R-flux string sigma model and algebroid duality on Lie 3-algebroids

Marc Andre Heller

Tohoku University

Based on joint work with Taiki Bessho (Tohoku University),
Noriaki Ikeda (Ritsumeikan University) and
Satoshi Watamura (Tohoku University)

arXiv:1511.03425
further paper in progress

Introduction and Motivation

1. There exist various fluxes in string theory, e.g. NS H-flux, F-flux
2. Non-geometric fluxes Q and R are conjectured from T-duality considerations, but their proper description is still obscure [Shelton-Taylor-Wecht]

Introduction and Motivation

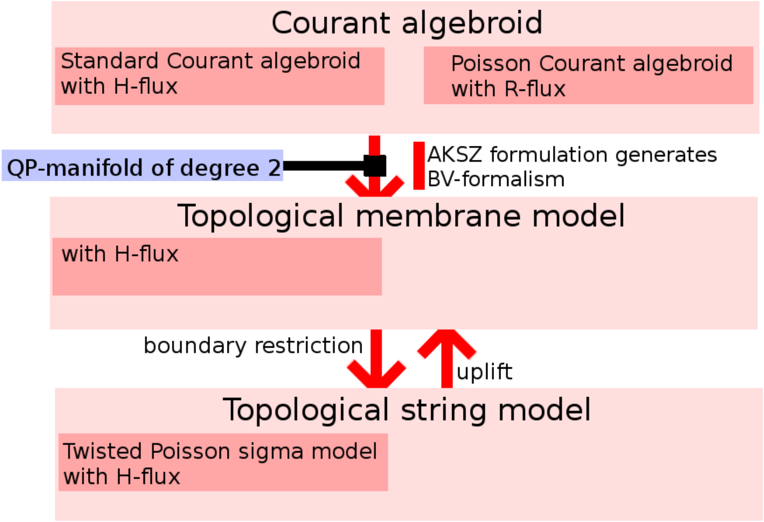
1. There exist various fluxes in string theory, e.g. NS H-flux, F-flux
2. Non-geometric fluxes Q and R are conjectured from T-duality considerations, but their proper description is still obscure [Shelton-Taylor-Wecht]
3. Topological T-duality [Bouwknegt-Evslin-Mathai] concerns the transformation between H- and F-flux and can well be described using generalized geometry (**standard Courant algebroid** on $TM \otimes T^*M$)
4. [Asakawa-Muraki-Sasa-Watamura] proposed a variant of generalized geometry based on a Courant algebroid, defined on a Poisson manifold with Poisson tensor θ, (**Poisson Courant algebroid**, see Muraki-san's talk) that can describe the transformation between Q and R

Introduction and Motivation

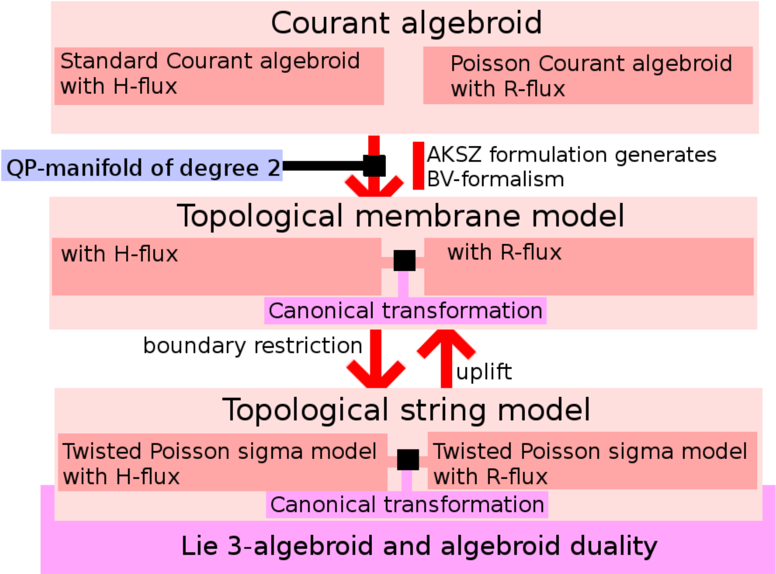
1. There exist various fluxes in string theory, e.g. NS H-flux, F-flux
2. Non-geometric fluxes Q and R are conjectured from T-duality considerations, but their proper description is still obscure [Shelton-Taylor-Wecht]
3. Topological T-duality [Bouwknegt-Evslin-Mathai] concerns the transformation between H- and F-flux and can well be described using generalized geometry (**standard Courant algebroid** on $TM \otimes T^*M$)
4. [Asakawa-Muraki-Sasa-Watamura] proposed a variant of generalized geometry based on a Courant algebroid, defined on a Poisson manifold with Poisson tensor θ, (**Poisson Courant algebroid**, see Muraki-san's talk) that can describe the transformation between Q and R
5. Our goal is to **construct a topological string theory with R-flux** and to **describe the transformation between H and R and find a complete generalization of topological T-duality incorporating all fluxes**

