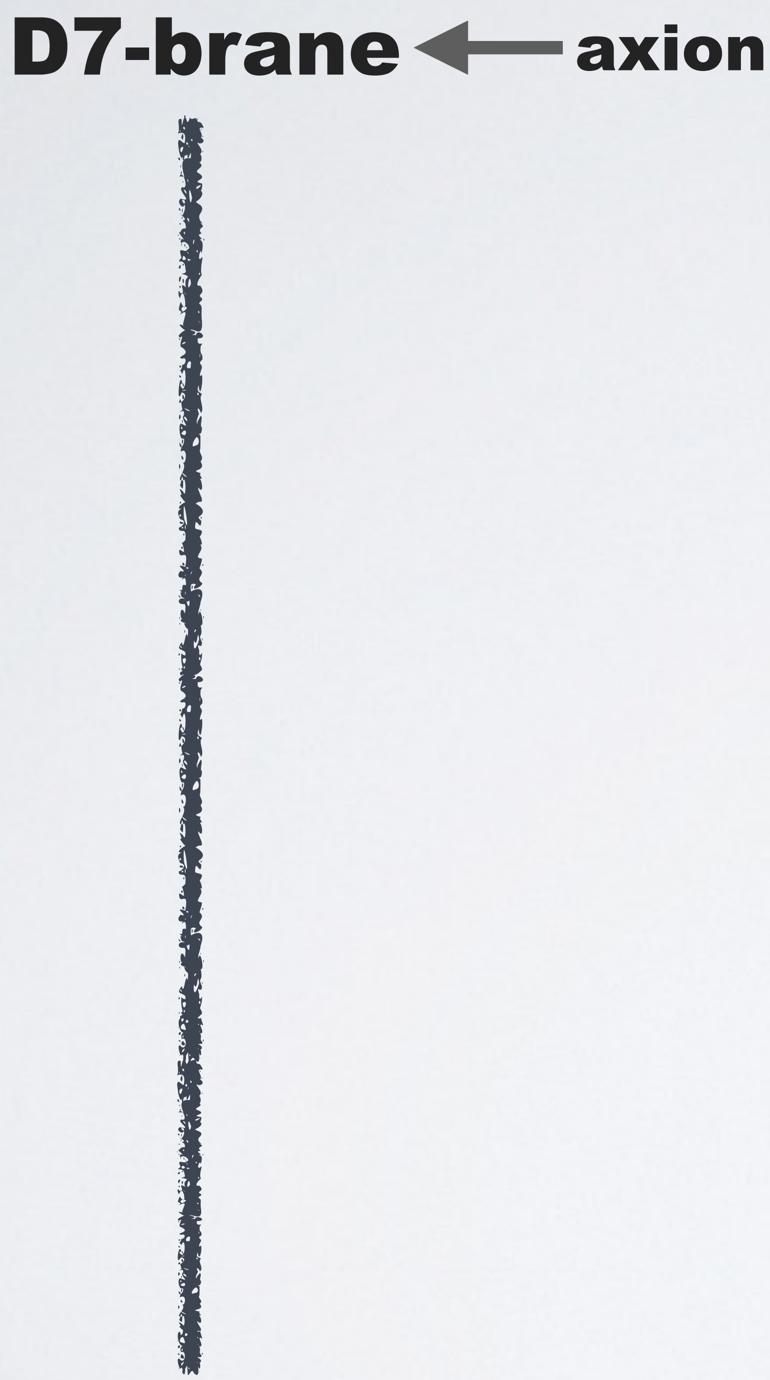
~~??~~

Rank-one theory $\mathbb{F}_0$

- we will use 7-brane picture → **duality**

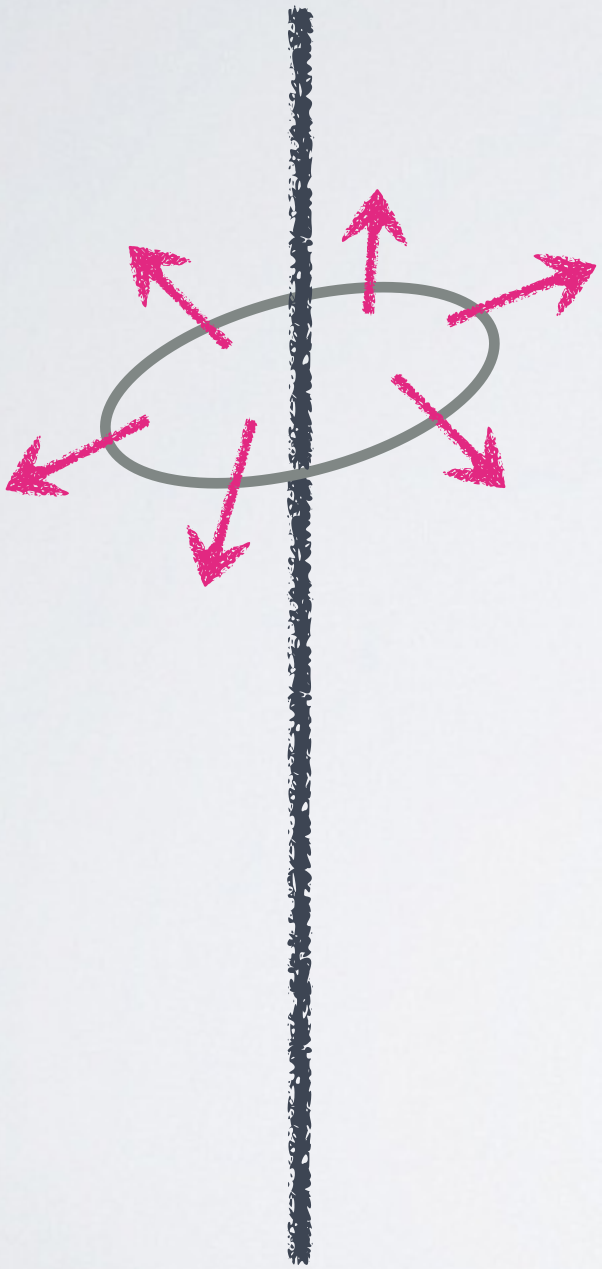


7-brane monodormy



7-brane monodormy

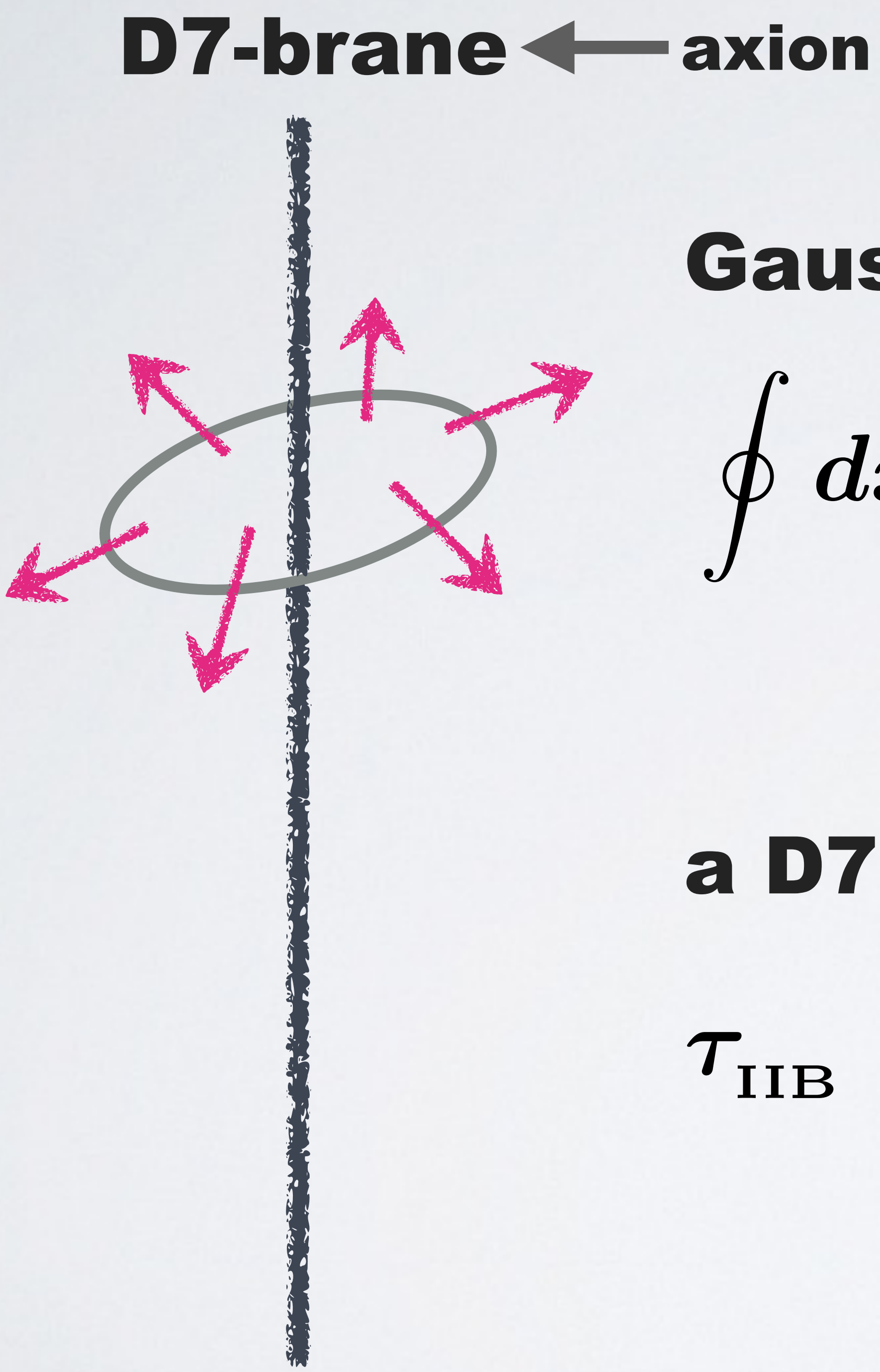
D7-brane ← axion



Gauss law

$$\oint dx^{\mu}\,\partial_{\mu} a(x) = N_{D7}$$

7-brane monodormy



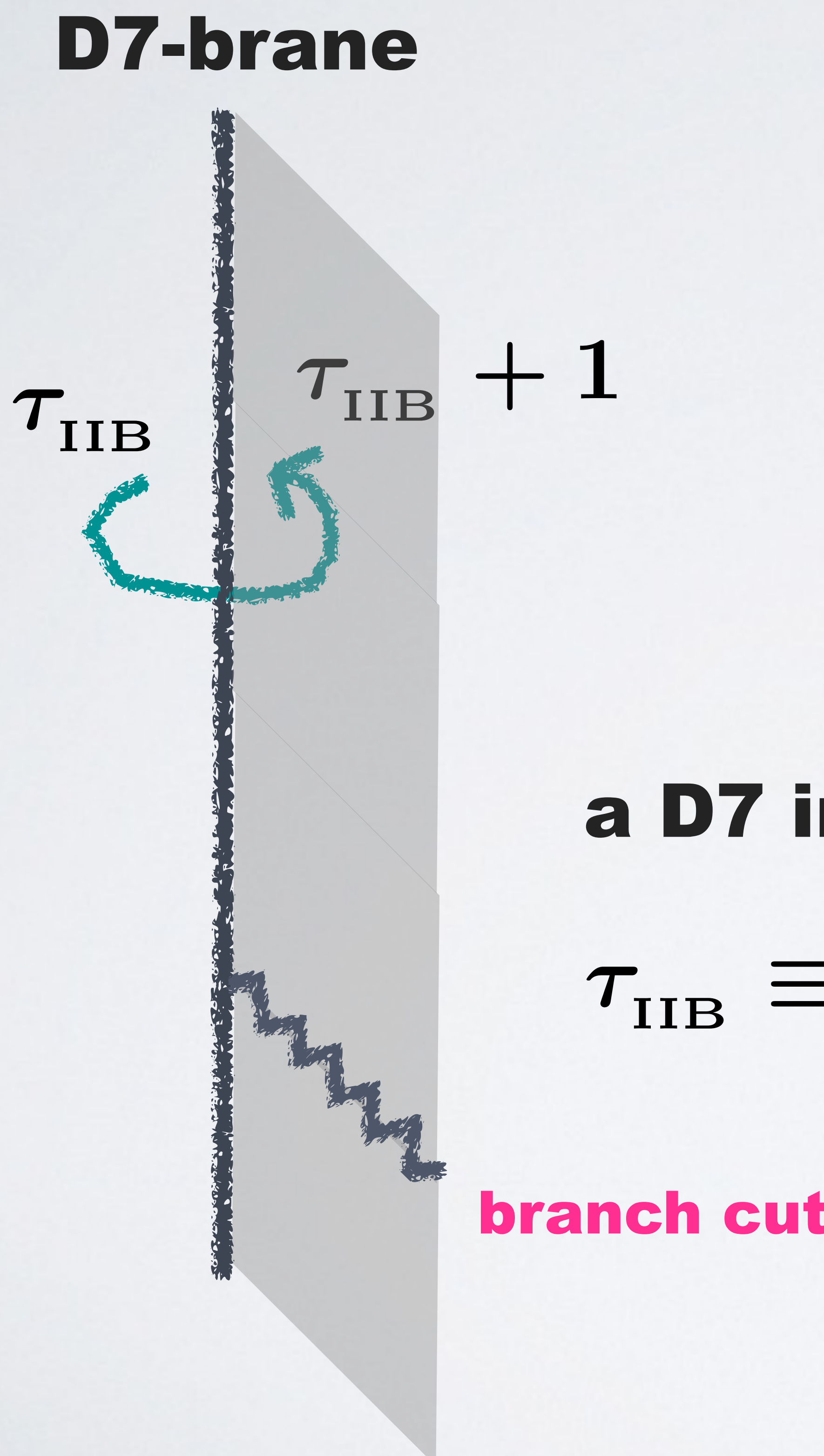
Gauss law

$$\oint dx^\mu\, \partial_\mu a(x) = N_{D7}$$

a D7 induces

$$\tau_{\rm IIB} \equiv a(x) + i e^{-\phi(x)} = \frac{1}{2\pi i}\log z$$

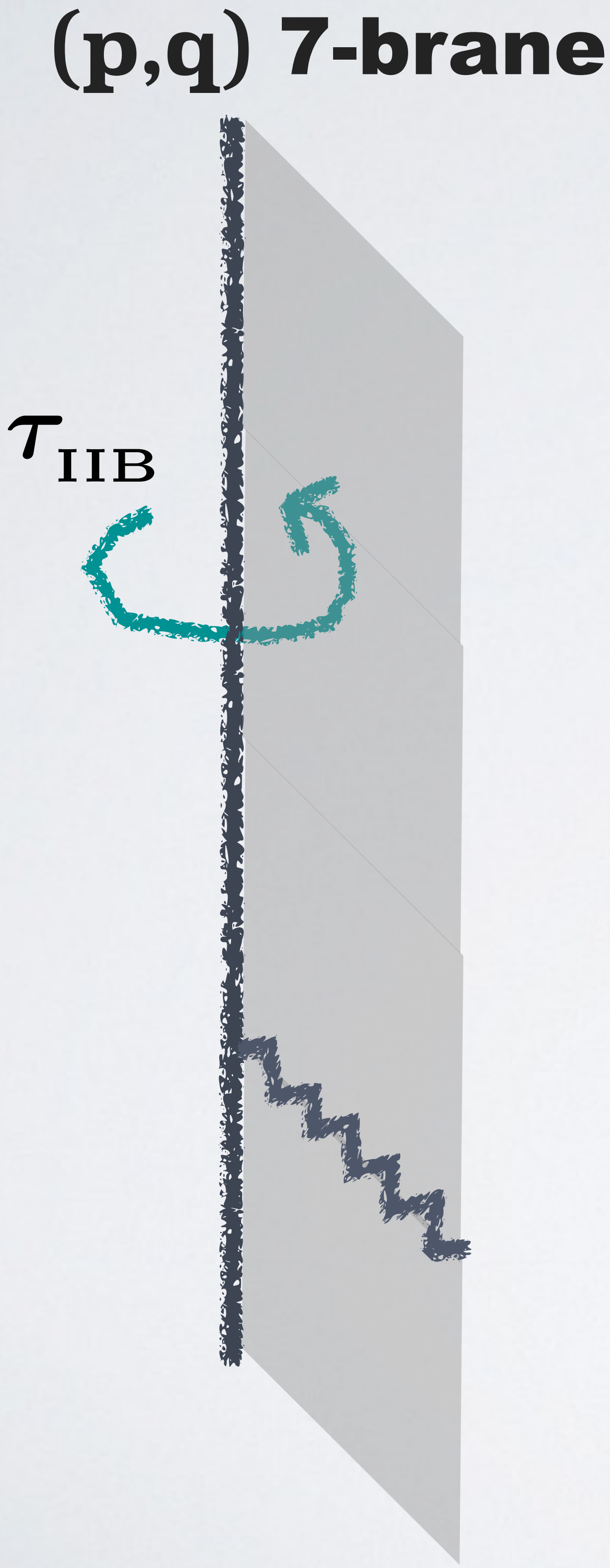
7-brane monodormy



a D7 induces

$$\tau_{\text{IIB}} \equiv a(x) + ie^{-\phi(x)} = \frac{1}{2\pi i} \log z$$

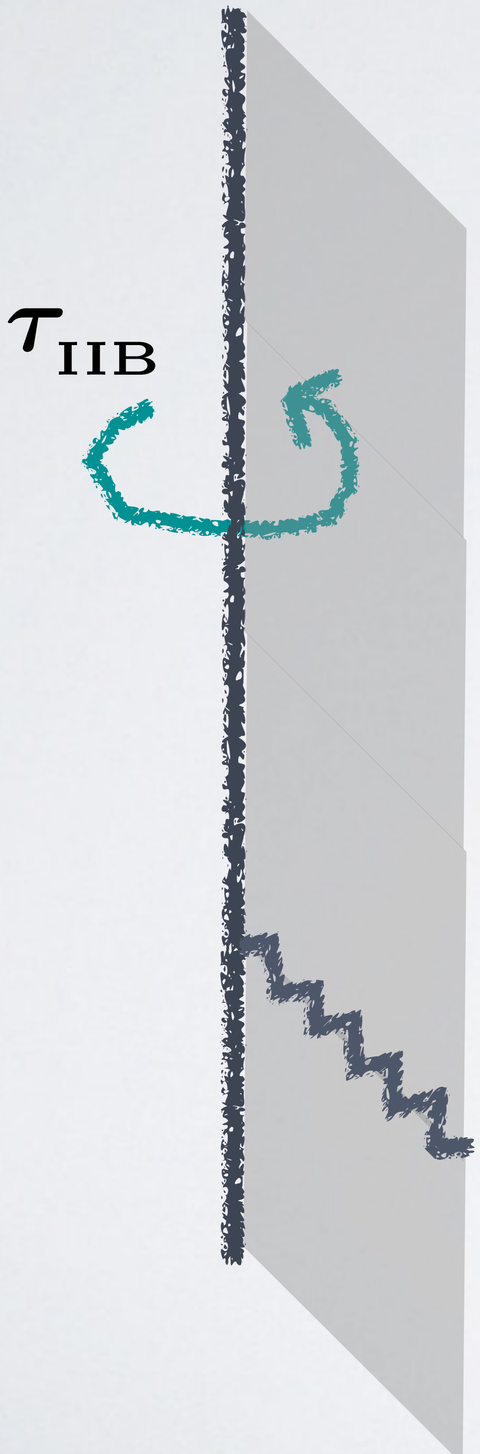
7-brane monodormy



$$\tau_{\mathrm{IIB}} \longrightarrow \frac{a\,\tau_{\mathrm{IIB}} + b}{c\,\tau_{\mathrm{IIB}} + d}$$

7-brane monodormy

(p,q) 7-brane

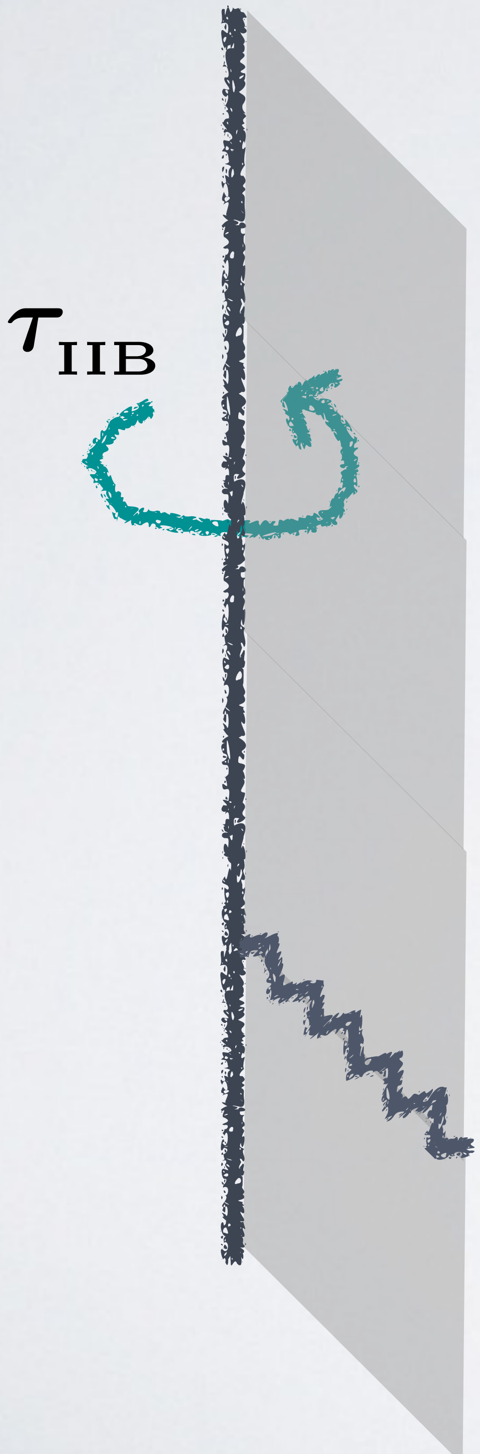


$$\tau_{\mathrm{IIB}} \longrightarrow \frac{a\,\tau_{\mathrm{IIB}} + b}{c\,\tau_{\mathrm{IIB}} + d}$$

$$\begin{pmatrix} 1 + pq & -p^2 \\ q^2 & 1 - pa \end{pmatrix} \in SL(2, \mathbb{Z})$$

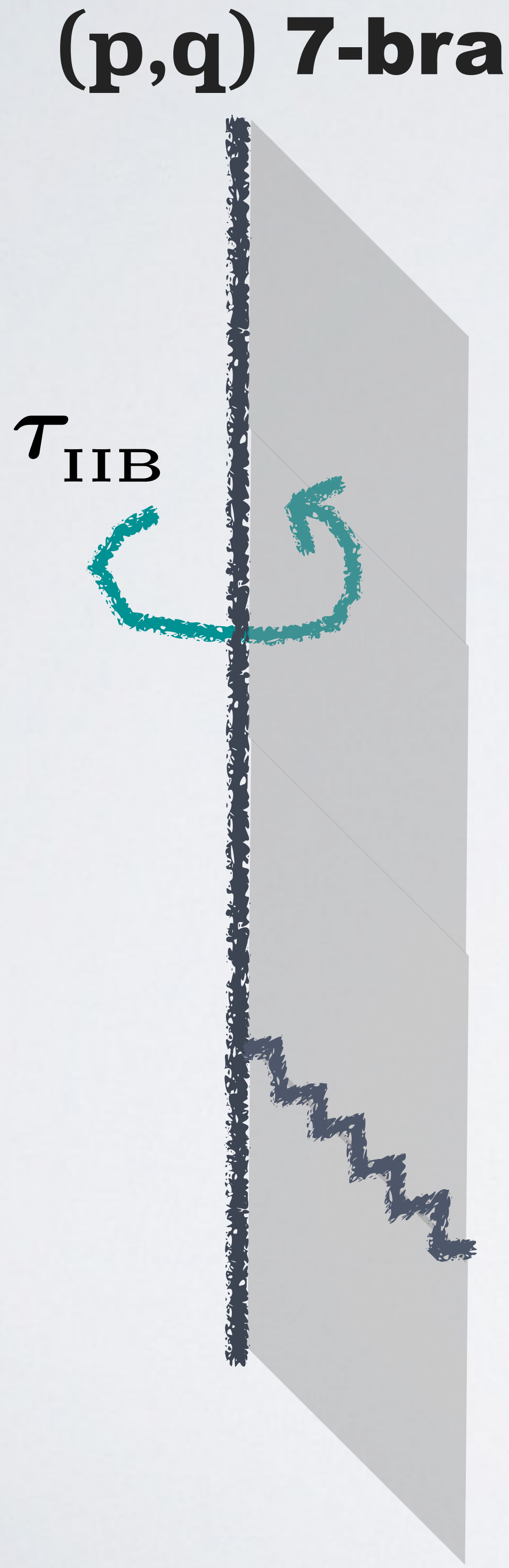
7-brane monodormy

(p,q) 7-brane



$$\tau_{\mathrm{IIB}} \longrightarrow \frac{a\,\tau_{\mathrm{IIB}} + b}{c\,\tau_{\mathrm{IIB}} + d} \xrightarrow{\text{SL(2,Z) duality}} \tau_{\mathrm{IIB}}$$