$$\underbrace{H \longrightarrow F}_{\text{Courant Alg.}} \longrightarrow \underbrace{Q \longrightarrow R}_{\text{Poisson C. Alg.}}$$

A sketch of what is known



What we developed



Preliminaries: Courant Algebroids and QP-Manifolds

Courant algebroid on vector bundle E

Vector bundle E over M with fiber metric $\langle\cdot\,,\cdot\rangle$, bundle map $\rho : E \longrightarrow TM$ and Dorfman bracket $[-,-]_D$ on $\Gamma(E)$ satisfying consistency conditions

Preliminaries: Courant Algebroids and QP-Manifolds

Courant algebroid on vector bundle E

Vector bundle E over M with fiber metric $\langle \cdot\,, \cdot \rangle$, bundle map $\rho : E \longrightarrow TM$ and Dorfman bracket $[-,-]_D$ on $\Gamma(E)$ satisfying consistency conditions

QP-manifold $(\mathcal{M}, \omega, \Theta)$

1. Nonnegatively graded manifold $\mathcal{M}$ with degree n symplectic structure ω, that induces a graded Poisson bracket $\{\cdot, \cdot\}$ on $\mathcal{C}^\infty(\mathcal{M})$
2. Hamiltonian function Θ such that the **classical master equation** $\{\Theta, \Theta\} = 0$ holds
3. Hamiltonian vector field $Q = \{\Theta, \cdot\}$, that obeys $\mathcal{L}_Q \omega = 0$

Preliminaries: Courant Algebroids and QP-Manifolds

Courant algebroid on vector bundle E

Vector bundle E over M with fiber metric $\langle \cdot \, , \cdot \rangle$, bundle map $\rho : E \longrightarrow TM$ and Dorfman bracket $[-,-]_D$ on $\Gamma(E)$ satisfying consistency conditions

QP-manifold $(\mathcal{M}, \omega, \Theta)$

1. Nonnegatively graded manifold $\mathcal{M}$ with degree n symplectic structure ω, that induces a graded Poisson bracket $\{\cdot, \cdot\}$ on $\mathcal{C}^\infty(\mathcal{M})$
2. Hamiltonian function Θ such that the **classical master equation** $\{\Theta, \Theta\} = 0$ holds
3. Hamiltonian vector field $Q = \{\Theta, \cdot\}$, that obeys $\mathcal{L}_Q \omega = 0$

The QP-manifold construction generates a BV-formalism, with coordinates of (ghost-)degree (*commuting* and *anticommuting*)

Preliminaries: Courant Algebroids and QP-Manifolds

Courant algebroid on vector bundle E

Vector bundle E over M with fiber metric $\langle \cdot\,, \cdot \rangle$, bundle map $\rho : E \longrightarrow TM$ and Dorfman bracket $[-,-]_D$ on $\Gamma(E)$ satisfying consistency conditions