7-brane monodormy

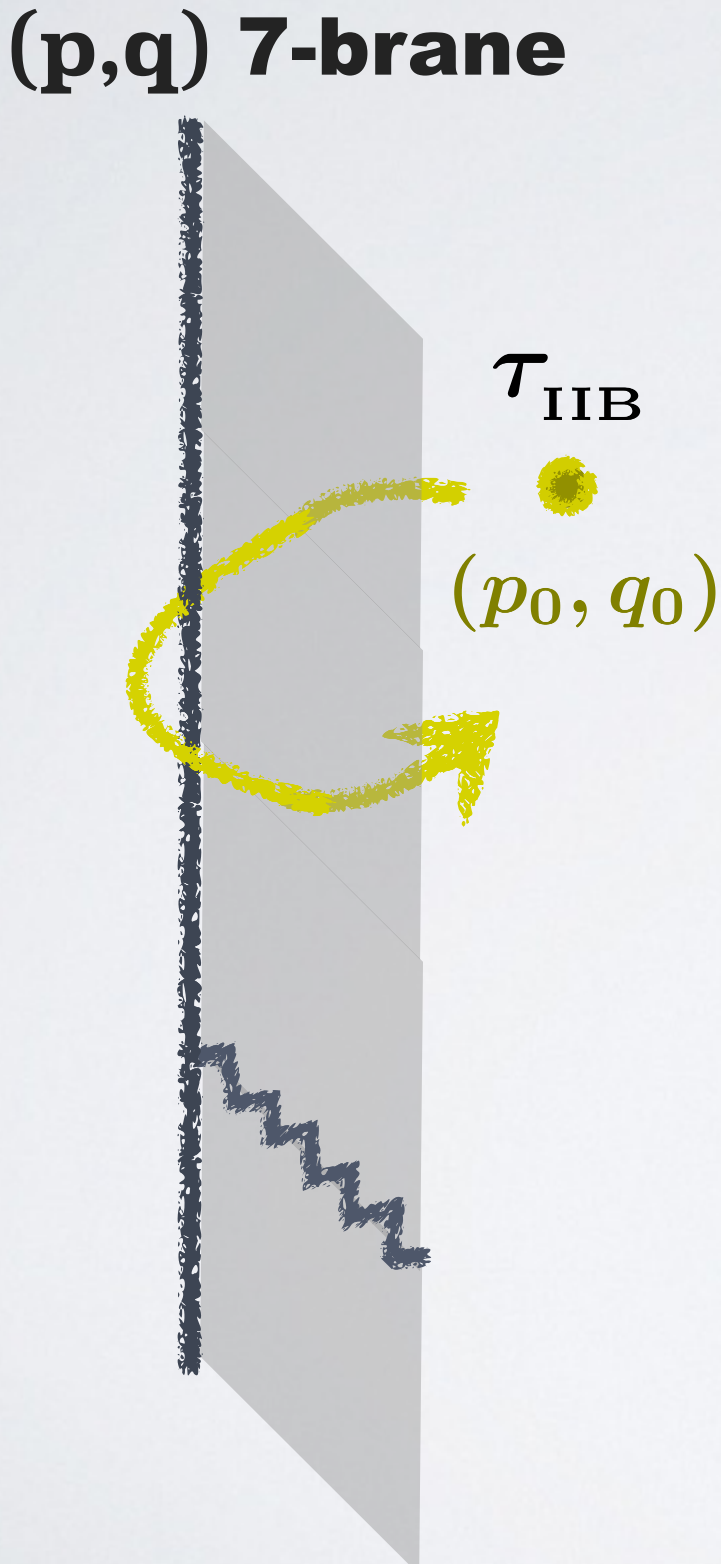


$$\tau_{\mathrm{IIB}} \longrightarrow \frac{a\,\tau_{\mathrm{IIB}} + b}{c\,\tau_{\mathrm{IIB}} + d} \quad \xrightarrow{\text{SL(2,Z) duality}} \quad \tau_{\mathrm{IIB}}$$

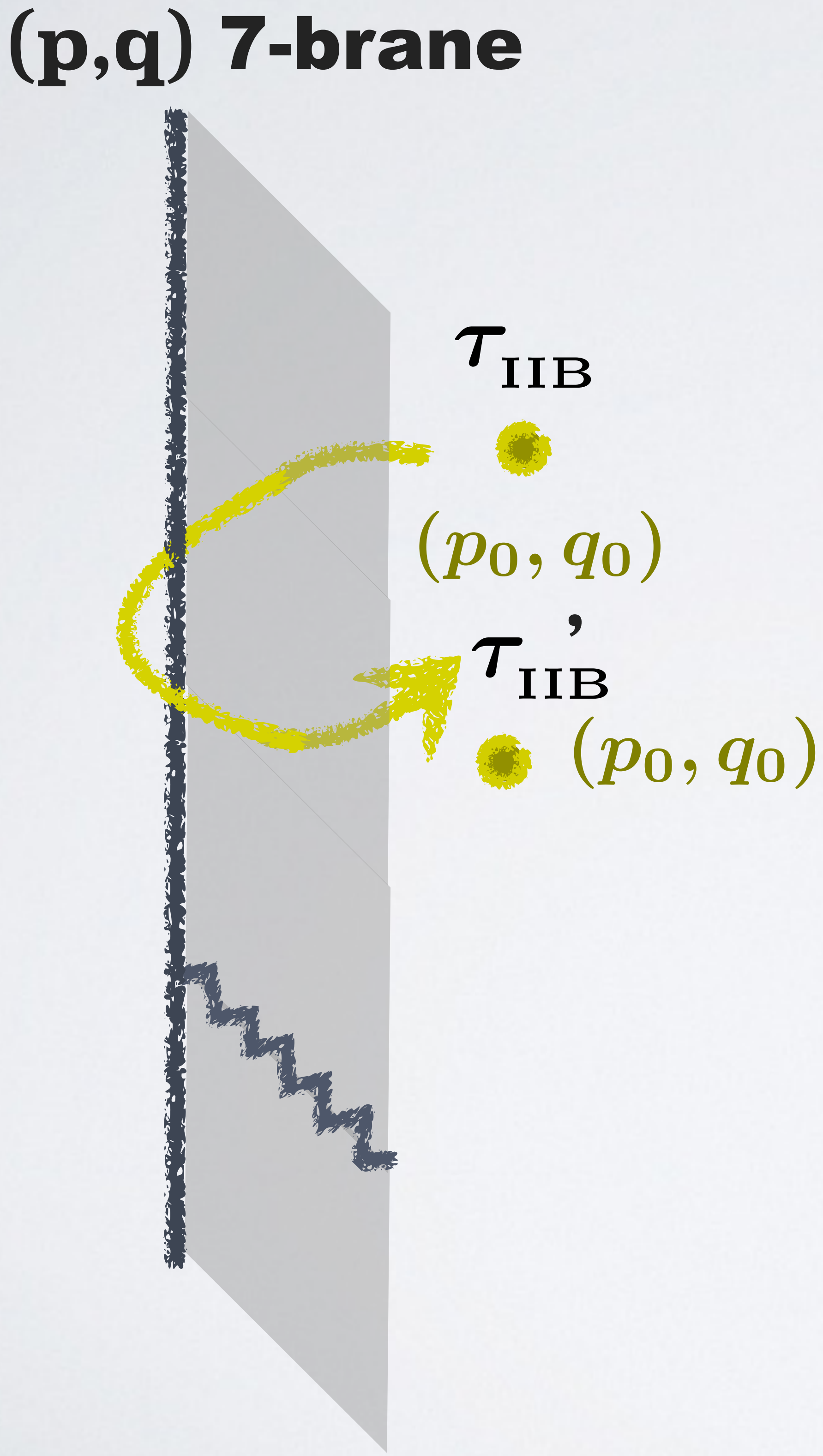
$\Downarrow$

the (p,q) charges change

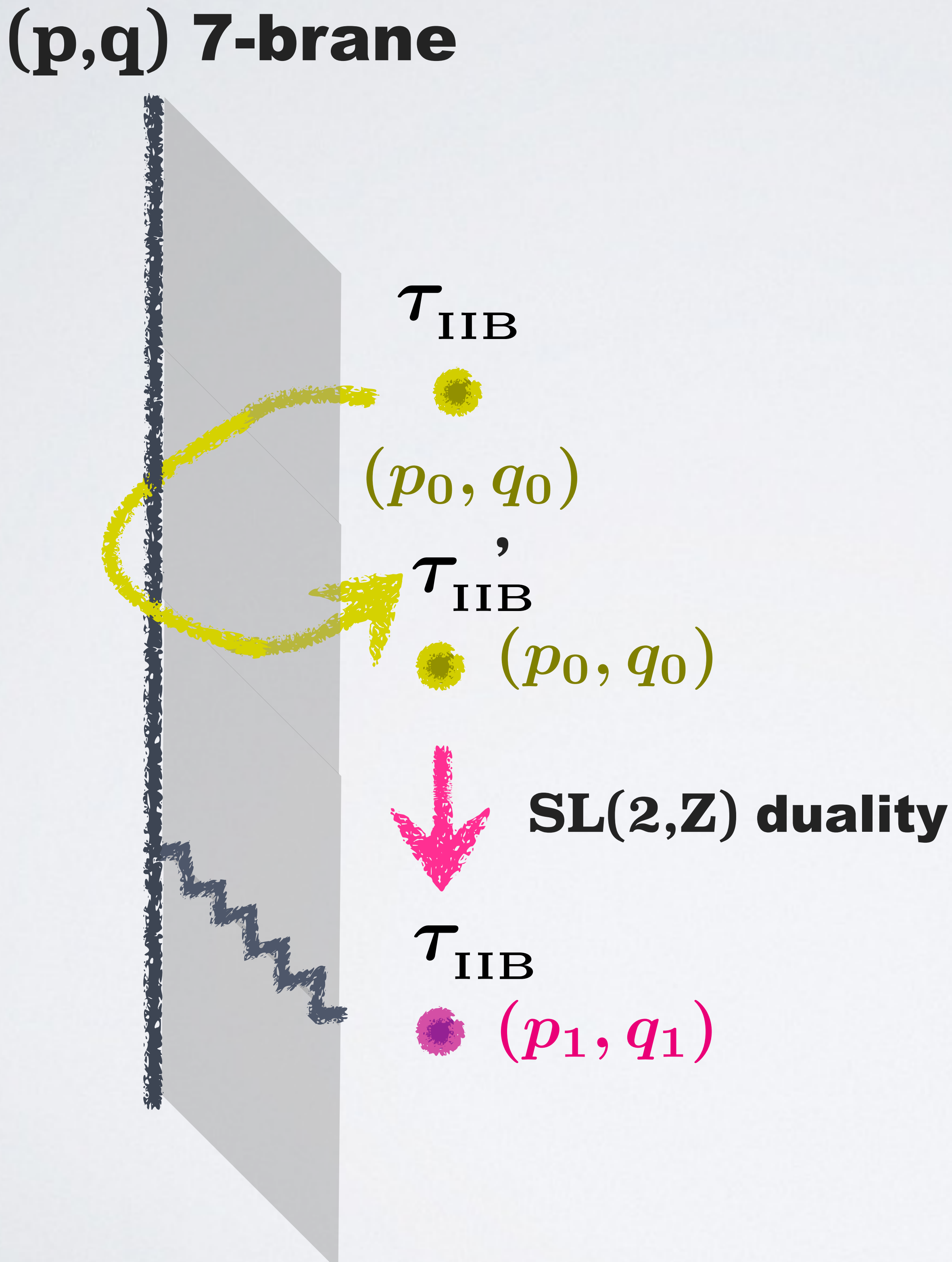
7-brane monodormy



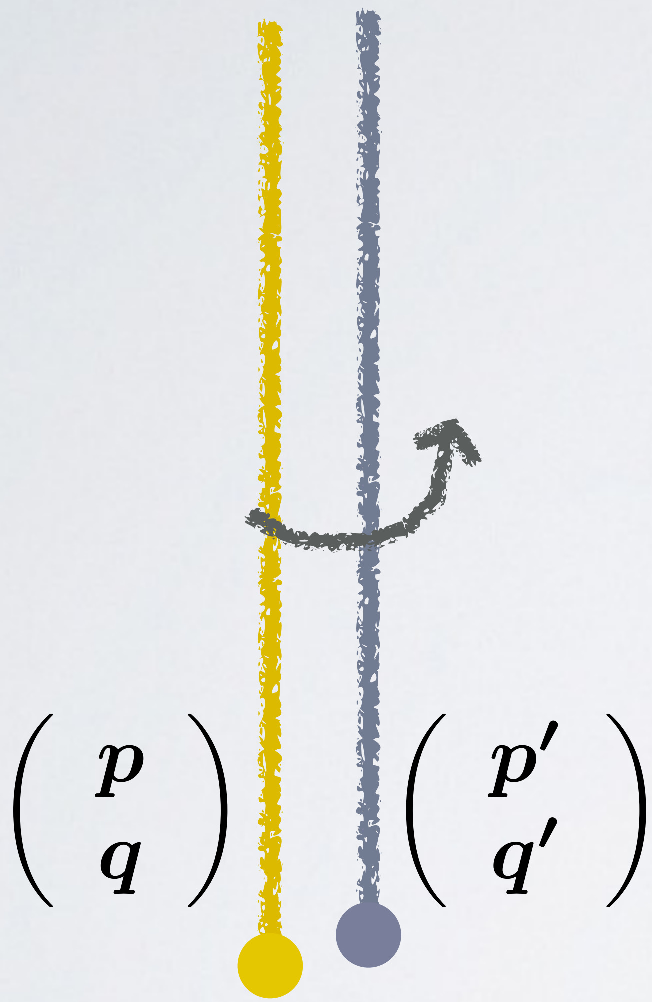
7-brane monodormy



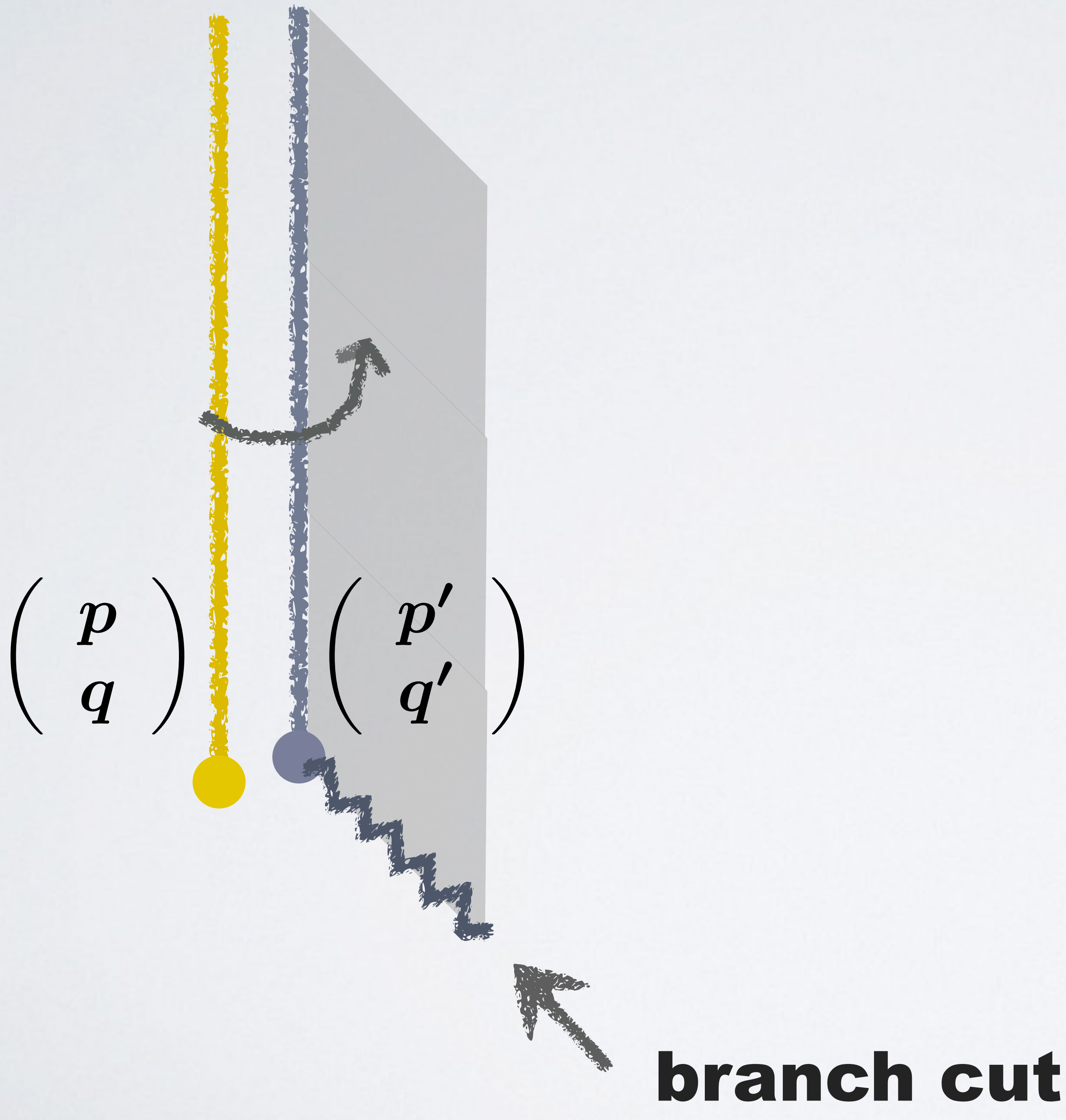
7-brane monodormy



7-brane monodormy

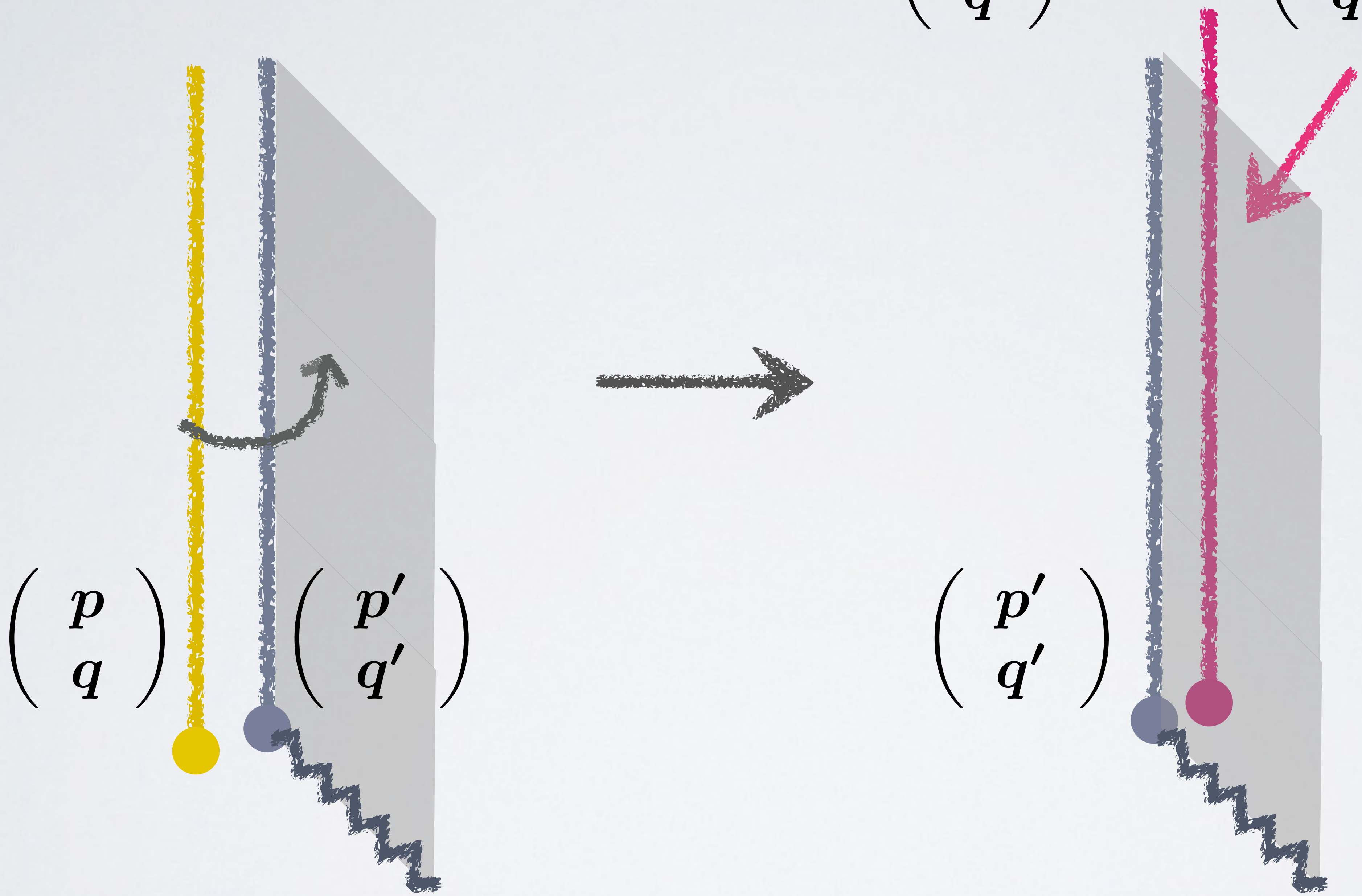


7-brane monodormy

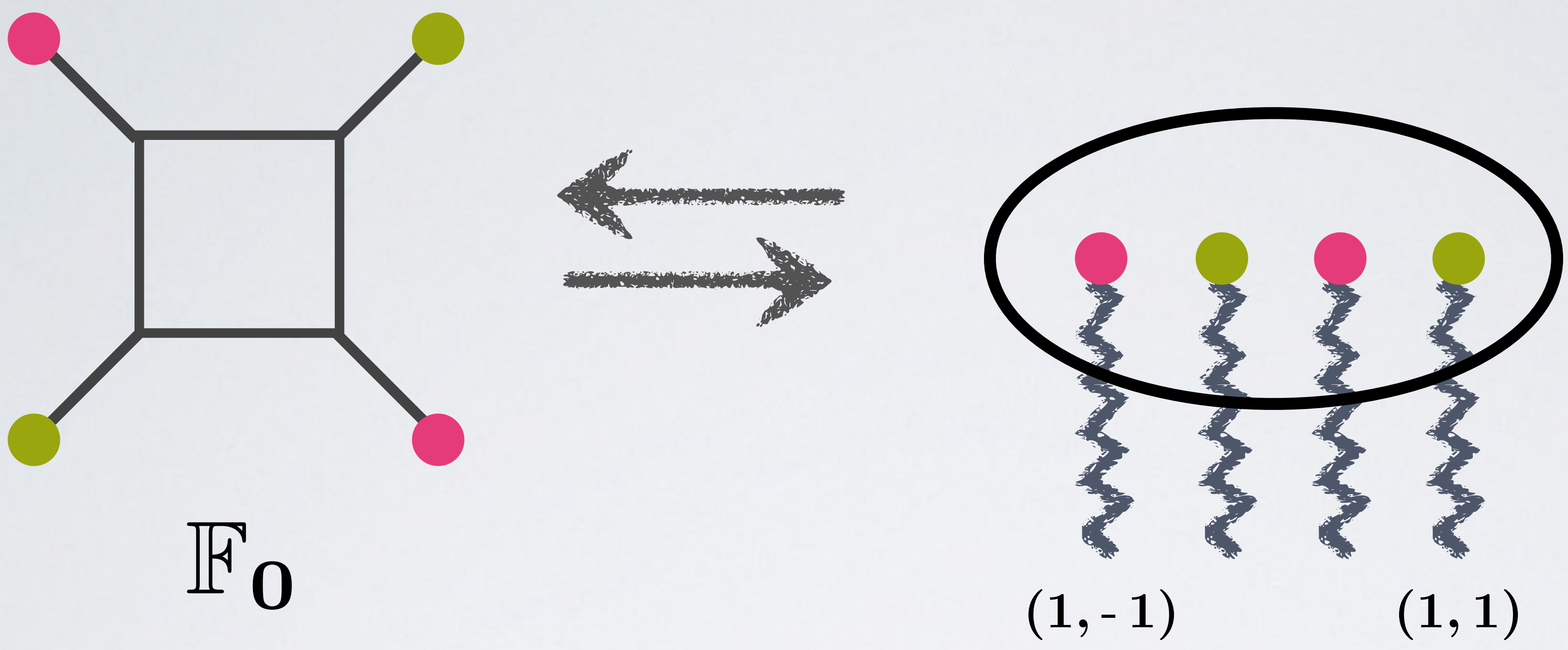


7-brane monodormy : Picard-Lefschetz tr.

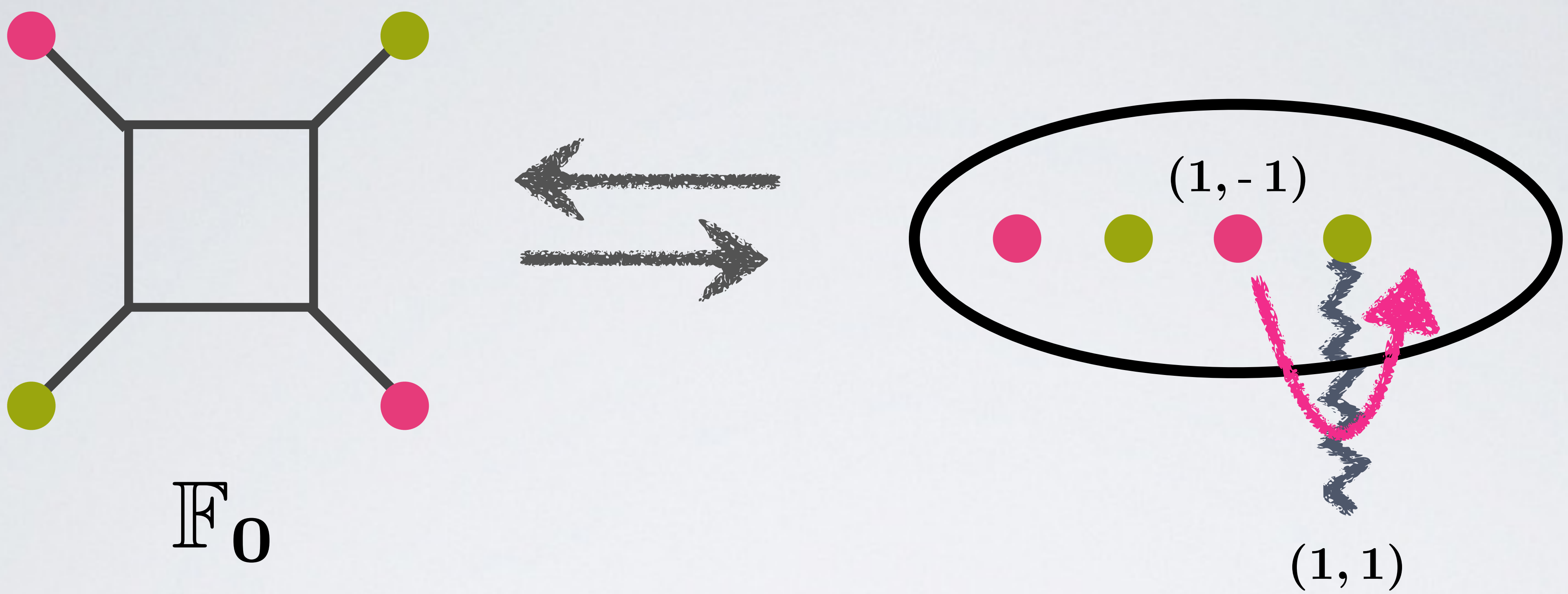
$$\begin{pmatrix} p \\ q \end{pmatrix} + \det \begin{pmatrix} p & p' \\ q & q' \end{pmatrix} \begin{pmatrix} p' \\ q' \end{pmatrix}$$



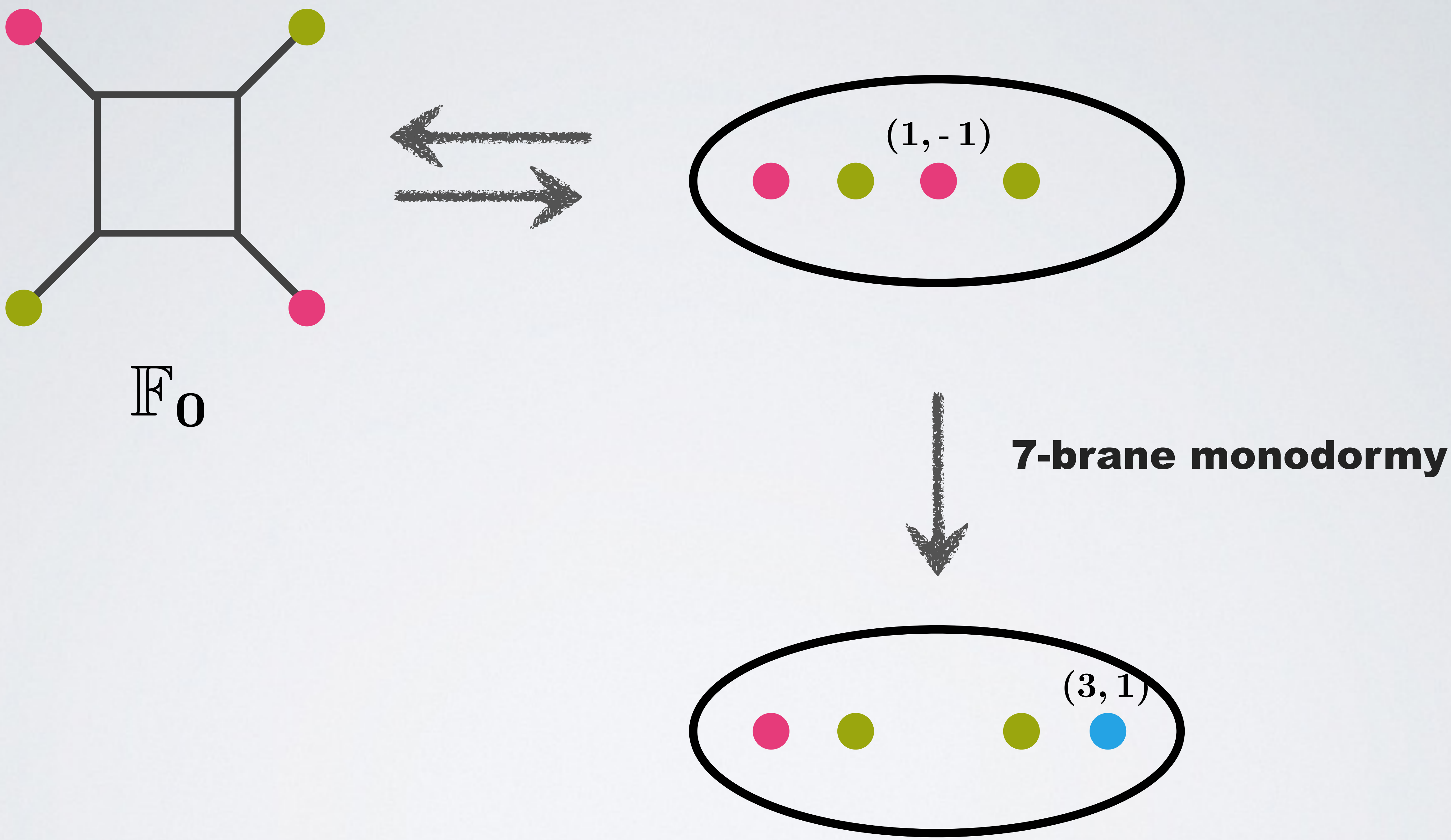
"Seiberg duality" for $\mathbb{F}_0$



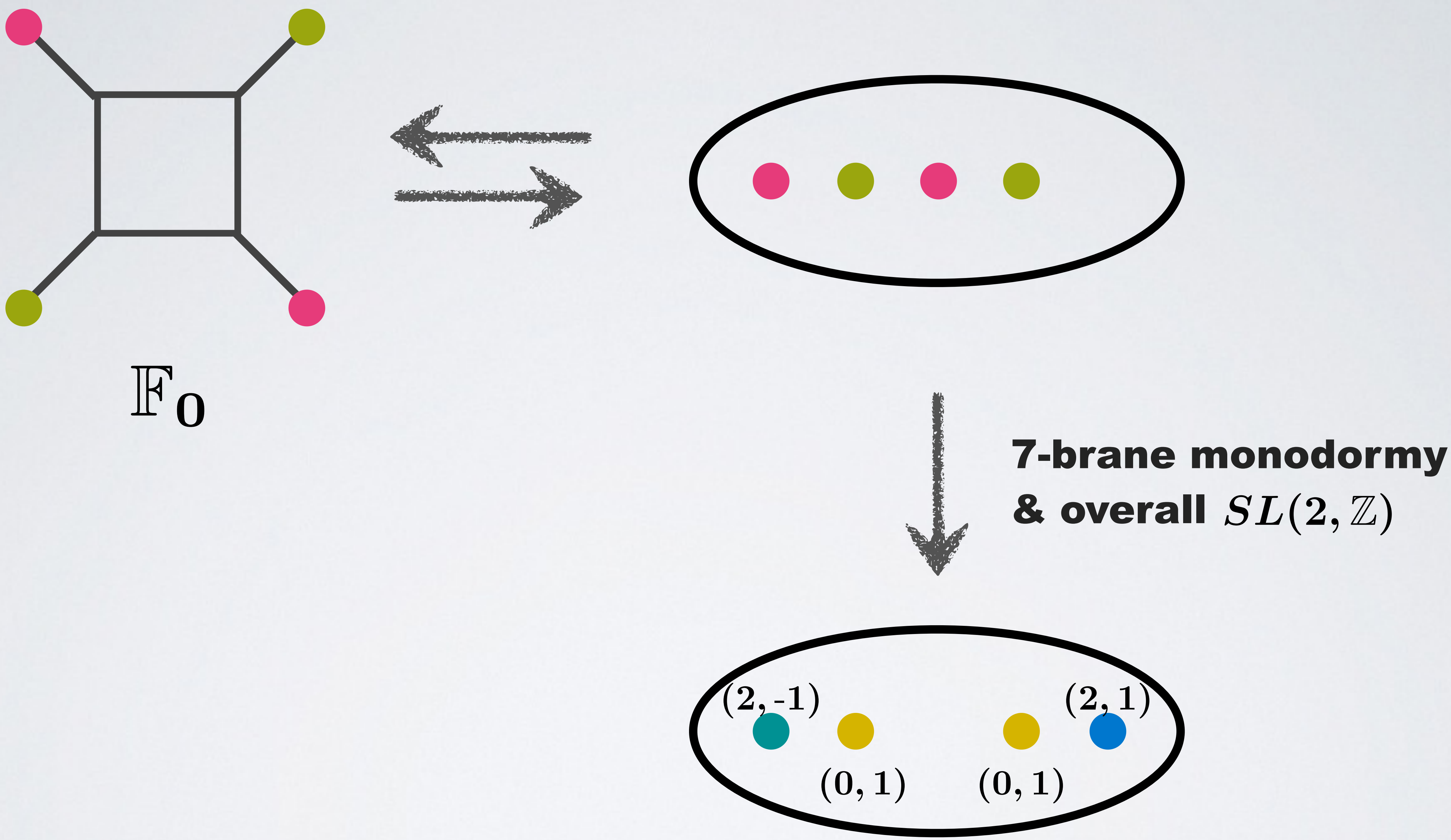
"Seiberg duality" for $\mathbb{F}_0$



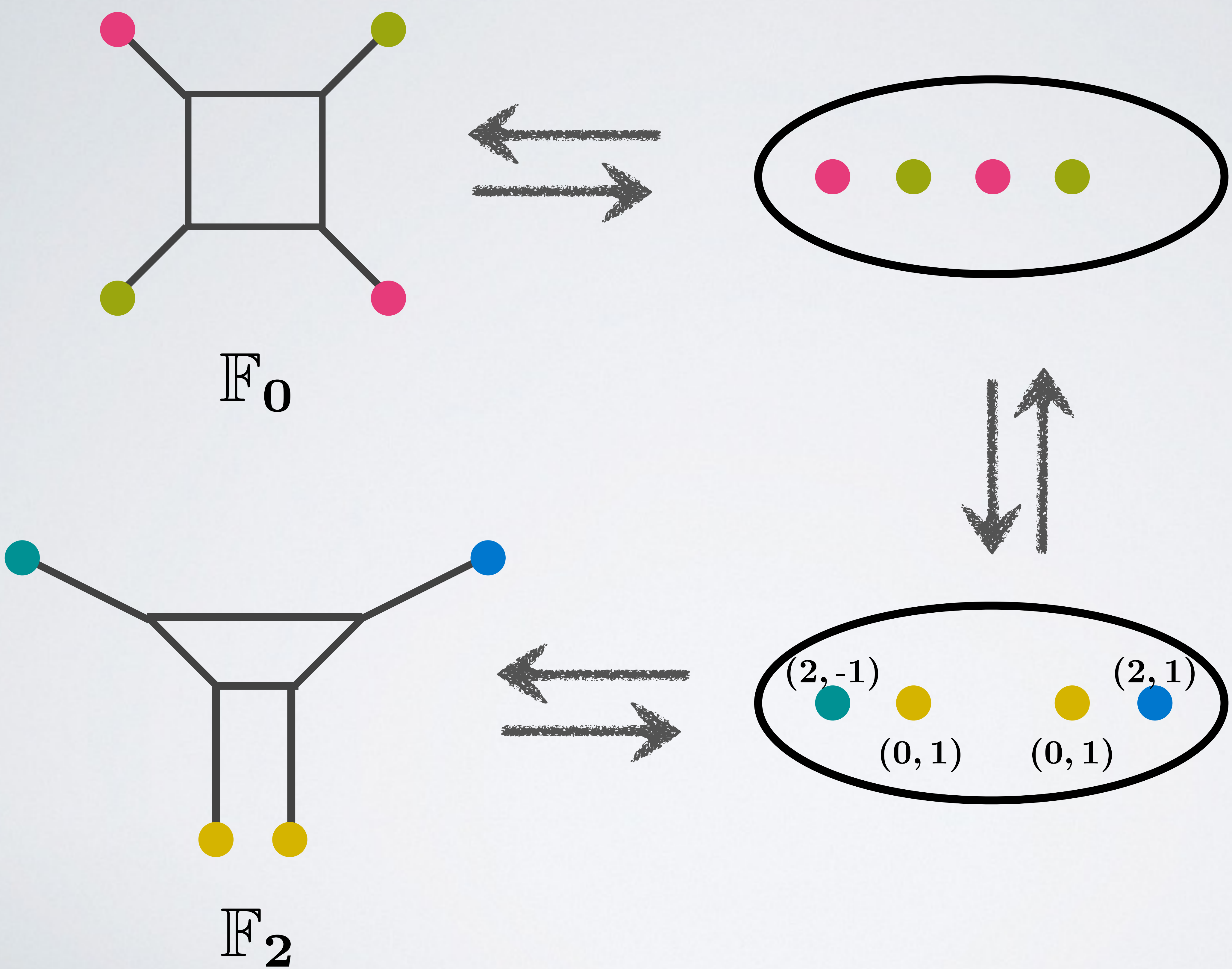
“Seiberg duality” for $\mathbb{F}_0$



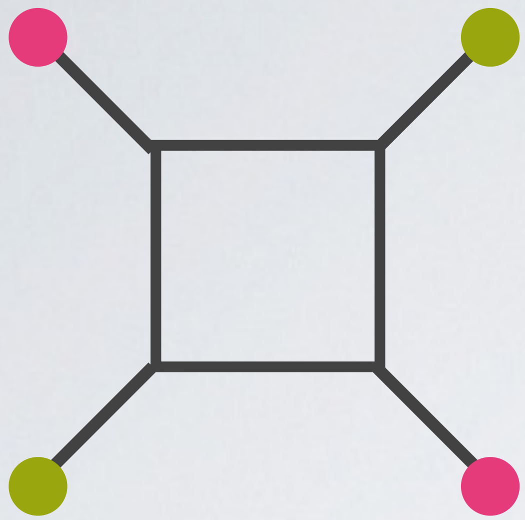
"Seiberg duality" for $\mathbb{F}_0$



"Seiberg duality" for $\mathbb{F}_0$

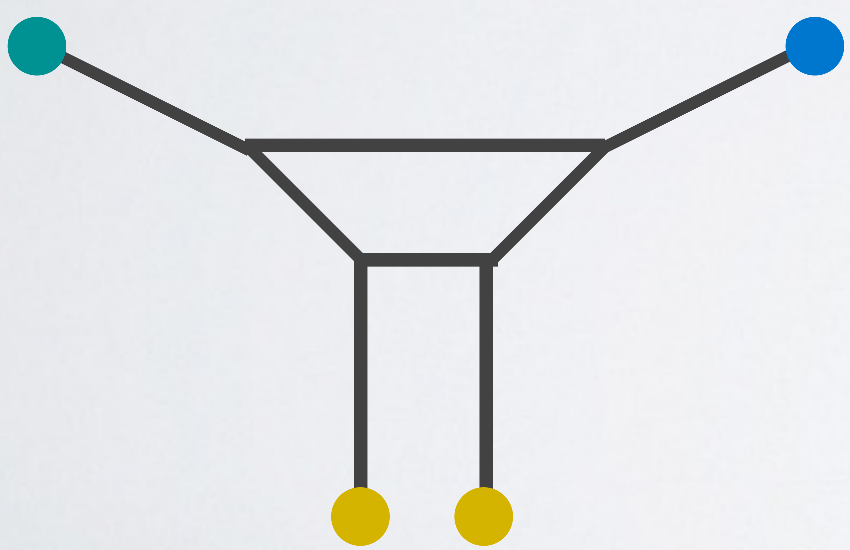


“Seiberg duality” for $\mathbb{F}_0$



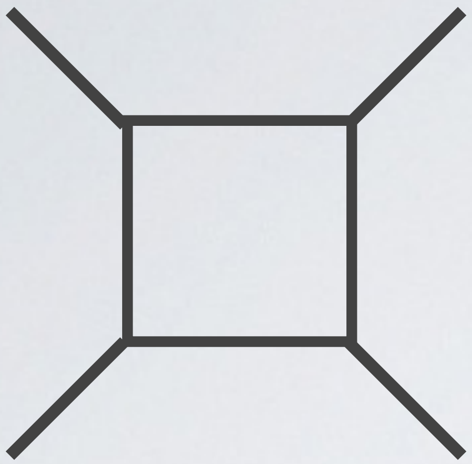
$\mathbb{F}_0$

same (5d) theory !!



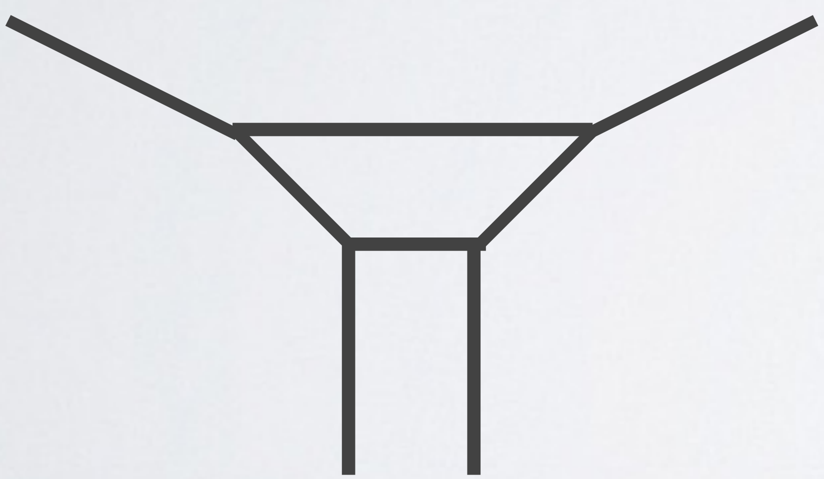
$\mathbb{F}_2$

“Seiberg duality” between webs themselves?



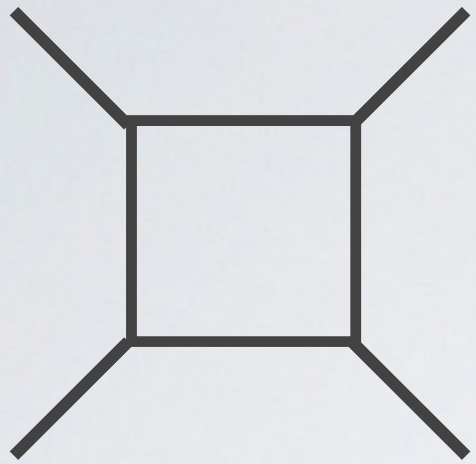
$\mathbb{F}_0$

BUT, a problem arises



$\mathbb{F}_2$

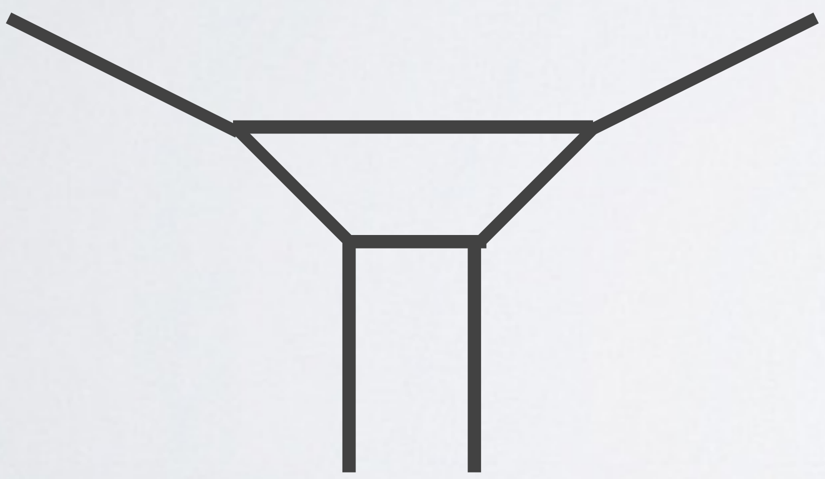
"Seiberg duality" between webs themselves?



$\mathbb{F}_0$

BUT, a problem arises

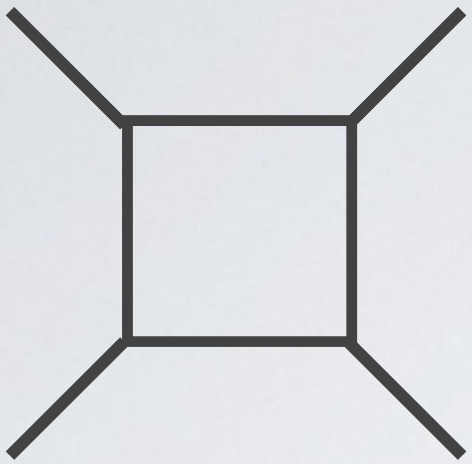
$$Z_{\mathbb{F}_0} \neq Z_{\mathbb{F}_2} \; !?$$



$\mathbb{F}_2$

This is a simplest version of SU(2) inconsistency

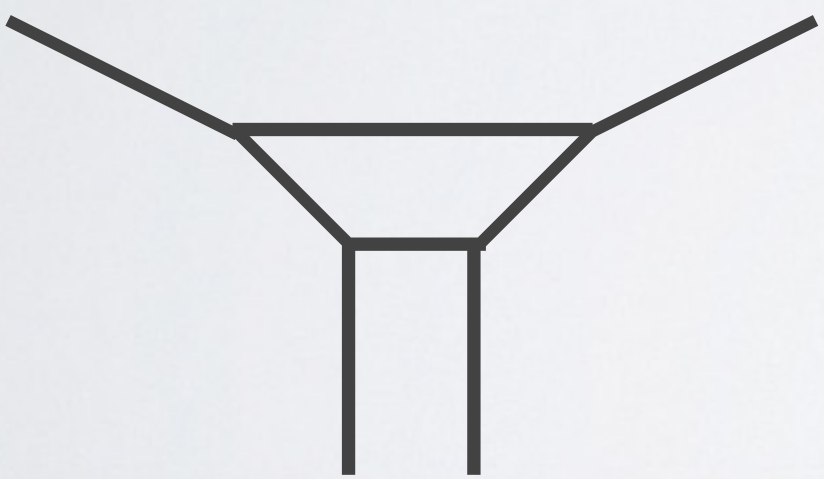
“Seiberg duality” between webs themselves?



$\mathbb{F}_0$

BUT, a problem arises 

$$\boxed{Z_{\mathbb{F}_0} \neq Z_{\mathbb{F}_2} !?}$$



$\mathbb{F}_2$

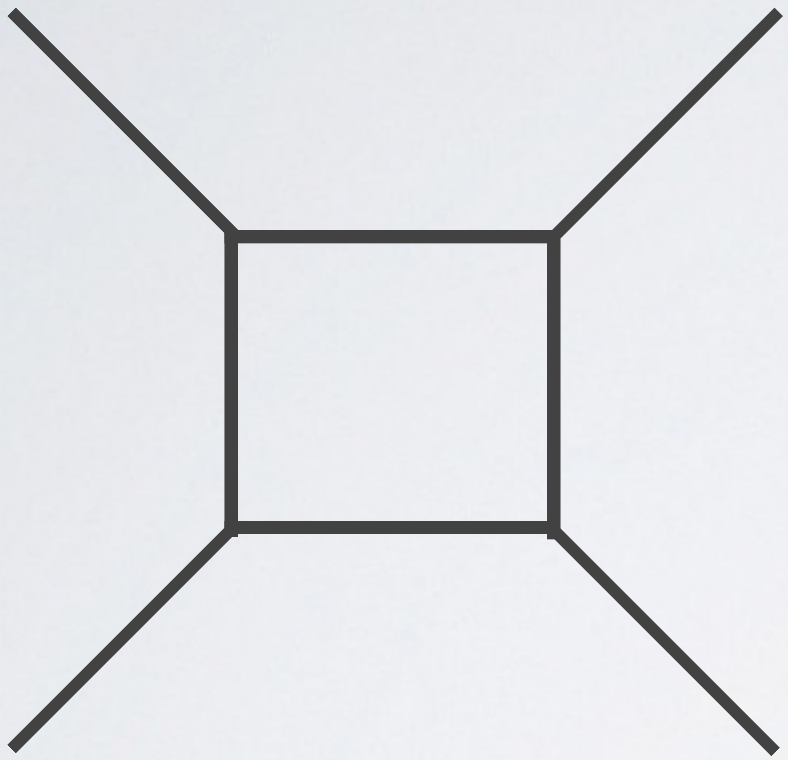
This is a simplest version of SU(2) inconsistency

We can resolve this inconsistency

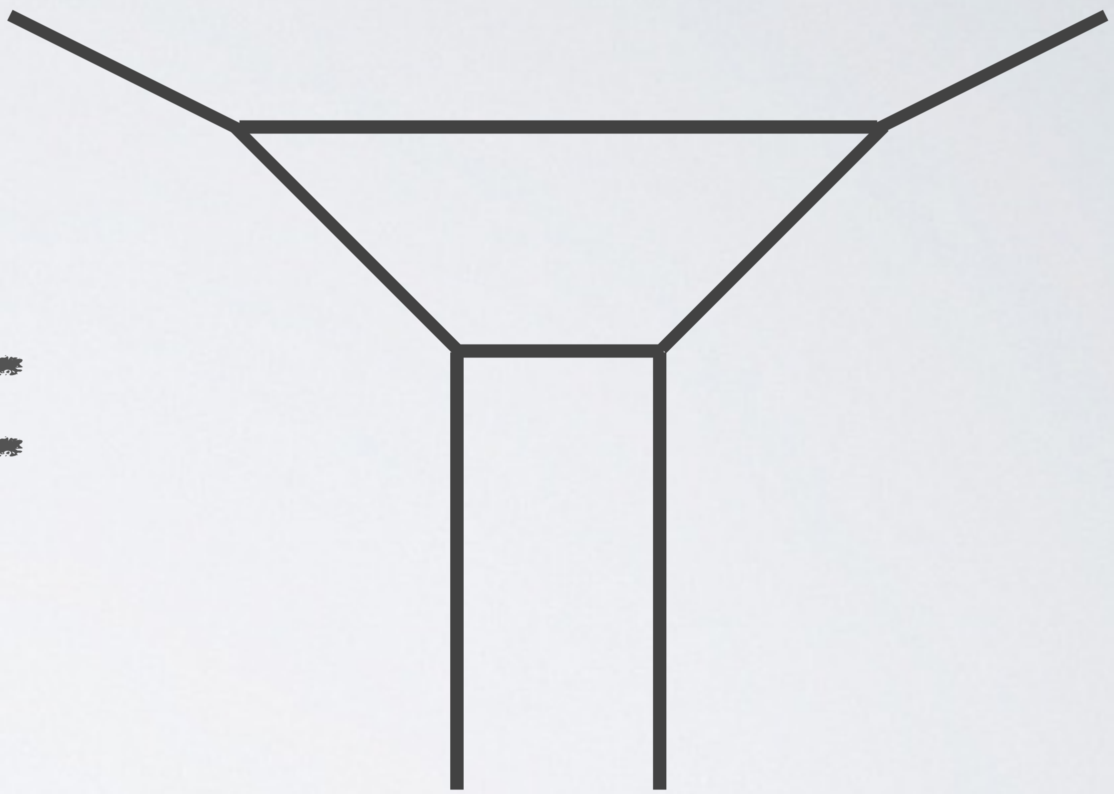
Why does SU(2) cause problem?

[MT, ’13, ’14]
[Bao-Mitev-Pomoni-MT-Yagi, ’13]
[Hayashi-Kim-Nishinaka, ’13]

5d gauge theory & extra contribution

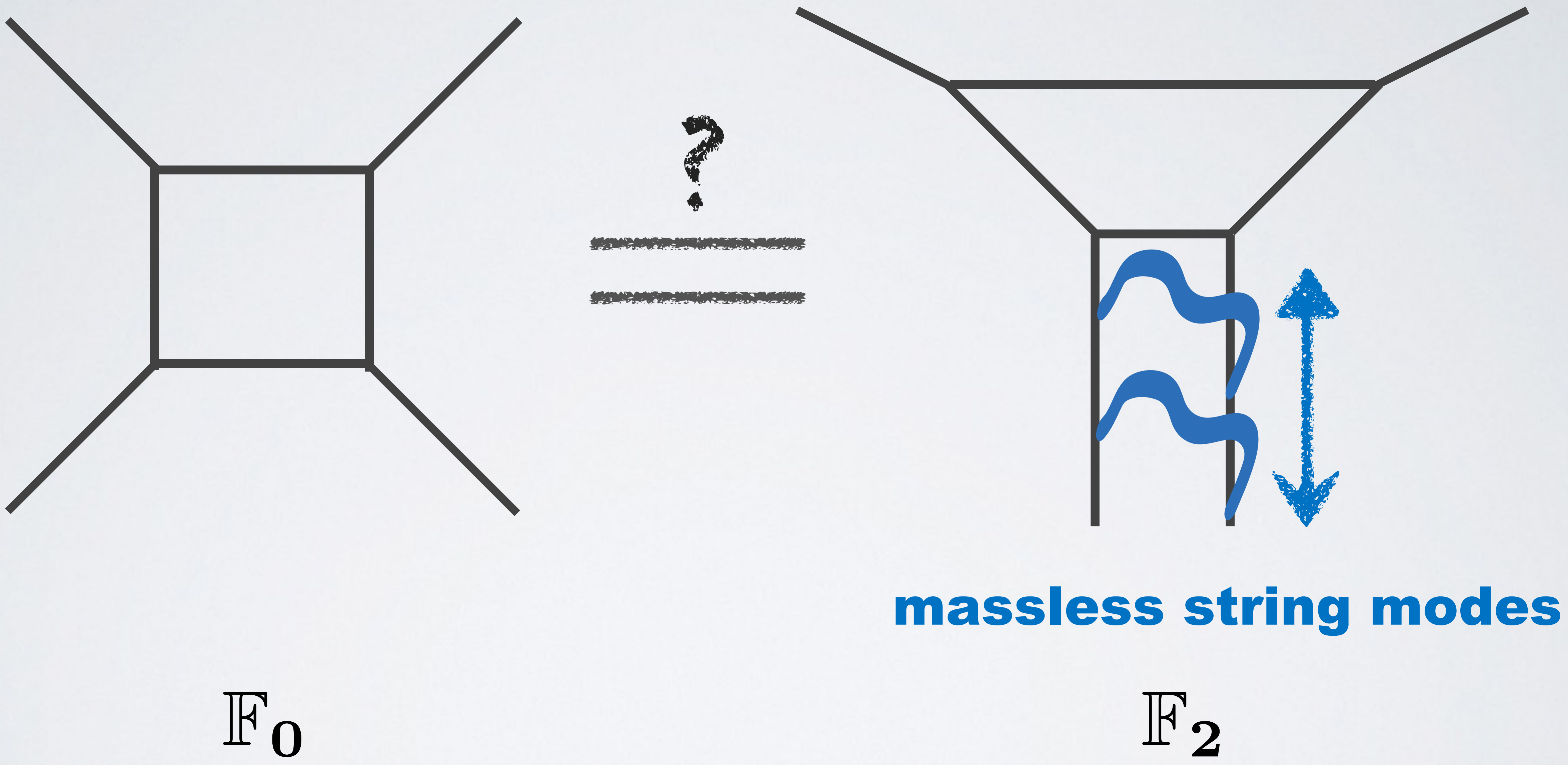


$$\stackrel{?}{=}$$

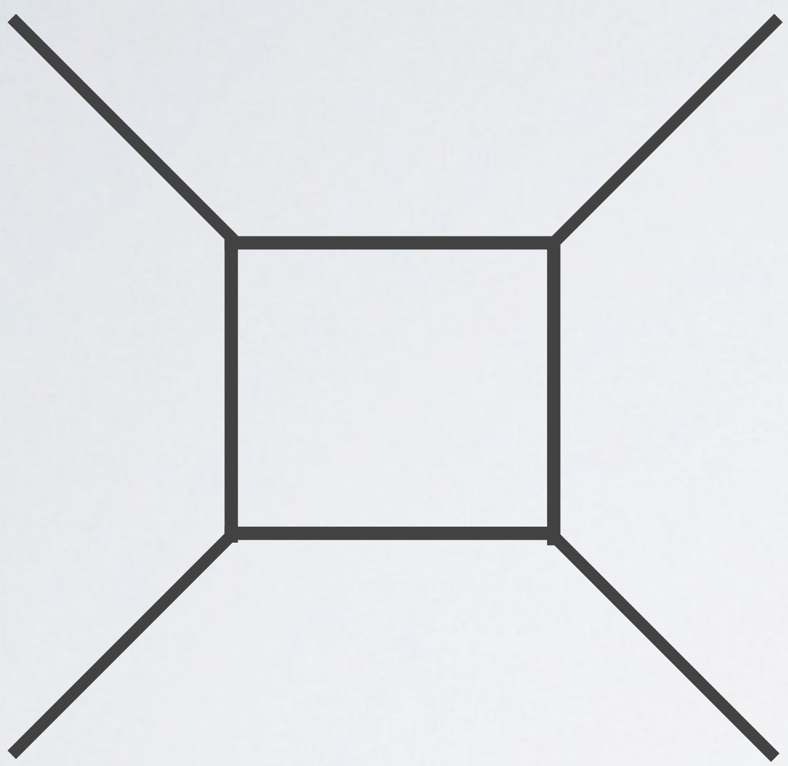


$\mathbb{F}_0$ $\mathbb{F}_2$

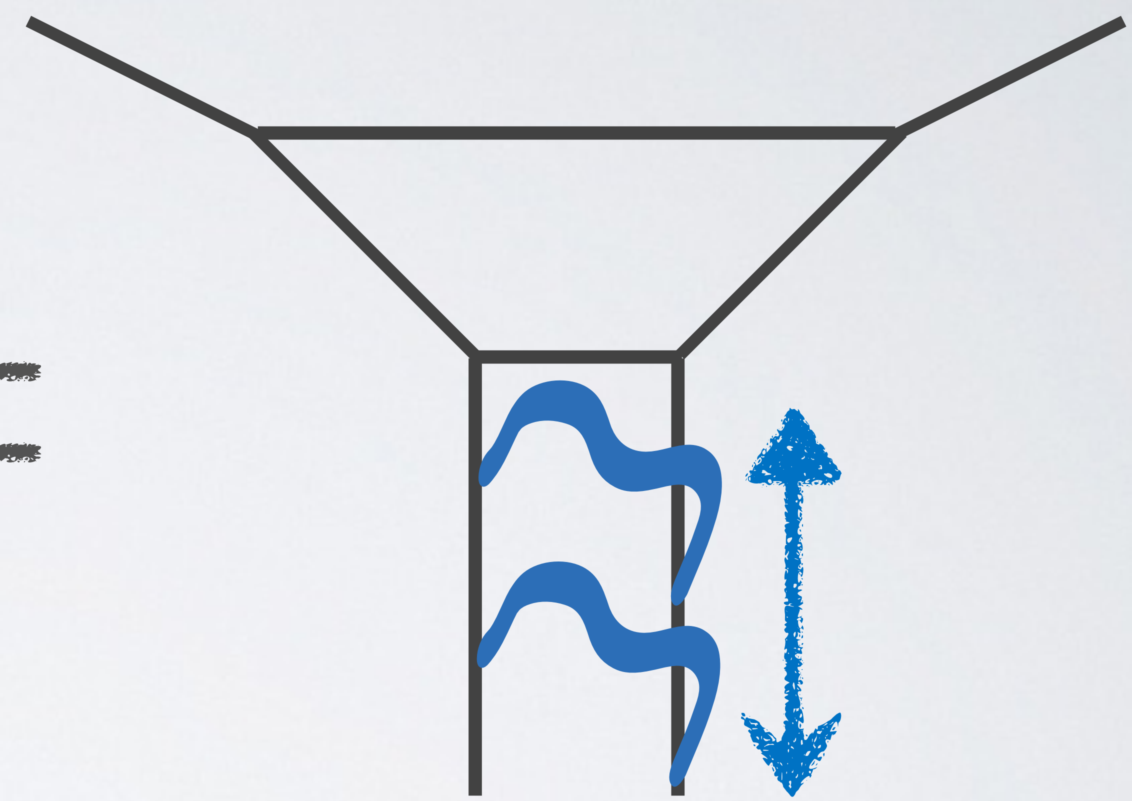
5d gauge theory & extra contribution



5d gauge theory & extra contribution



$\stackrel{?}{=}$



massless string modes

$\mathbb{F}_0$

$\mathbb{F}_2$

$= \mathbb{F}_0$ + extra contribution

5d gauge theory & extra contribution

Claim (factorization conjecture)

[BMPTY] [HKN]

$$Z^{SU(2)}_{Nek} = \frac{Z_{web}}{Z_{extra}}$$

extra contribution is **factored out** ("decoupled")

5d gauge theory & extra contribution

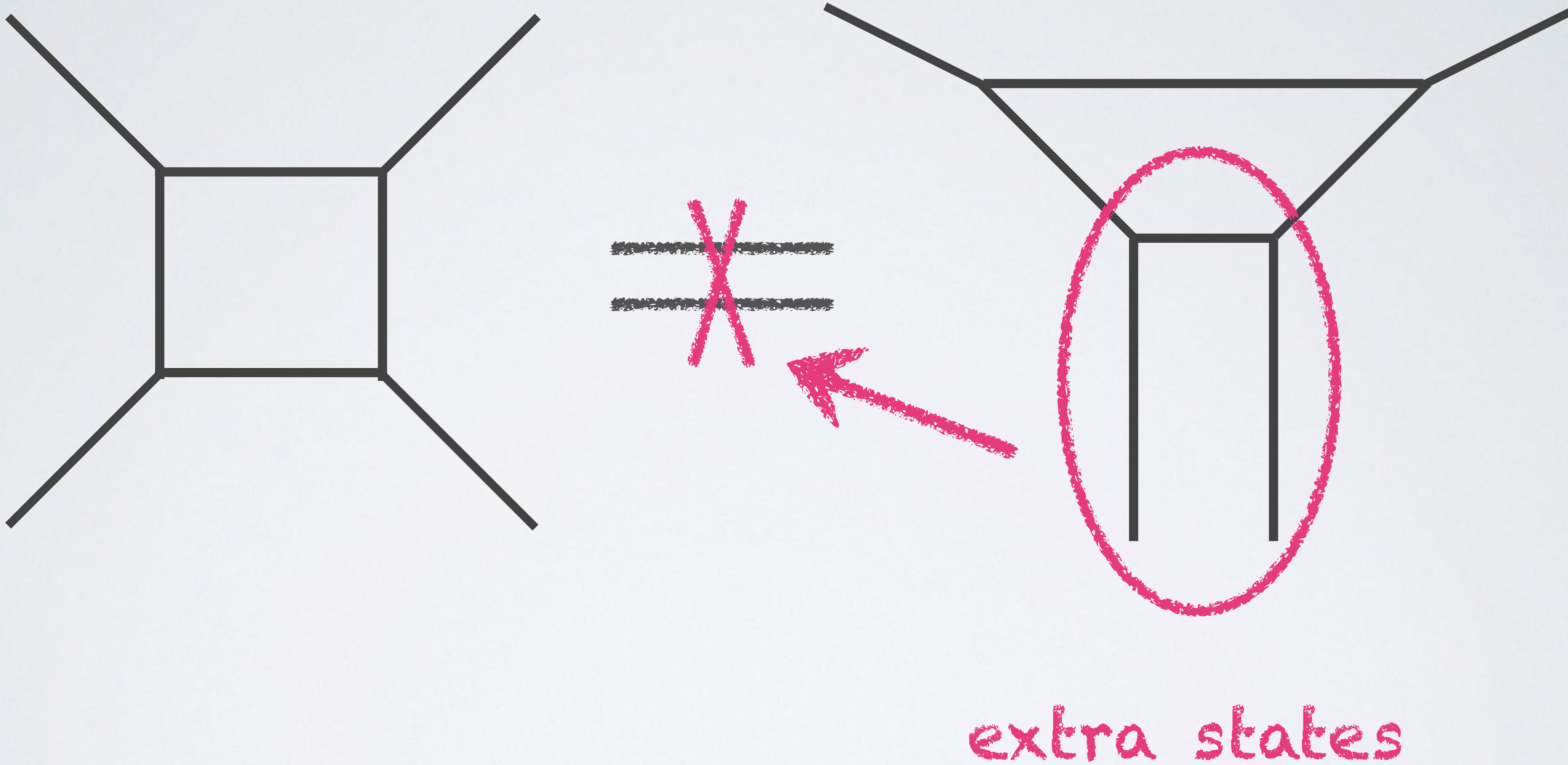
Claim (factorization conjecture)

[BMPTY] [HKN]

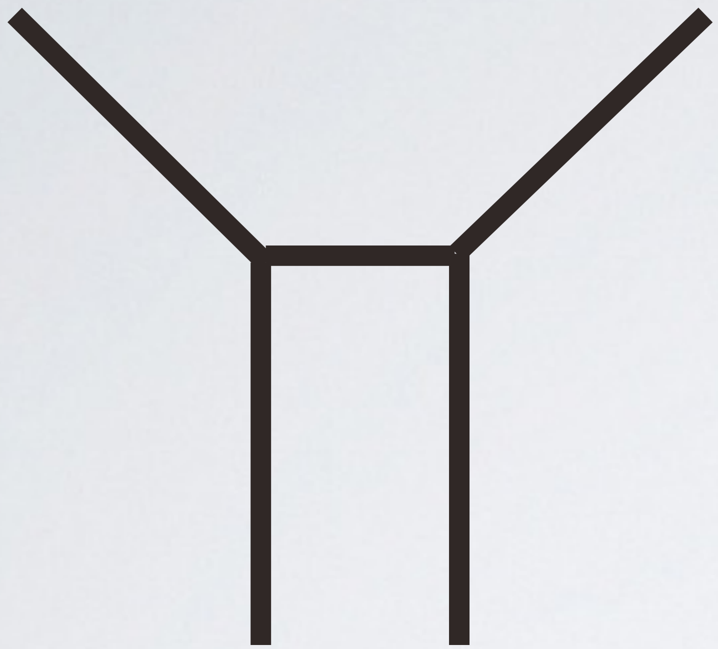
$$\boxed{Z^{SU(2)}_{Nek} = \frac{Z_{web}}{Z_{extra}}}$$

Q. How can we compute Z_{extra}??

Extra contribution: typical example

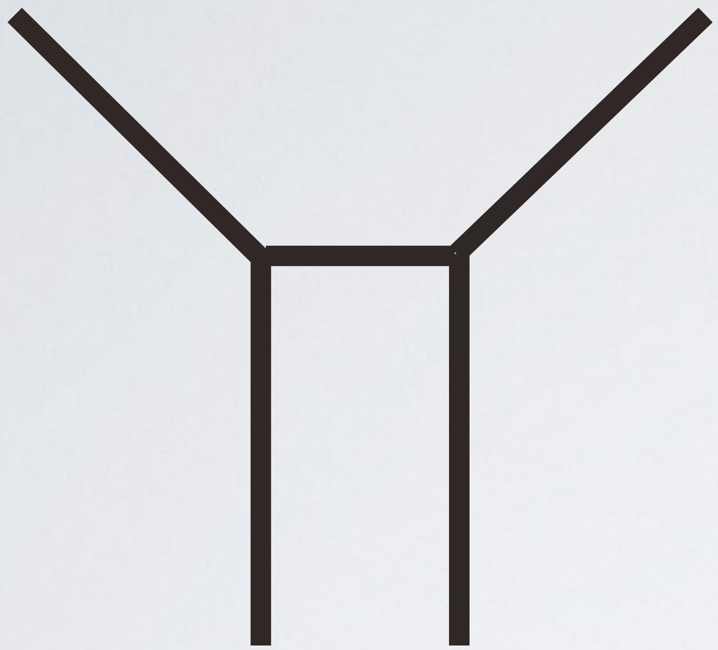


Extra contribution: typical example



$$= Z_{extra}$$

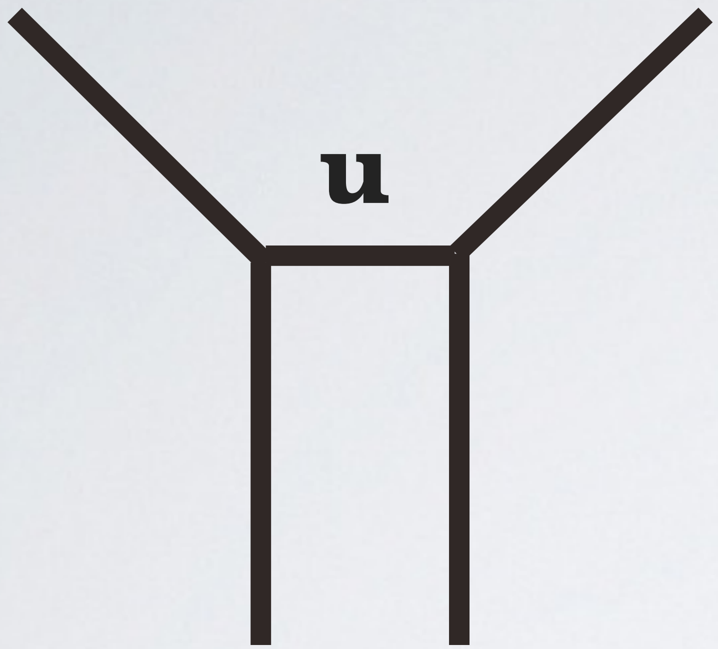
Extra contribution: typical example



$$= Z_{extra}$$

we can employ topological string 

Extra contribution: typical example



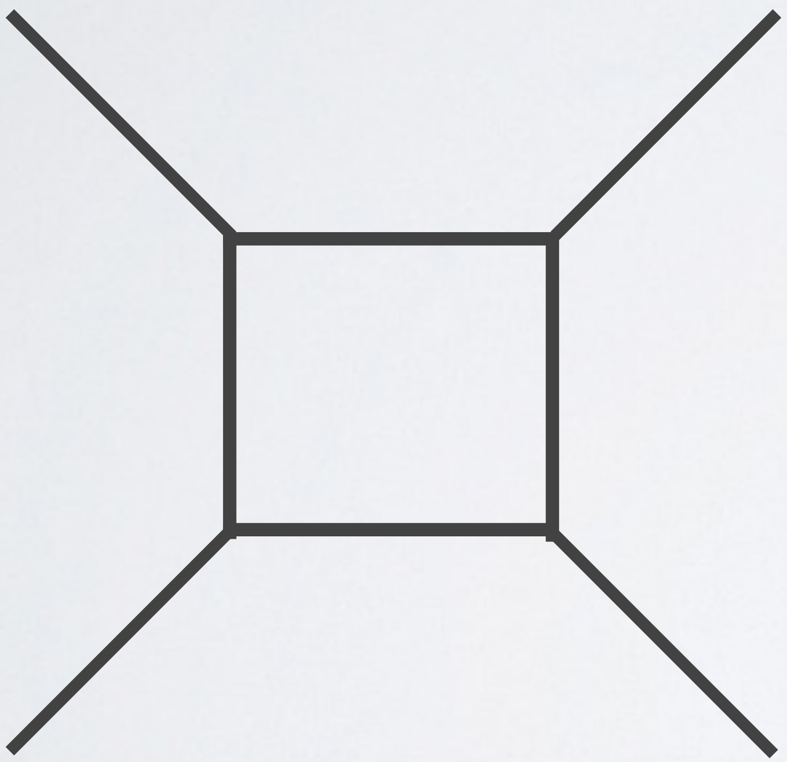
$$= Z_{extra}$$

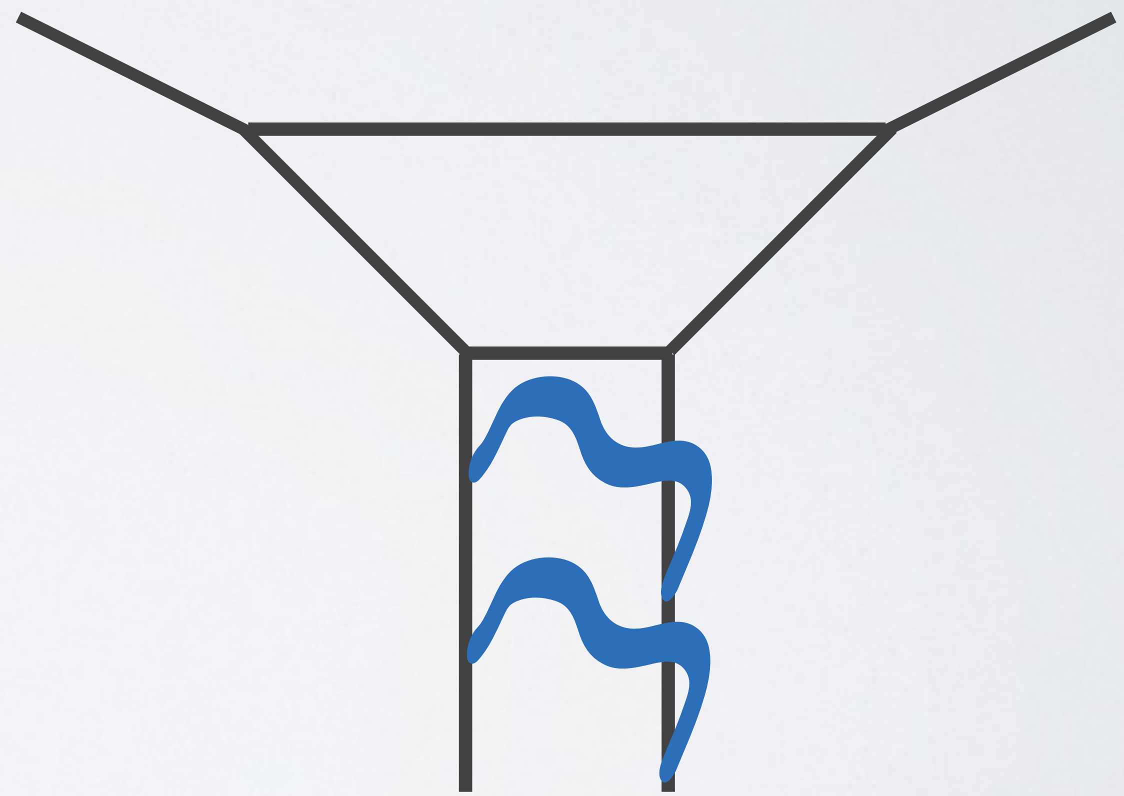
$$= \sum_{\lambda} \left(-u \sqrt{\frac{q}{t}} \right)^{|\lambda|} P_{\lambda^t}(t^{\rho}; q; t) P_{\lambda}(q^{\rho}; t; q)$$

$$= \prod_{i,j=1}^{\infty} \frac{1}{1 - u\, t^{i-1}\, q^{j}}$$

Conjecture [MT, '13] [Bergman-Gomez-Zafrir,'13]

$$\boxed{Z_{\mathbb{F}_0} = \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^j) \times Z_{\mathbb{F}_2}}$$





Conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

$$\boxed{Z_{\mathbb{F}_0} = \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^j) \times Z_{\mathbb{F}_2}}$$

I don't have mathematical proof

$$Z_{\mathbb{F}_0} = \sum_{R_{1,2}} \left(u \frac{q}{t}\right)^{|\vec{R}|} Z_{\vec{R}}^{\text{vect.}}(Q_F; t, q)$$

$$Z_{\mathbb{F}_2} = \sum_{R_{1,2}} \left(u \frac{q}{t}\right)^{|\vec{R}|} Z_{\vec{R}}^{\text{CS}, m=2}(Q_F; t, q)\, Z_{\vec{R}}^{\text{vect.}}(Q_F; t, q)$$

Conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

$$\boxed{Z_{\mathbb{F}_0} = \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^{j}) \times Z_{\mathbb{F}_2}}$$

I don't have mathematical proof

$$Z = \sum_{k=0}^{\infty} u^k\, Z^{\,k\text{-inst}}$$

Conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

Let us test at 1-instanton level

$$Z^{\text{1-inst}}_{\mathbb{F}_0}(Q_F;t,q) = u\frac{q}{t}\frac{1+\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

$$Z^{\text{1-inst}}_{\mathbb{F}_2}(Q_F;t,q) = u\left(\frac{q}{t}\right)^2\frac{Q_F+1+\frac{1}{Q_F}-\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

Conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

Let us test at 1-instanton level

$$Z_{\mathbb{F}_0}^{\text{1-inst}}(Q_F;t,q) = u\frac{q}{t}\frac{1+\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

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$$\longrightarrow \quad Z_{\mathbb{F}_0}(Q_F;t,q) - Z_{\mathbb{F}_2}(Q_F;t,q) = -u\frac{q}{(1-q)(1-t)}.$$

Conjecture [MT, ’13][Bergman-Gomez-Zafrir,’13]

Let us test at 1-instanton level

$$Z_{\mathbb{F}_0}^{\text{1-inst}}(Q_F;t,q) = u\frac{q}{t}\frac{1+\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

$$Z_{\mathbb{F}_2}^{\text{1-inst}}(Q_F;t,q) = u\left(\frac{q}{t}\right)^2\frac{Q_F+1+\frac{1}{Q_F}-\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

$$\longrightarrow \quad Z_{\mathbb{F}_0}(Q_F;t,q) - Z_{\mathbb{F}_2}(Q_F;t,q) = \boxed{-u\frac{q}{(1-q)(1-t)}}.$$

$$\longleftrightarrow \quad \prod_{i,j=1}^{\infty}(1-u\,t^{i-1}\,q^j) = 1 \boxed{- u\frac{q}{(1-q)(1-t)}} + \cdots$$

Conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

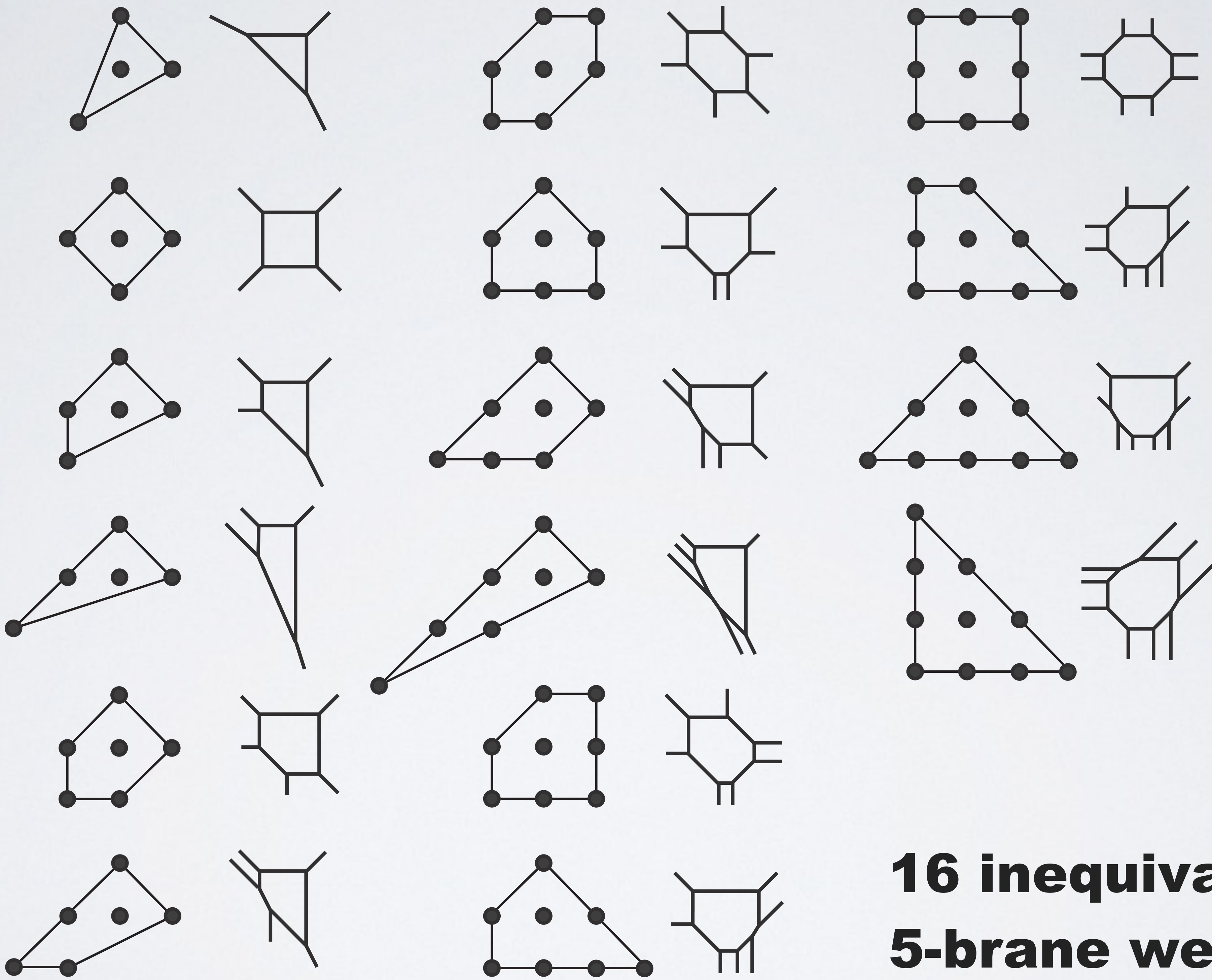
Let us test at 1-instanton level

$$\boxed{Z_{\mathbb{F}_0} = \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^j) \times Z_{\mathbb{F}_2}}$$

$$Z_{\mathbb{F}_0}(Q_F; t, q) - Z_{\mathbb{F}_2}(Q_F; t, q) = \boxed{-u \frac{q}{(1-q)(1-t)}}.$$

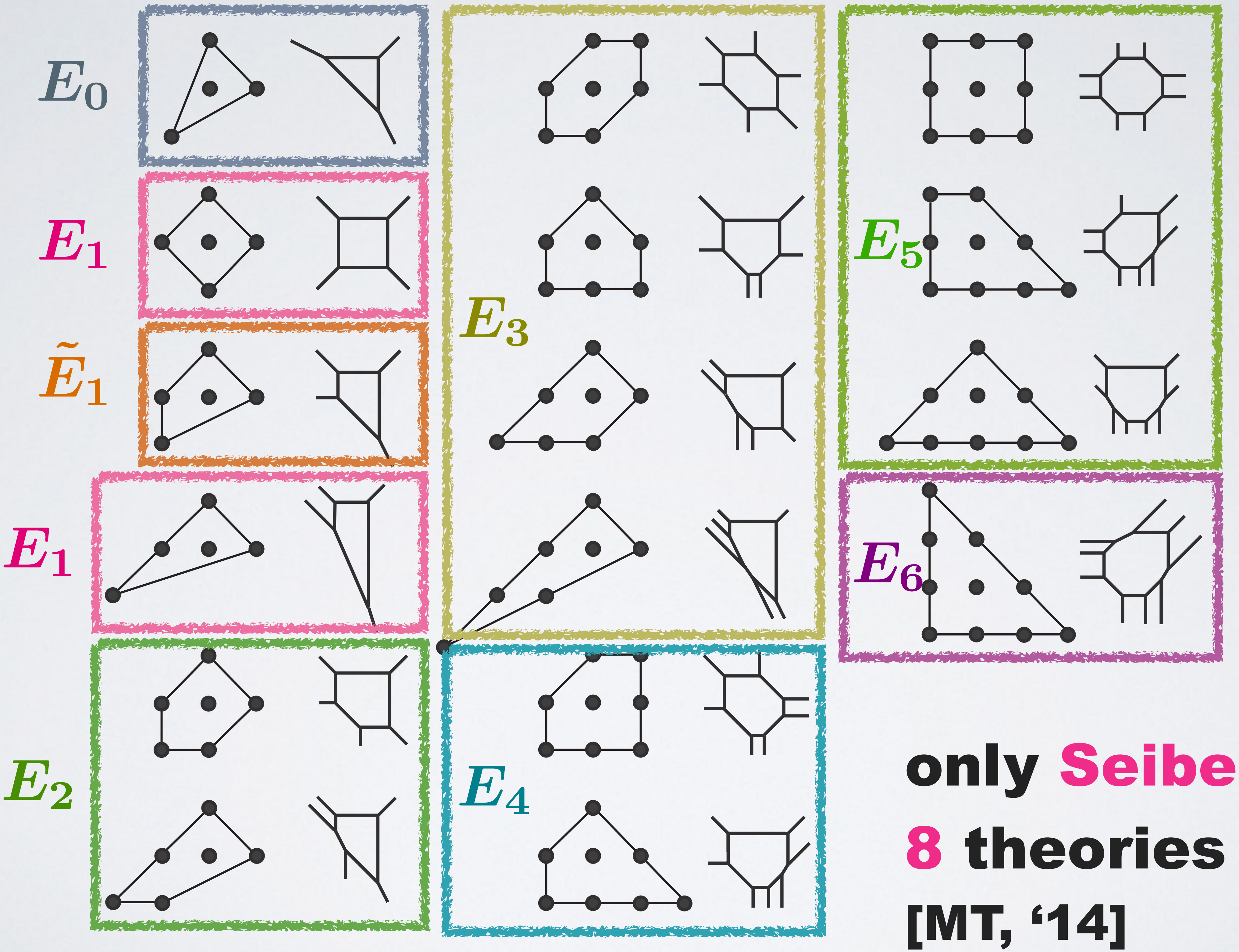
$$\Longleftrightarrow \quad \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^j) = 1 \boxed{- u \frac{q}{(1-q)(1-t)}} + \cdots$$

Generalization: all the webs with 1d Coulomb branch



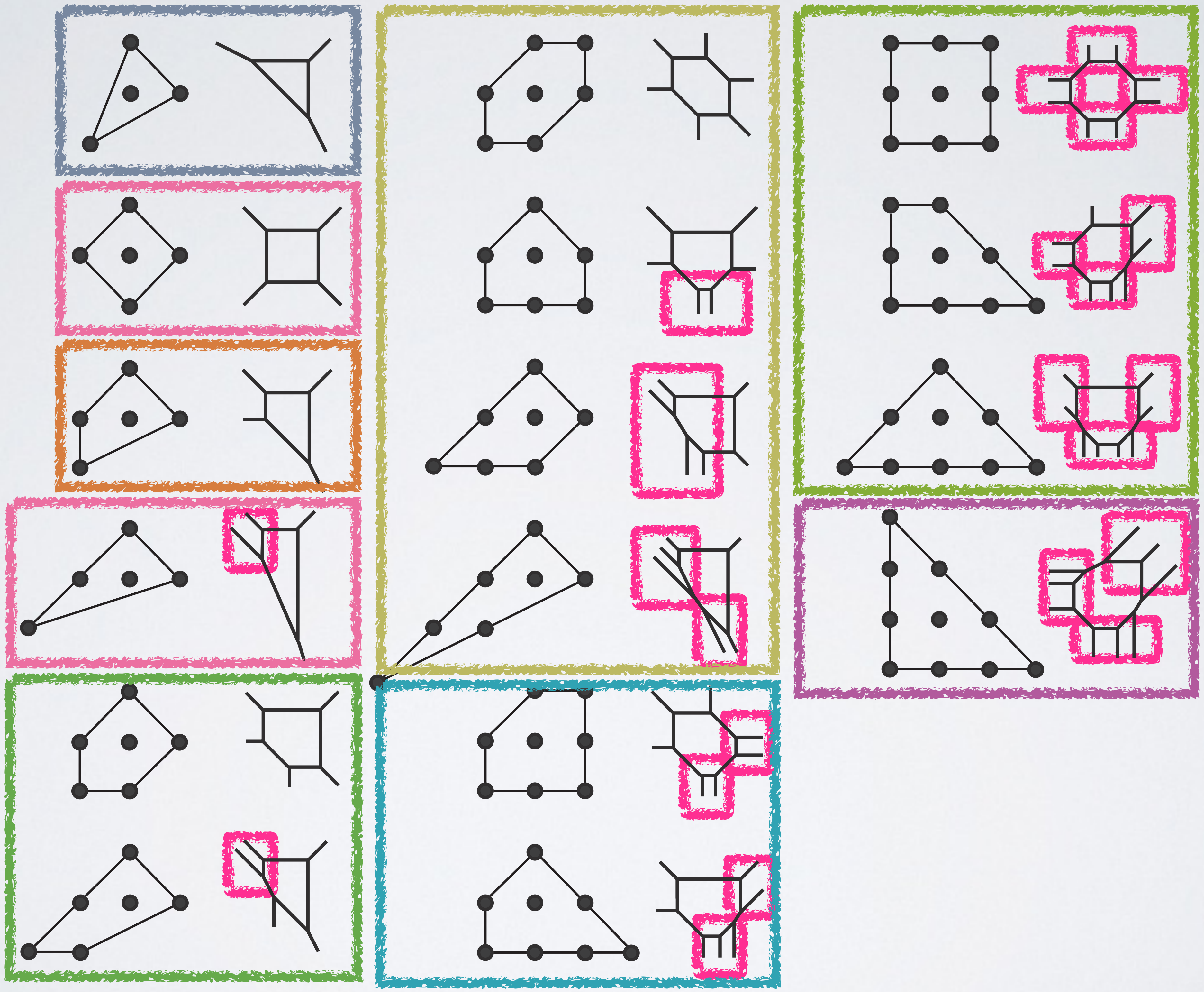
16 inequivalent 5-brane webs

7-brane picture leads to ... [MT, '14]



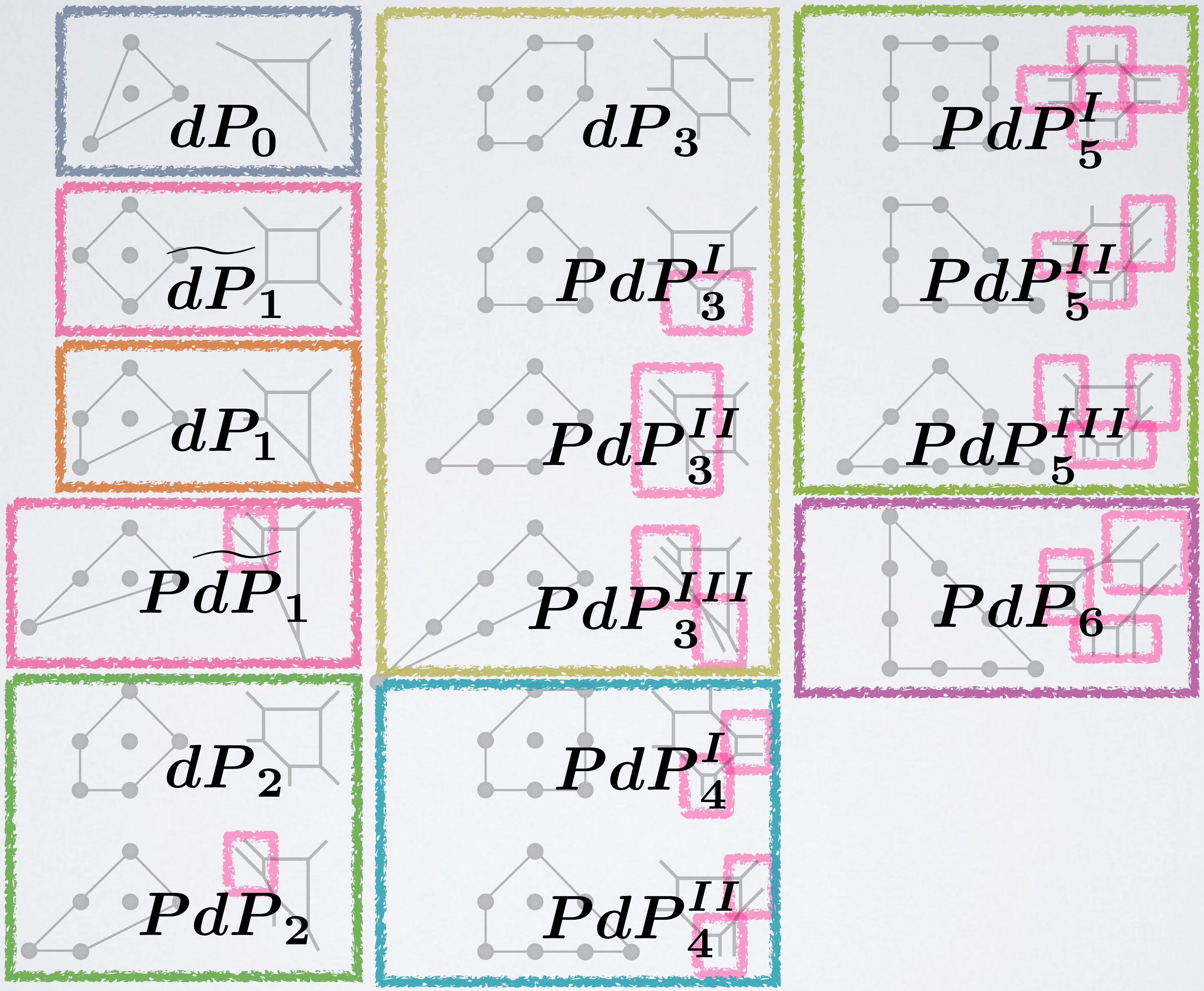
only Seiberg's 8 theories !!
[MT, '14]

7-brane picture leads to duality [MT, '14]



some of them contain extra d.o.f.

7-brane picture leads to duality [MT, '14]



Conjecture

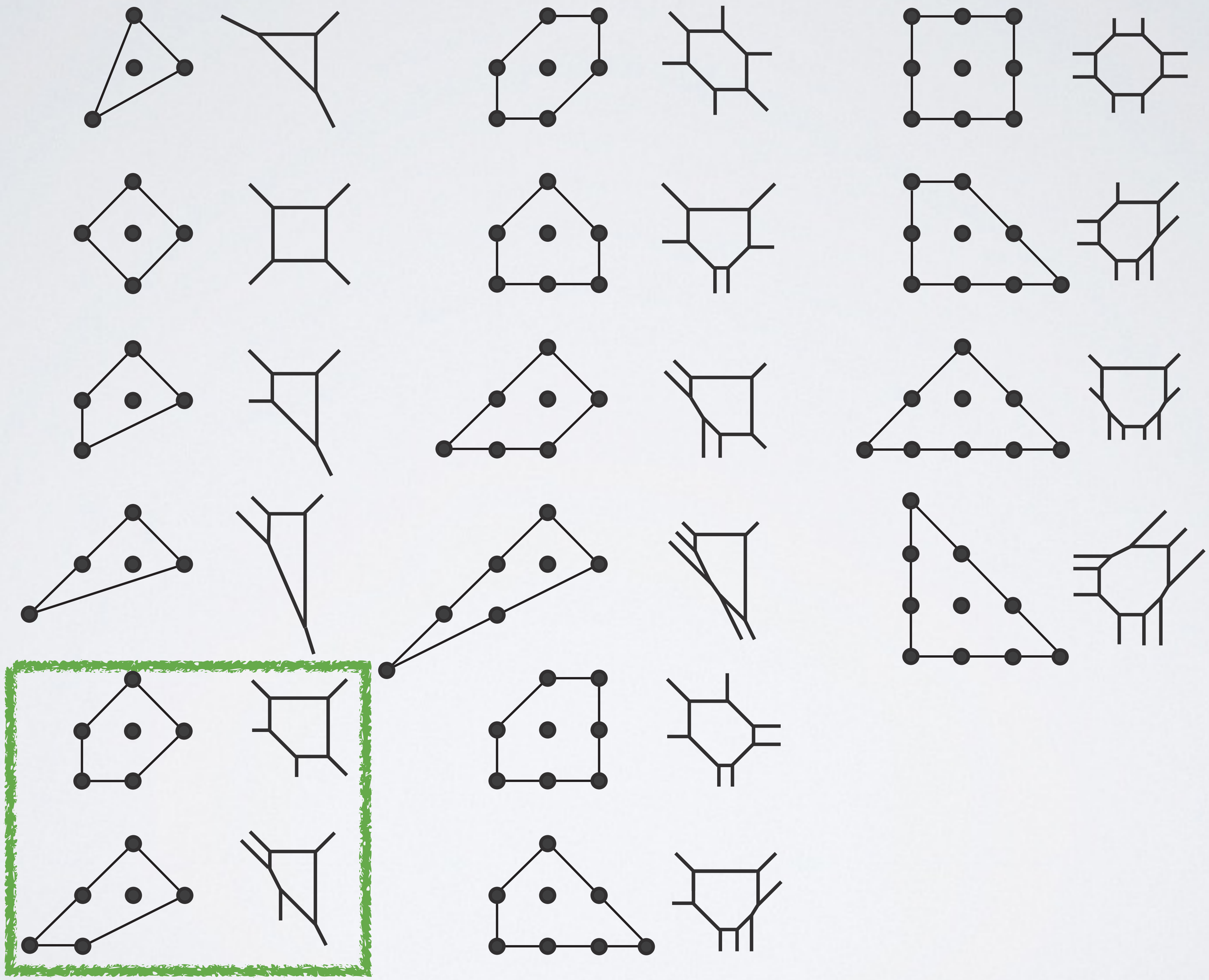
[MT, '13, '14]
[Bao-Mitev-Pomoni-MT-Yagi, "13]
[Hayashi-Kim-Nishinaka, "13]

$$Z_{dP_k} = \prod_{\ell} \prod_{i,j=1}^{\infty} (1 - u_\ell \, q^i \, t^{j-1}) \cdot Z_{PdP_k}^{N}$$

$N = I, II, \cdots$: degeneracy of the pseudo dP

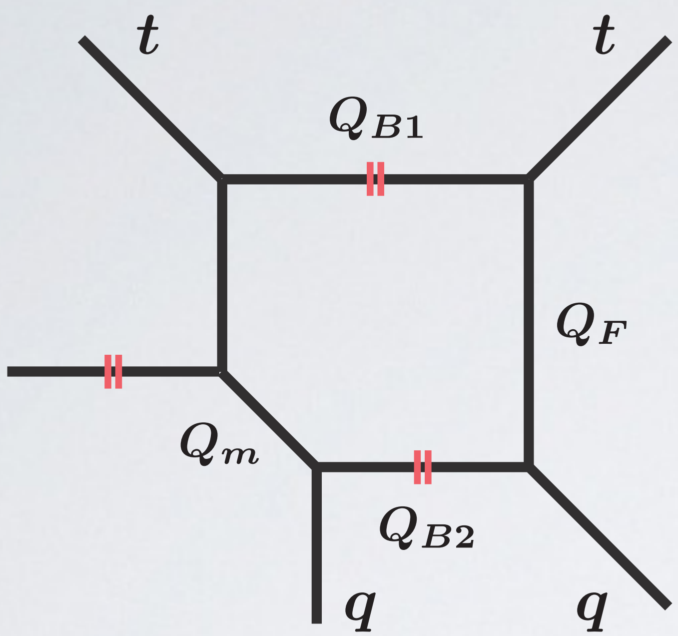
u_ℓ is a monomial of parameters that is independent of Coulomb branch moduli $Q_F = e^{2Ra}$

$\mathbf{E}_2$ case [MT, '14] [BMPTY, '13] [HKN, '13]

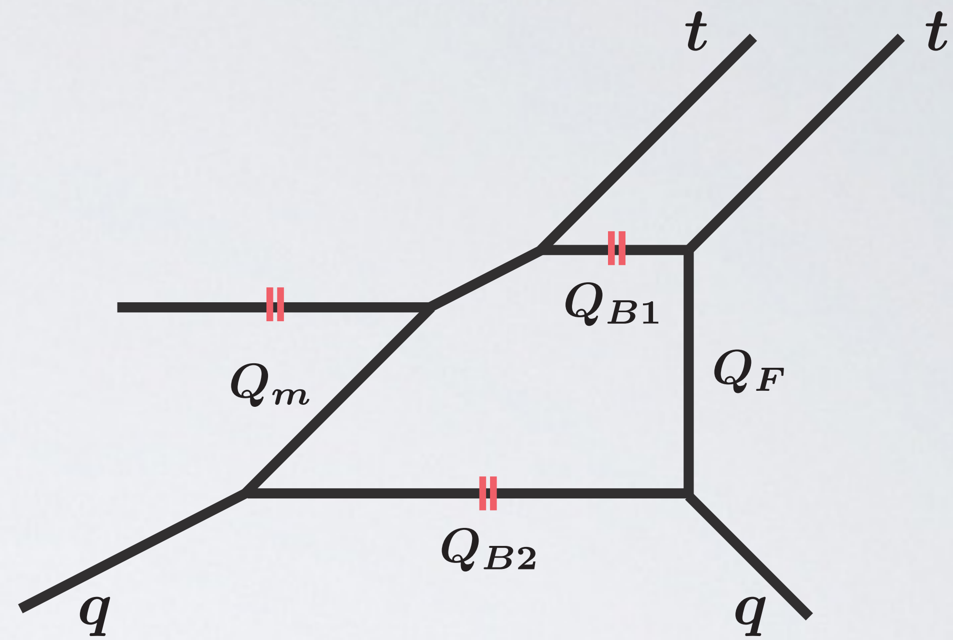


E_2

$\mathbf{E_2}$ case [MT, '14] [BMPTY, '13] [HKN, '13]

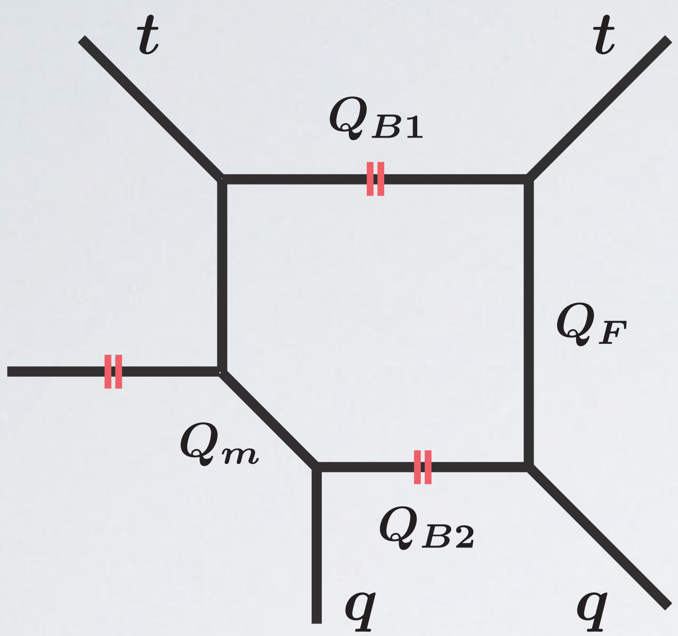


dP_2

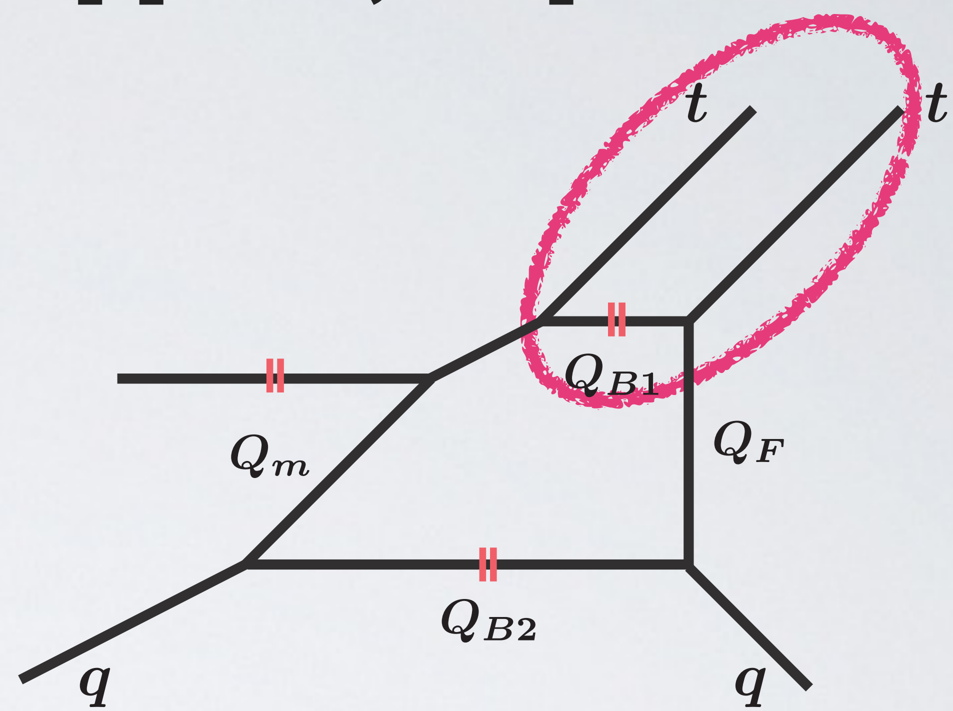


PdP_2

$\mathbf{E_2}$ case [MT, '14] [BMPTY, '13] [HKN, '13]



dP_2

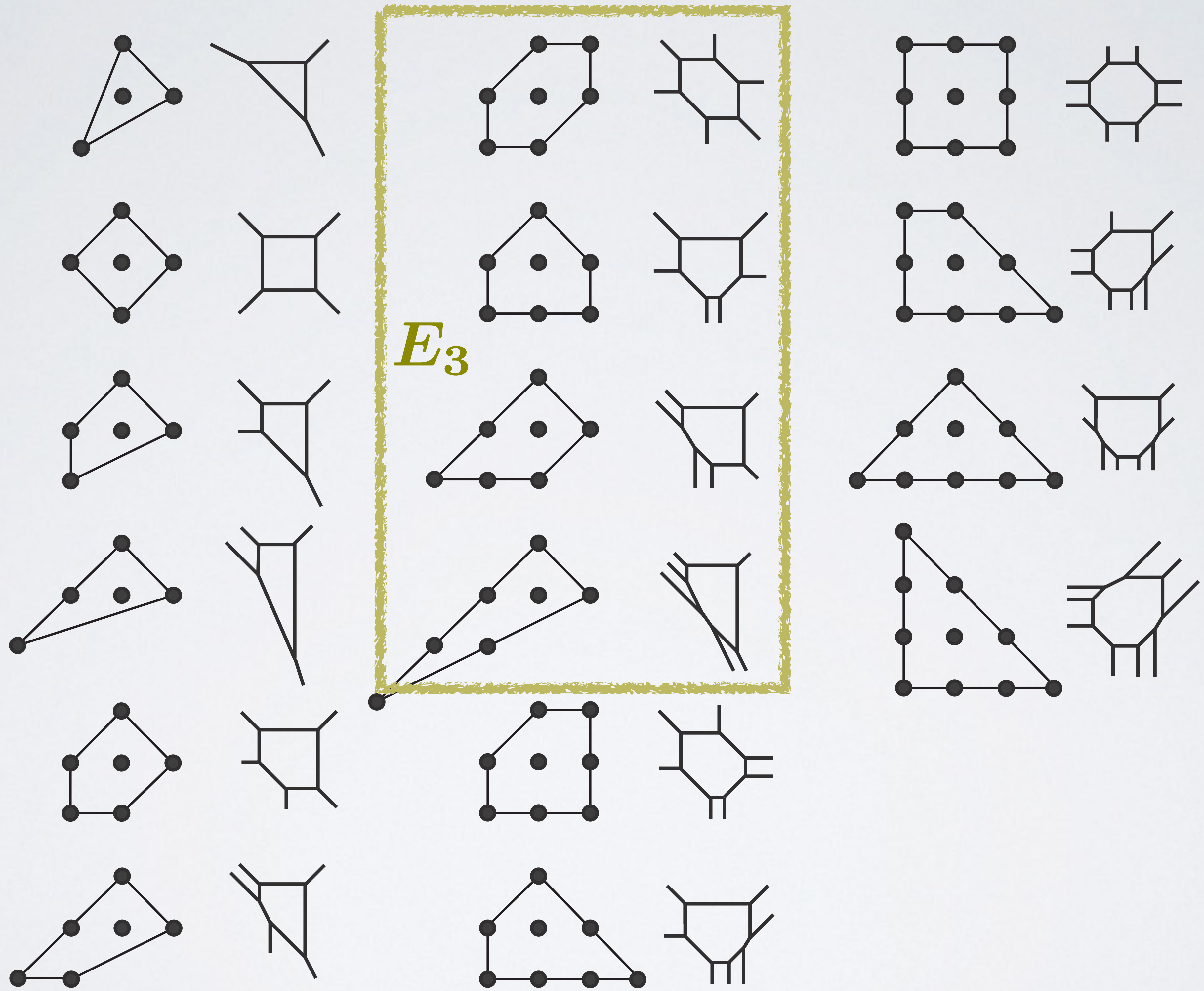


PdP_2

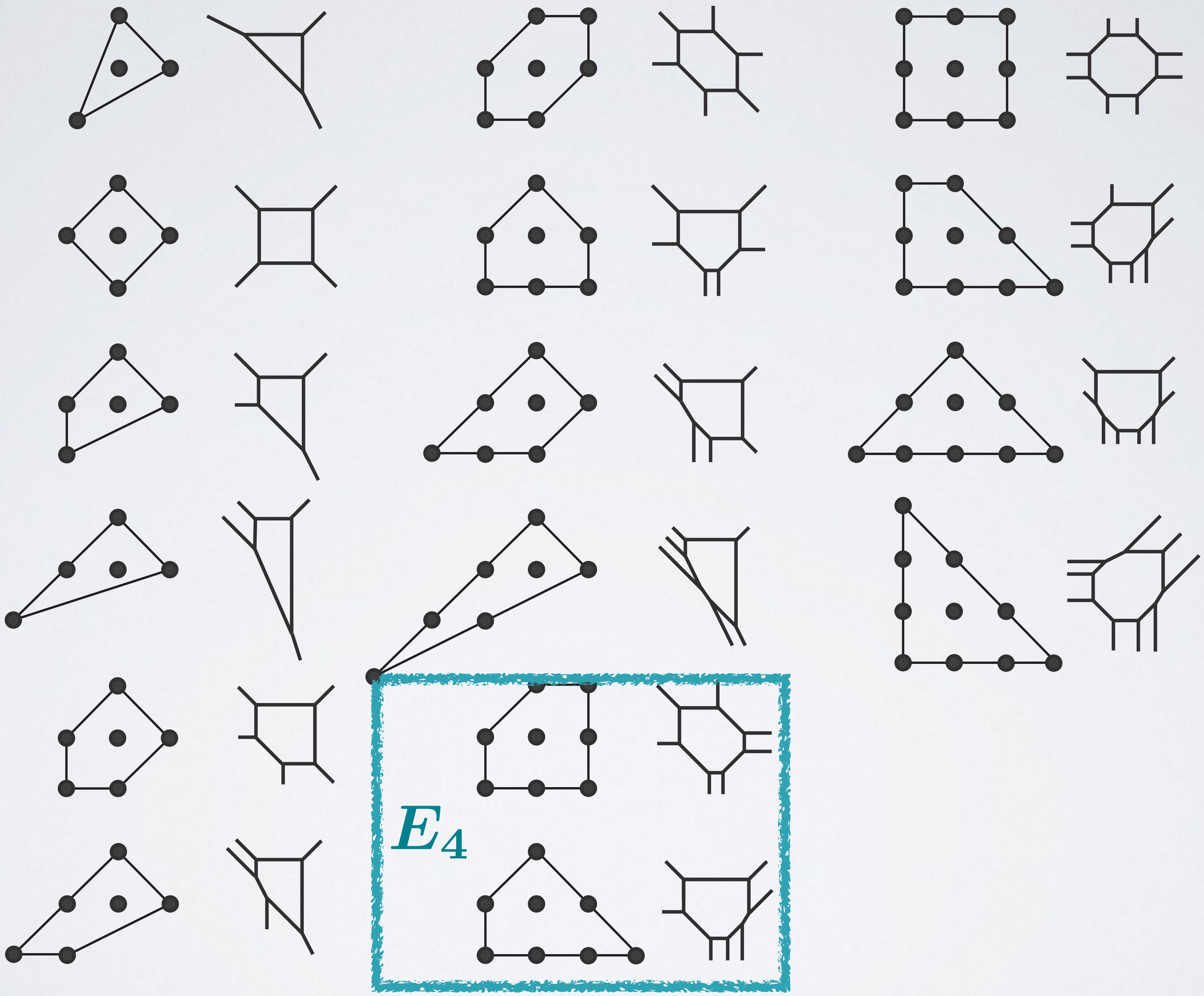
$$Z_{dP_2} = \prod_{i,j=1}^{\infty} (1 - u\, t^i\, q^{j-1}) \times Z_{PdP_2}$$

1- & 2-instanton test is straightforward.

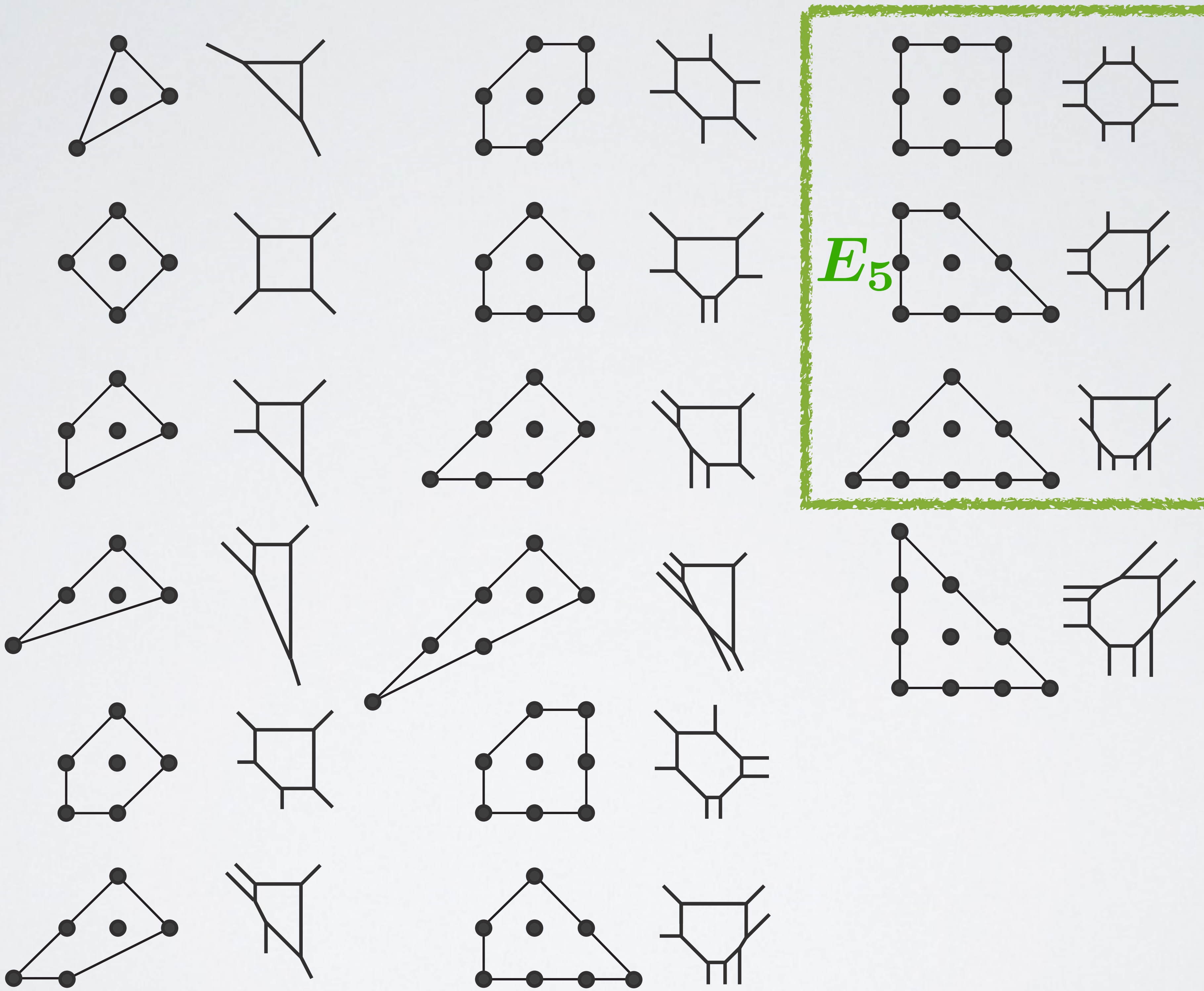
$\mathbf{E_3}$ case [MT, '14]



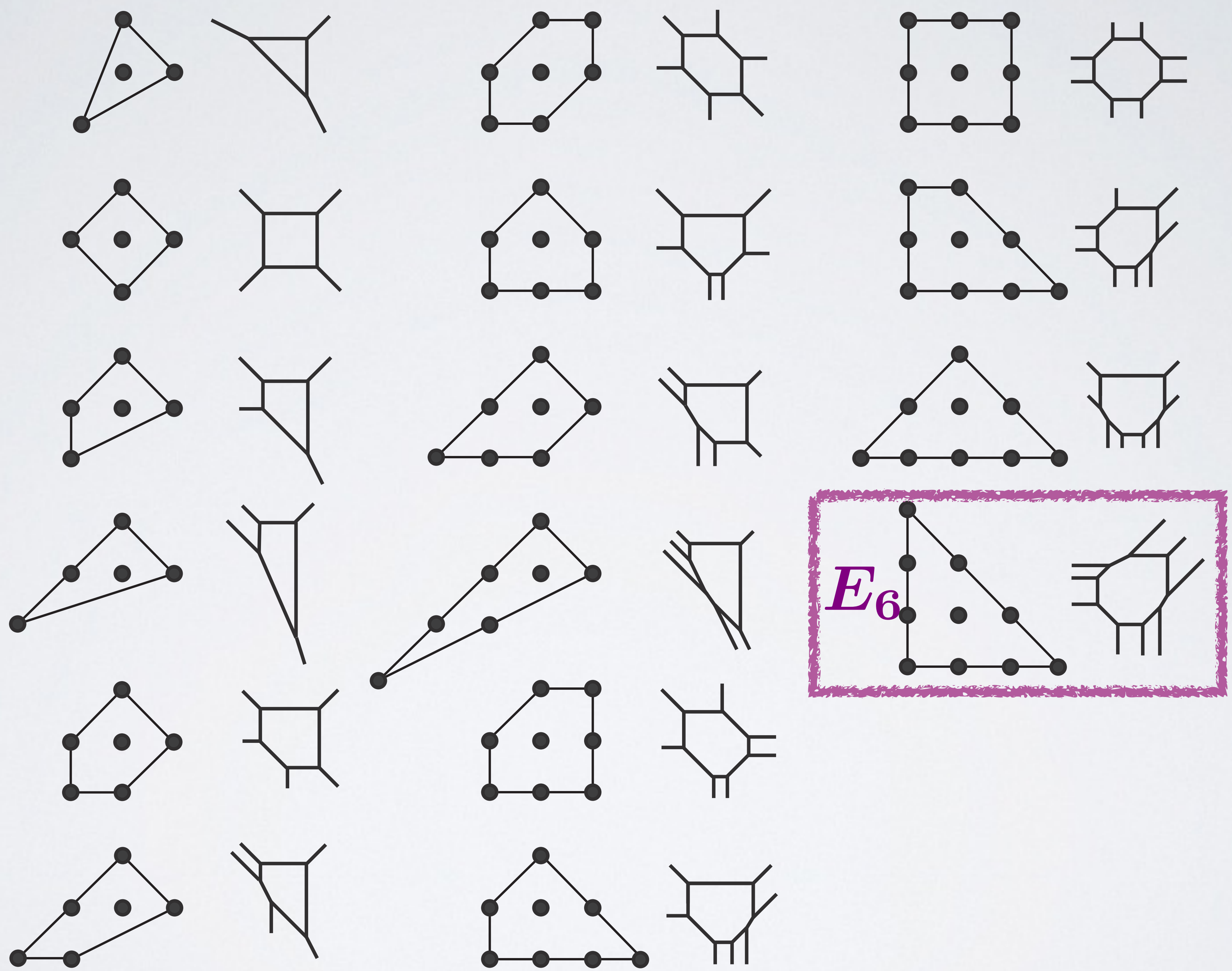
$\mathbf{E}_4$ case [MT, '14]



$\mathbf{E}_5$ case [MT, '14]



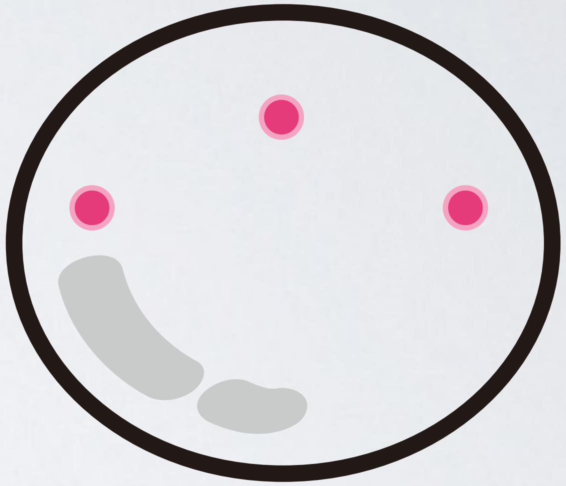
$\mathbf{E_6}$ case [BMPTY, '13] [HKN, '13]



E_6 SCFT [Benini-Benvenuti-Tachikawa,'09]

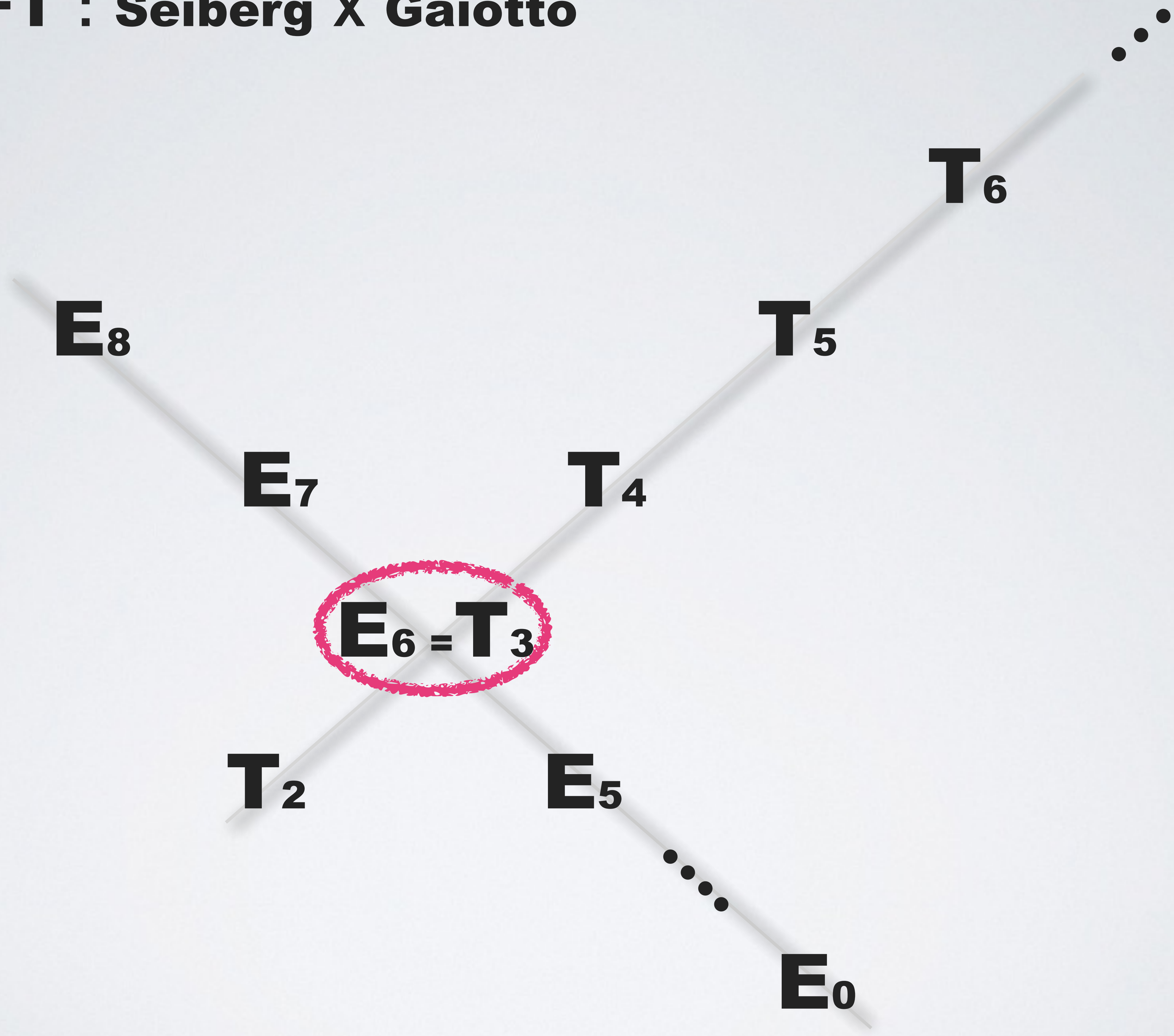
three M5-branes on sphere (Gaiotto's 4d T_3)

- ✔ 5d uplift of the Gaiotto thy

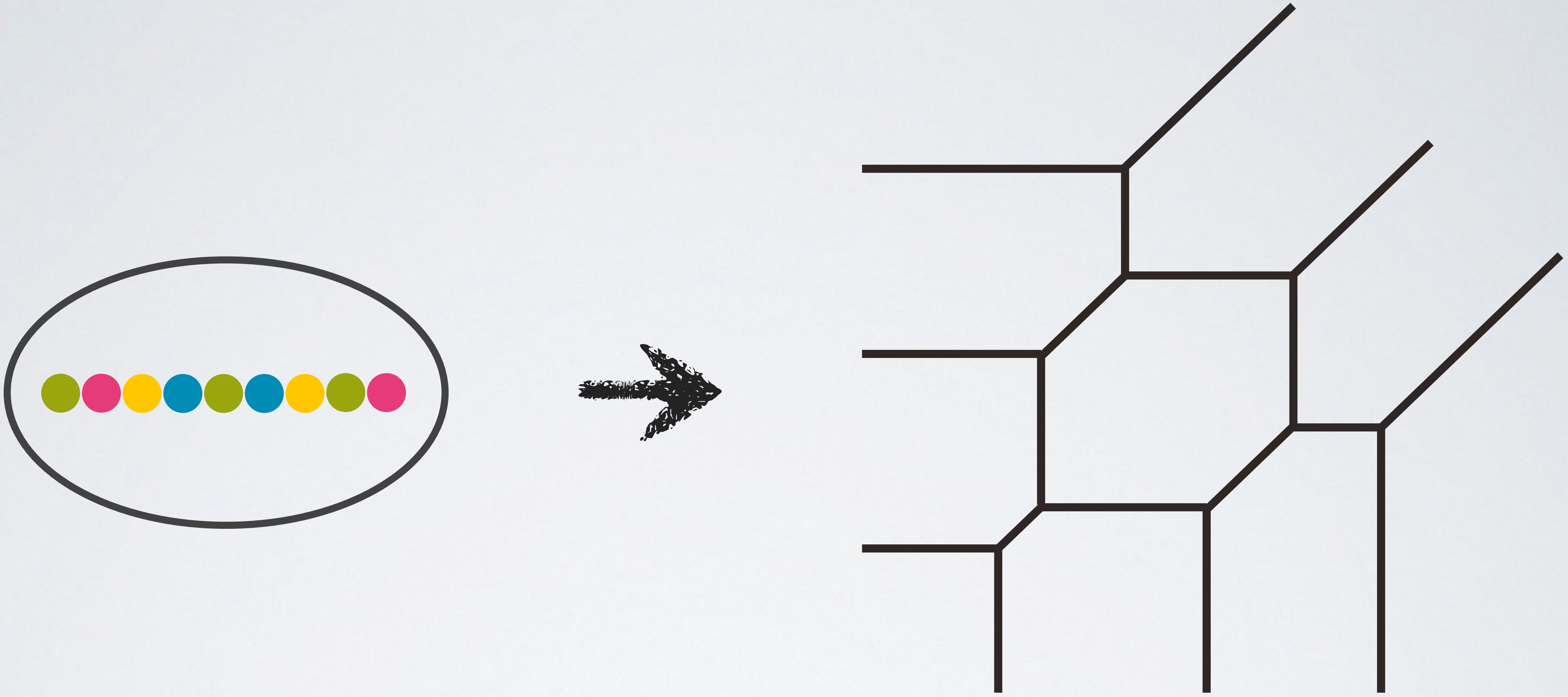


- ✔ N M5s gives T_N theory

E_6 SCFT : Seiberg X Gaiotto

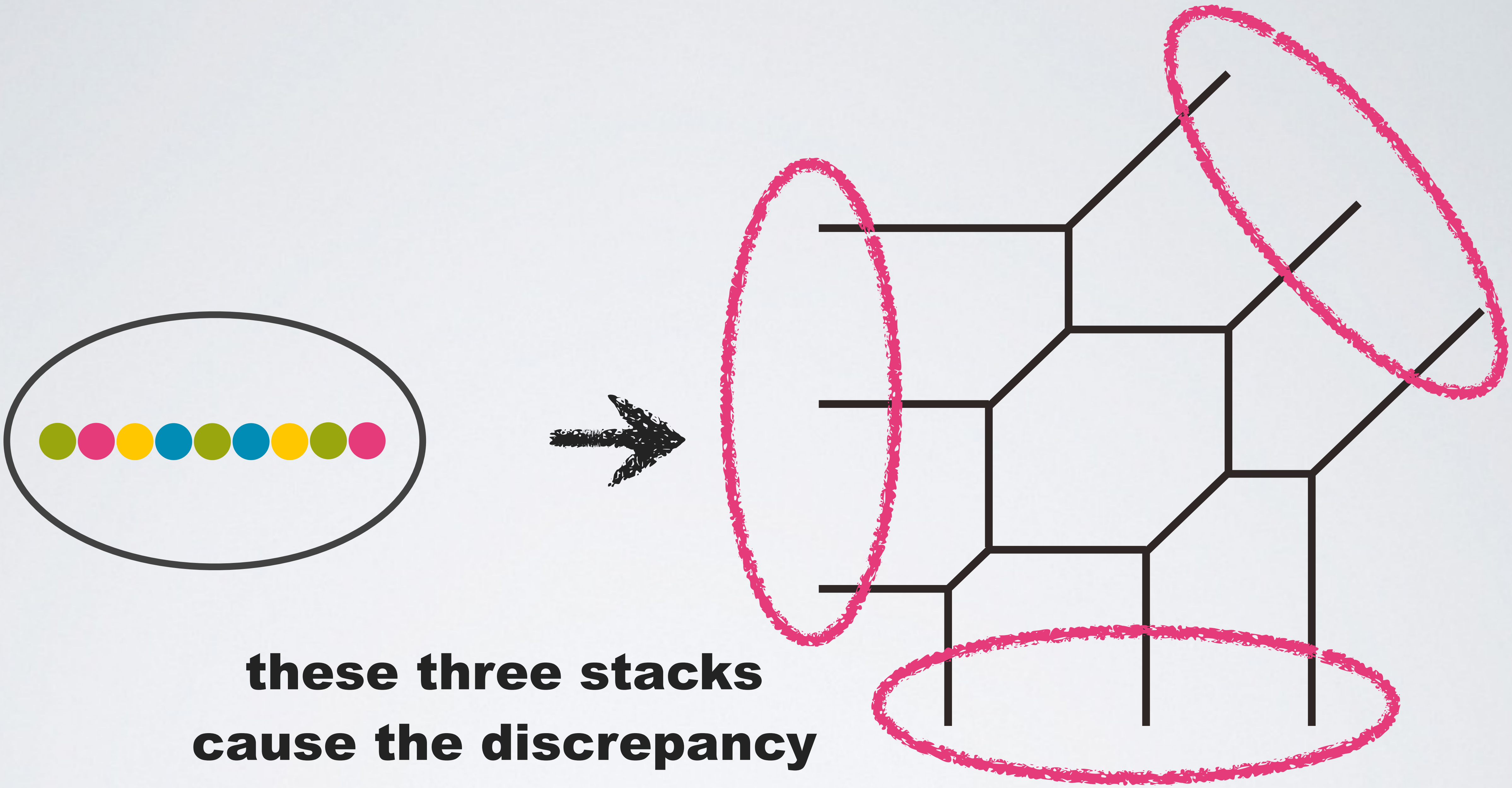


E_6 theory via brane web



$$PdP_6 = \mathbb{C}^3/\mathbb{Z}_3 \times \mathbb{Z}_3$$

$\mathbf{E_6}$ theory via brane web

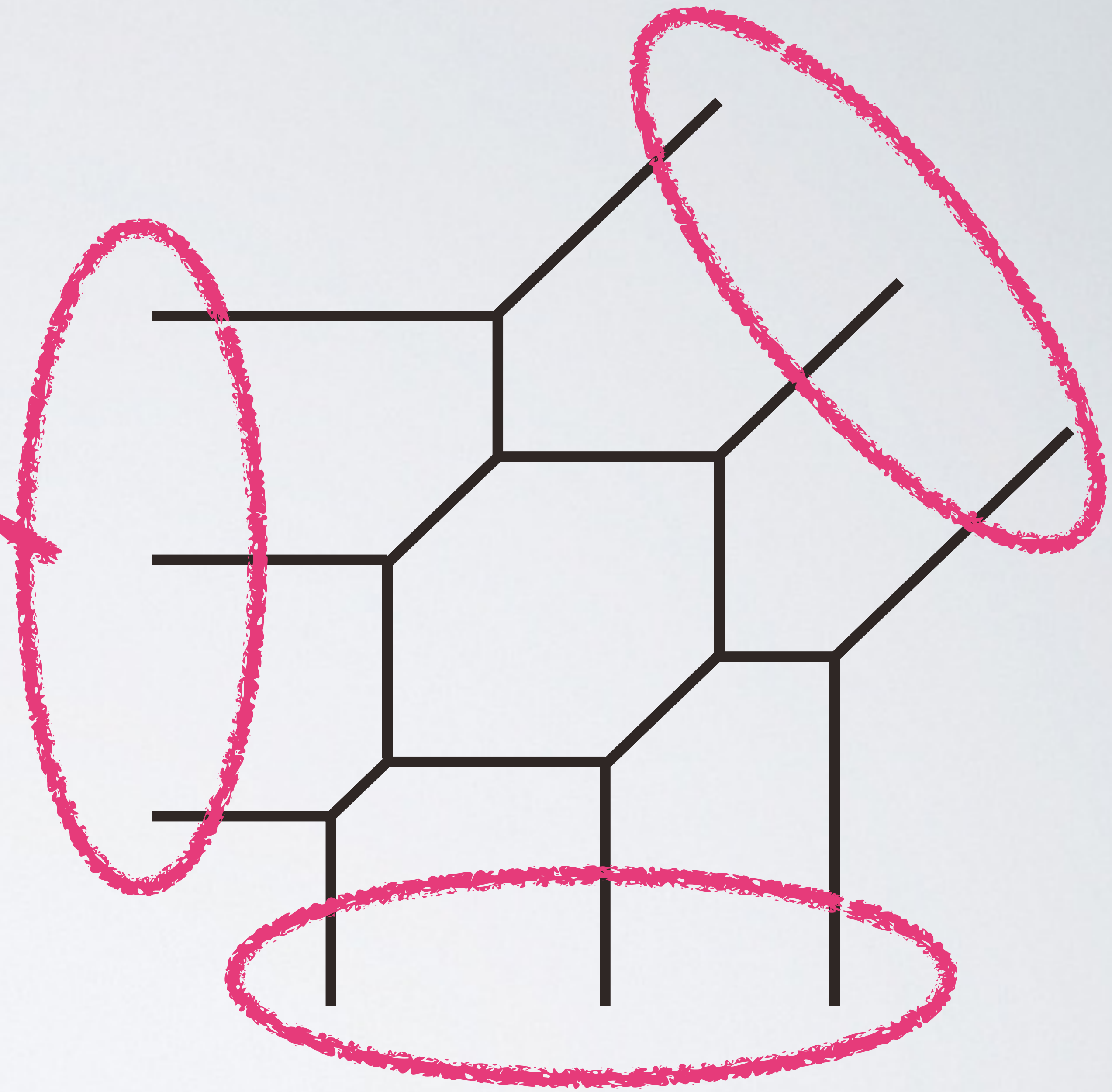


these three stacks
cause the discrepancy

$$PdP_6 = \mathbb{C}^3/\mathbb{Z}_3 \times \mathbb{Z}_3$$

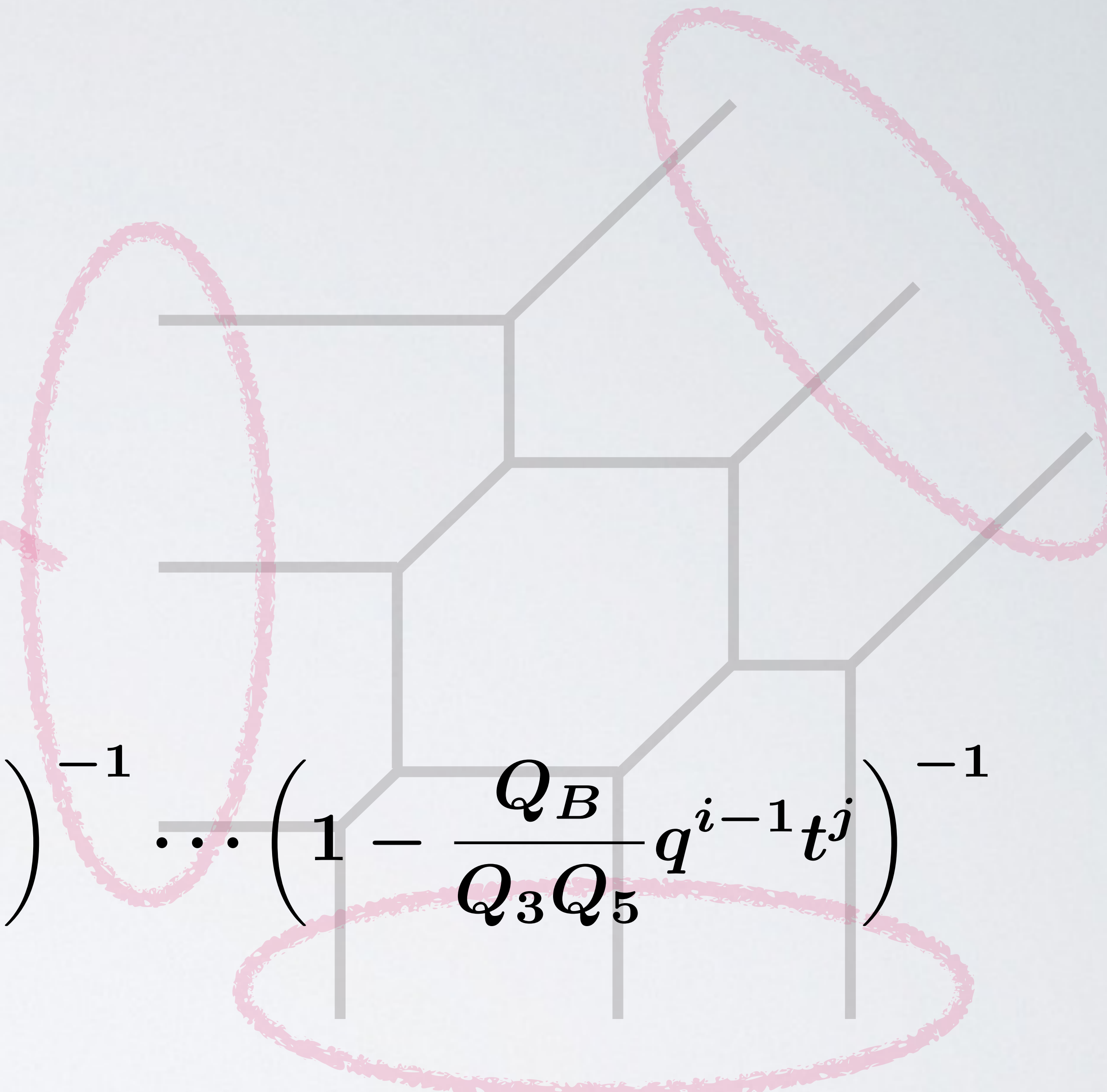
$\mathbf{E_6}$ theory via brane web

$$Z_{dP_6} = \frac{Z_{PdP_6}}{Z_{\mathrm{extra}}}$$



E_6 theory via brane web

$$Z_{dP_6} = \frac{Z_{PdP_6}}{Z_{\mathrm{extra}}}$$



$$Z_{extra} = \prod_{i,j=1}^{\infty} \left(1 - \frac{Q_2}{Q_1} q^i t^{j-1}\right)^{-1} \cdots \left(1 - \frac{Q_B}{Q_3 Q_5} q^{i-1} t^j\right)^{-1}$$

$\mathbf{E_6}$ theory

ugly **asymmetric** function of masses

$$Z_{dP_6} = \frac{Z_{PdP_6}}{Z_{\text{extra}}}$$

E6 theory

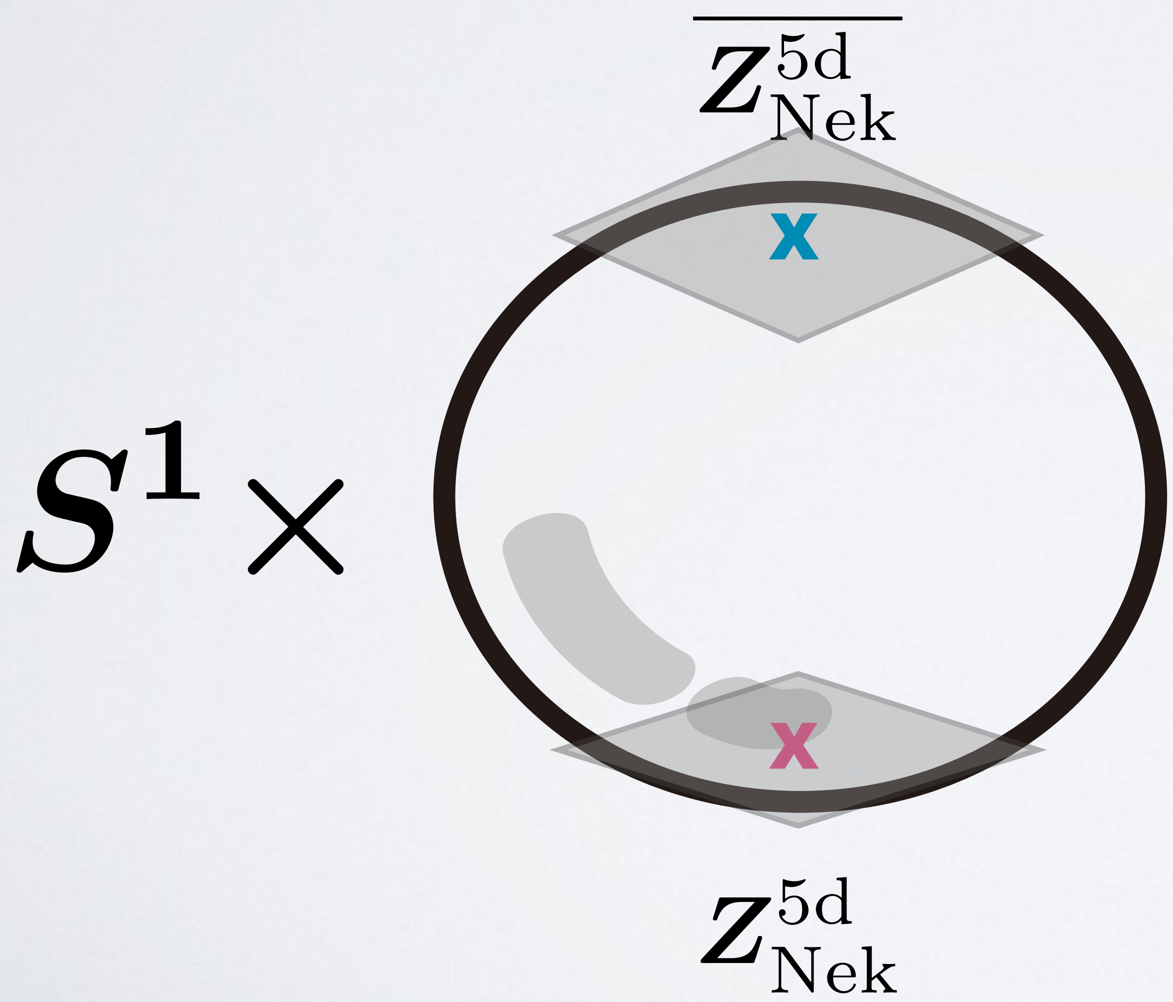
ugly asymmetric function of masses

$$Z_{dP_6} = \frac{Z_{PdP_6}}{Z_{\text{extra}}}$$

$$Z^{\text{1-inst.}}_{dP_6} = u\frac{q}{t}\frac{1}{(1-q)(1-t^{-1})(1-Q_F t^{-1} q)(1-Q_F^{-1} t^{-1} q)}$$
$$\times\Bigg[\left(1+\frac{q}{t}\right)\left(1+\sum_{f_1\neq f_2}\frac{1}{Q_{mf_1}Q_{mf_2}}+\sum_{f=1}^{5}\frac{Q_F^2 Q_{mf}}{Q_{m1}Q_{m2}Q_{m3}Q_{m4}Q_{m5}}\right)$$
$$-\sqrt{\frac{q}{t}}(1+Q_F)\left(Q_F^2+\sum_{f_1\neq f_2}\frac{Q_F Q_{mf_1}Q_{mf_2}}{Q_{m1}Q_{m2}Q_{m3}Q_{m4}Q_{m5}}+\sum_{f=1}^{5}\frac{1}{Q_{mf}}\right)\Bigg]$$

Superconformal Index [Kim-Kim-Lee, '12], [Iqbal-Vafa, '12]

$$I(x,y,m,u) = \int [da] \left| Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a) \right|^2$$



E_6 superconformal index [BMPTY] [HKN]

No E_6!

$$I_{dP_6} = \frac{I_{PdP_6}}{I_{\text{extra}}}$$

$$= 1 + \chi^{E_6}_{\mathbf{78}} x^2 + (1 + \chi^{E_6}_{\mathbf{78}})\chi_1(y) x^3$$

$$+ (1 + (1 + \chi^{E_6}_{\mathbf{78}})\chi_2(y) + \chi^{E_6}_{\mathbf{2430}}) x^4 + \cdots$$

This resolves the Iqbal-Vafa discrepancy

E6 superconformal index [BMPTY] [HKN]

No E6 !

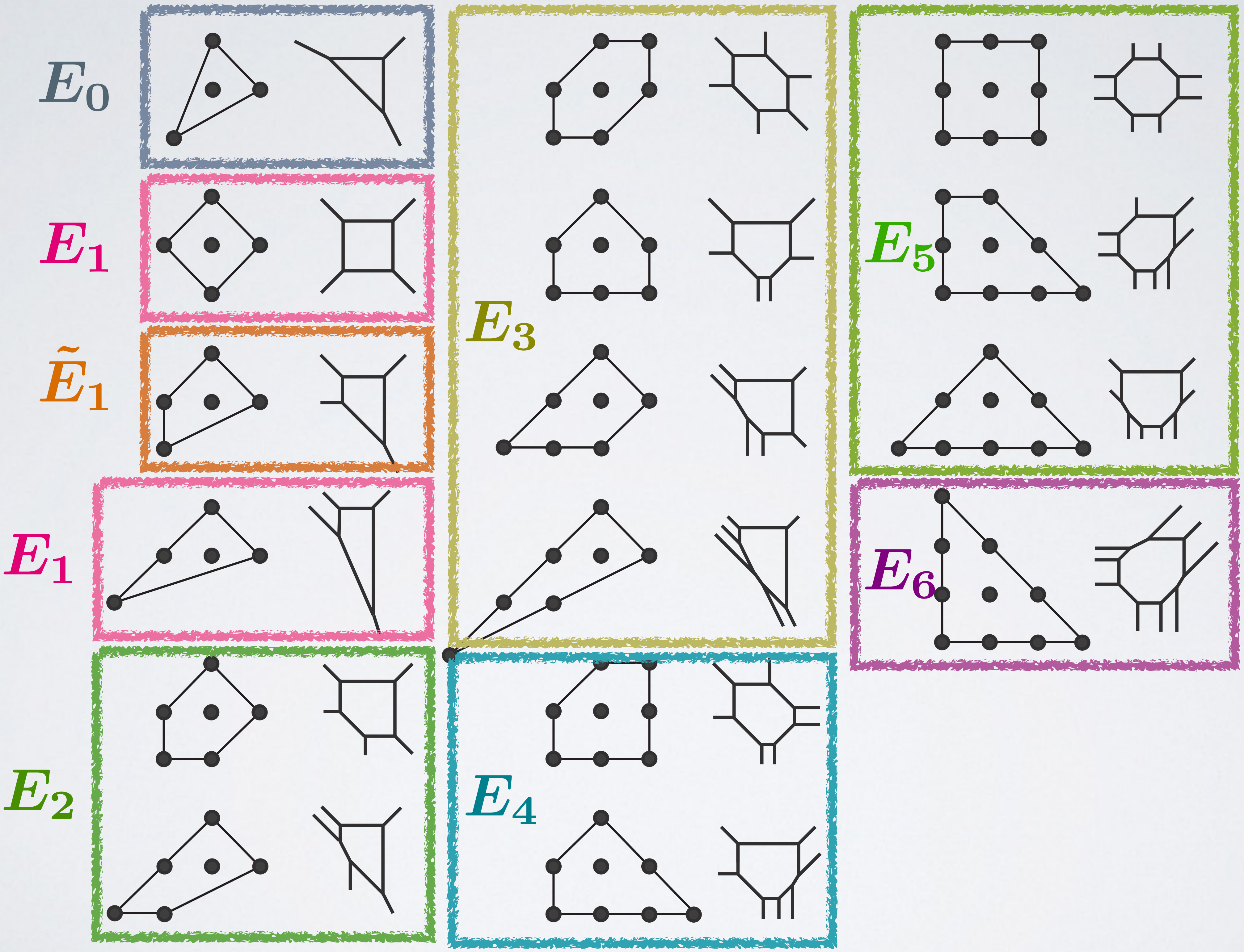
$$I_{dP_6} = \frac{I_{PdP_6}}{I_{\mathrm{extra}}}$$

$$= 1 + \chi^{E_6}_{\mathbf{78}} x^2 + (1 + \chi^{E_6}_{\mathbf{78}})\chi_1(y) x^3$$

$$+ (1 + (1 + \chi^{E_6}_{\mathbf{78}})\chi_2(y) + \chi^{E_6}_{\mathbf{2430}})x^4 + \cdots$$

agrees with [Kim-Kim-Lee]'s E6 index !!

These web systems reflect expected sym.



5. Summary
&
Open Problems

we revisit 5d SCFT from the perspective of 7-branes & modified Nekrasov function.

→ new duality

we revisit 5d SCFT from the perspective of 7-branes & modified Nekrasov function.

$\longrightarrow$ new duality

refined topological vertex for non-toric web

$$Z_{\text{delPezzo}} = Z_{\text{toric delPezzo}} \div Z_{\text{extra}}$$

Proof our conjecture on Nekrasov function ?

$$Z_{\text{delPezzo}} = Z_{\text{toric delPezzo}} \div Z_{\text{extra}}$$

Proof our conjecture on Nekrasov function ?

$$Z_{\text{delPezzo}} = Z_{\text{toric delPezzo}} \div Z_{\text{extra}}$$

Relation to 5d AGT [Awata-Yamada] ?

Proof our conjecture on Nekrasov function ?

$$Z_{\text{delPezzo}} = Z_{\text{toric delPezzo}} \div Z_{\text{extra}}$$

Relation to 5d AGT [Awata-Yamada] ?

The cases with multiple dim Coulomb branch
[work in progress]

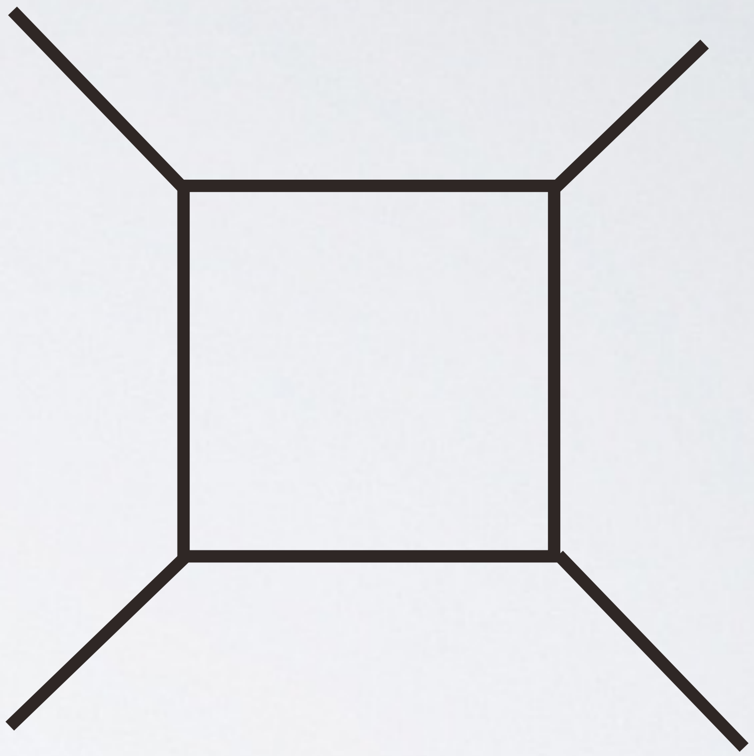
. . . . .

FIN

0. Supplementary

pure SU(2) Yang-Mills theory

local Hirzebruch surface :

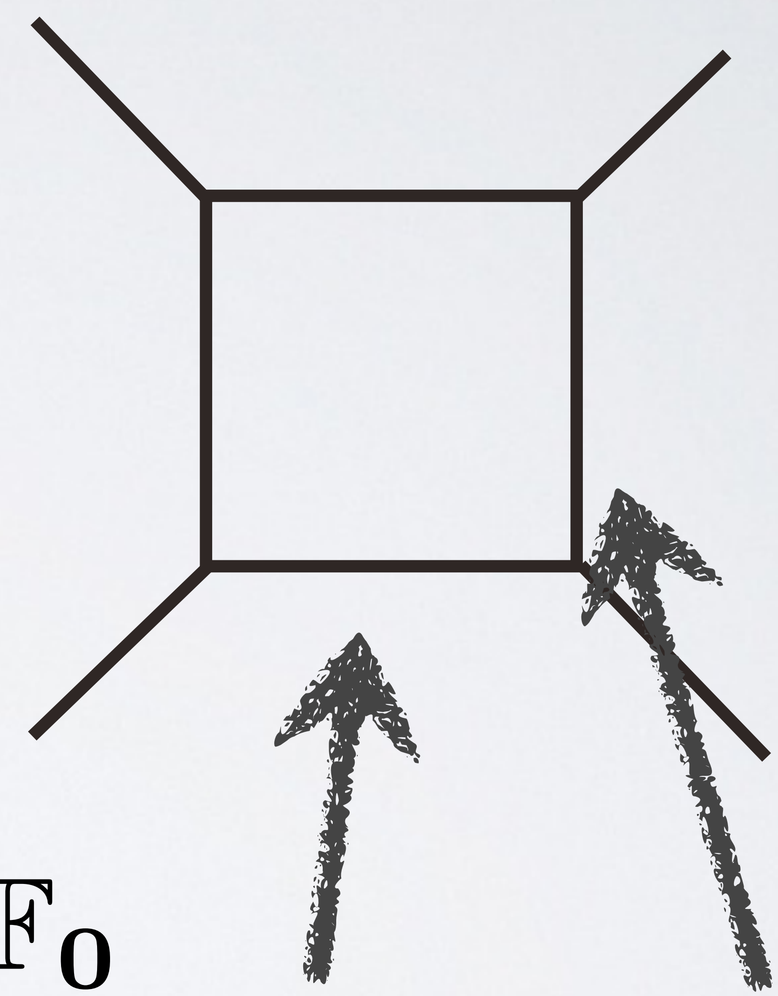


canonical line bundle over $\mathbb{F}_0$

$$= \mathbb{CP}^1 \times \mathbb{CP}^1$$

pure SU(2) Yang-Mills theory

local Hirzebruch surface :



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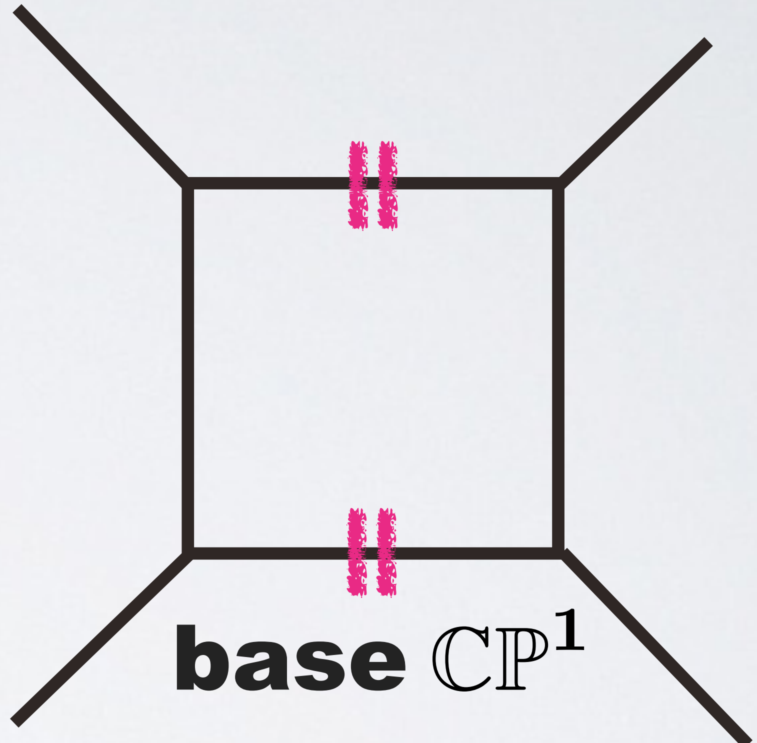

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pure SU(2) Yang-Mills theory

$$Q_F u = e^{-t_B}$$

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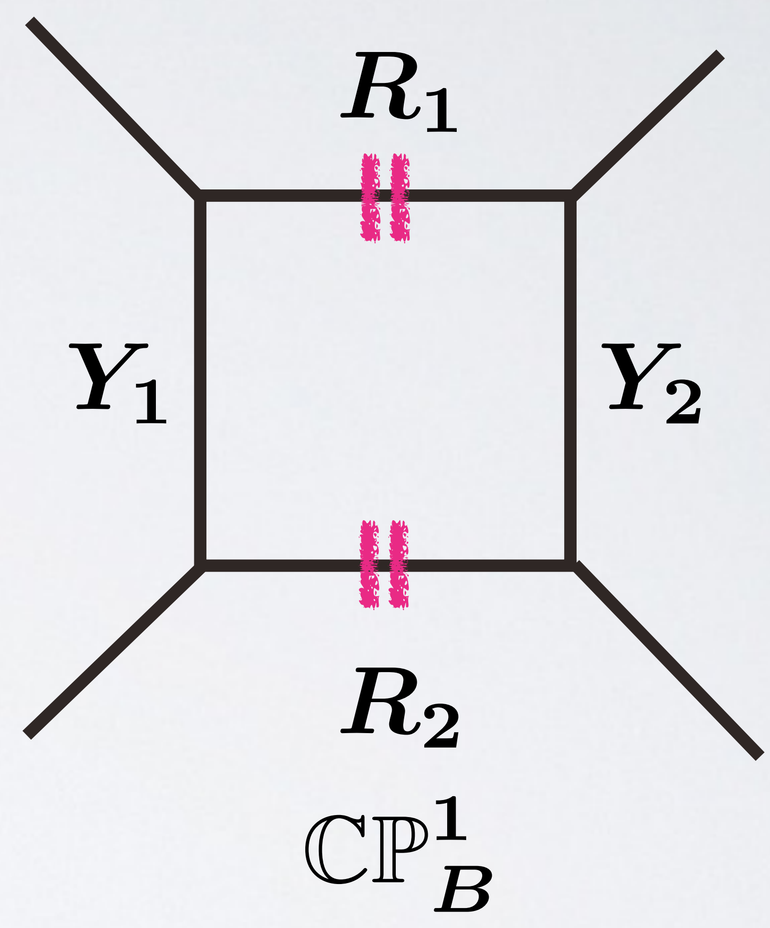
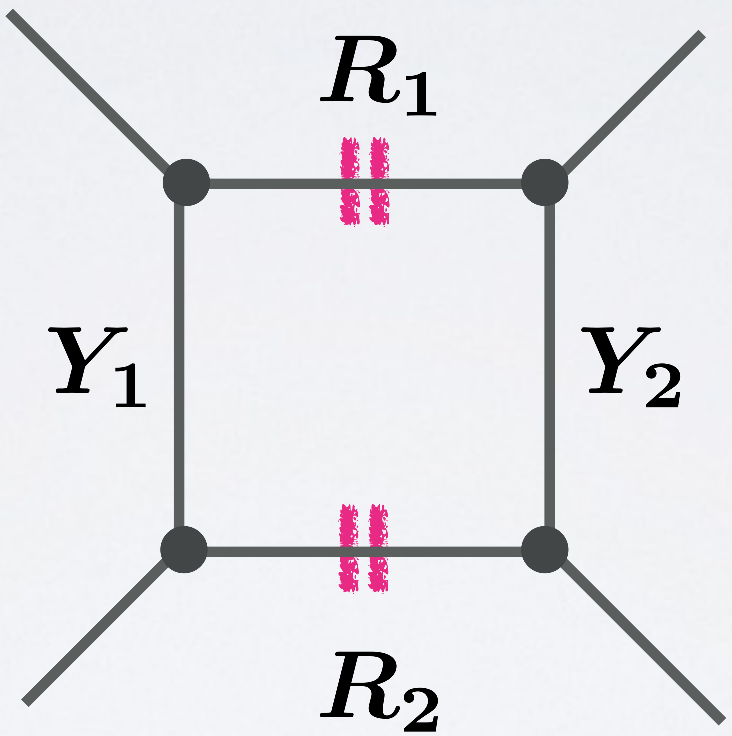


pure SU(2) Yang-Mills theory

$$P_{R_1}(q^{-\rho};t,q)S_{\mathbf{Y}_1}(q^{-\rho}t^{-R_1^t}) \qquad P_{R_1^t}(t^{-\rho};q,t)S_{\mathbf{Y}_2}(t^{-\rho}q^{-R_1})$$



$$P_{R_2}(q^{-\rho};t,q)S_{\mathbf{Y}_1}(q^{-\rho}t^{-R_2^t}) \qquad P_{R_2^t}(t^{-\rho};q,t)S_{\mathbf{Y}_2}(t^{-\rho}q^{-R_2})$$

pure SU(2) Yang-Mills theory

$$Z_{\mathbb{F}_0} = \sum_{R_{1,2}} (uQ_F)^{|R_1|+|R_2|} \sum_{Y_{1,2}} (Q_F)^{|Y_1|+|Y_2|}$$

$$P_{R_1}(q^{-\rho}; t, q) S_{Y_1}(q^{-\rho} t^{-R_1^t})$$

$$P_{R_1^t}(t^{-\rho}; q, t) S_{Y_2}(t^{-\rho} q^{-R_1})$$

$$P_{R_2^t}(t^{-\rho}; q, t) S_{Y_2}(t^{-\rho} q^{-R_2})$$

$$P_{R_2}(q^{-\rho}; t, q) S_{Y_1}(q^{-\rho} t^{-R_2^t})$$

Nekrasov formula on $\mathbb{R}^4 \times S^1$

- contribution from SU(2) gauge field (instanton)

$$Z^{\text{vect.}}_{\vec{R}}(Q_F; t, q) = \left(\frac{q}{t}\right)^{|\vec{R}|} \frac{1}{\prod_{\alpha,\beta=1,2} N_{R_\alpha R_\beta}(Q_{\beta\alpha}; t, q)},$$

$$Q_{21} = Q_{12}^{-1} = Q_F \equiv e^{2Ra}, \quad Q_{11} = Q_{22} = 1$$

$$N_{R_\alpha R_\beta}(Q; t, q) = \prod_{s \in R_\alpha} \left(1 - Q\, t^{\ell_{R_\beta}(s)} q^{a_{R_\alpha}(s)+1}\right) \prod_{t \in Y_\beta} \left(1 - Q\, t^{-(\ell_{R_\alpha}(t)+1)} q^{-a_{R_\beta}(t)}\right)$$

$$\boxed{Z_{\mathbb{F}_0} = \sum_{R_{1,2}} \left(u \frac{q}{t}\right)^{|\vec{R}|} Z^{\text{vect.}}_{\vec{R}}(Q_F; t, q)}$$