QP-manifold $(\mathcal{M}, \omega, \Theta)$

1. Nonnegatively graded manifold $\mathcal{M}$ with degree n symplectic structure ω, that induces a graded Poisson bracket $\{\cdot, \cdot\}$ on $\mathcal{C}^\infty(\mathcal{M})$
2. Hamiltonian function Θ such that the **classical master equation** $\{\Theta, \Theta\} = 0$ holds
3. Hamiltonian vector field $Q = \{\Theta, \cdot\}$, that obeys $\mathcal{L}_Q \omega = 0$

The QP-manifold construction generates a BV-formalism, with coordinates of (ghost-)degree (*commuting* and *anticommuting*)

QP-manifold of degree 2 $\equiv$ Courant algebroid with vector bundle E

Poisson Courant Algebroids From QP-Manifolds

Special case of a Courant algebroid: Courant algebroid on Poisson manifold, based on generalized geometry on the cotangent bundle. Interesting for the formulation of non-geometric fluxes in string theory

Poisson Courant Algebroids From QP-Manifolds

Special case of a Courant algebroid: Courant algebroid on Poisson manifold, based on generalized geometry on the cotangent bundle. Interesting for the formulation of non-geometric fluxes in string theory

Poisson Courant Algebroid

$$(E = TM \oplus T^*M, \langle -, - \rangle, [-, -]_D, \rho = 0 \oplus \theta^\sharp)$$

Vector bundle $E = TM \oplus T^*M \to M$
(M, θ) Poisson manifold with Poisson structure $\theta \in \Gamma(\wedge^2 TM)$
$R \in \Gamma(\wedge^3 TM)$ such that $[\theta, R]_S = 0$ (*Schouten bracket* on $\wedge^\bullet TM$)
Bundle map $\rho : TM \oplus T^*M \to TM$ defined by $\rho(X + \alpha) = \theta^{ij}\alpha_i(x)\frac{\partial}{\partial x^j}$
Bilinear operation

$$[X + \alpha, Y + \beta]^\theta_D \equiv [\alpha, \beta]_\theta + L^\theta_\alpha Y - \iota_\beta d_\theta X - \iota_\alpha \iota_\beta R,$$

where $X + \alpha, Y + \beta \in \Gamma(TM \oplus T^*M)$
Lie bracket on T^*M (Koszul bracket) $[-, -]_\theta : T^*M \times T^*M \to T^*M$
Inner product $\langle -, - \rangle$ on $TM \oplus T^*M$

Poisson Courant Algebroids From QP-Manifolds

QP-formulation of the Poisson Courant algebroid on E

Graded manifold $\mathcal{M} = T^*[2]T[1]M$, **embedding map** $j : E \otimes TM \to \mathcal{M}$
Local coordinates (x^i, ξ_i, q^i, p_i) of (ghost-)degree $(0, 2, 1, 1)$
Symplectic form $\omega = \delta x^i \wedge \delta \xi_i + \delta q^i \wedge \delta p_j$ induces graded P. bracket $\{\cdot, \cdot\}$

Poisson Courant Algebroids From QP-Manifolds

QP-formulation of the Poisson Courant algebroid on E

Graded manifold $\mathcal{M} = T^*[2]T[1]M$, **embedding map** $j : E \otimes TM \to \mathcal{M}$
Local coordinates (x^i, ξ_i, q^i, p_i) of (ghost-)degree $(0, 2, 1, 1)$
Symplectic form $\omega = \delta x^i \wedge \delta\xi_i + \delta q^i \wedge \delta p_j$ induces graded P. bracket $\{\cdot, \cdot\}$
Hamiltonian function

$$\Theta = \theta^{ij}(x)\xi_i p_j - \frac{1}{2}\frac{\partial\theta^{jk}}{\partial x^i}(x)q^i p_j p_k + \frac{1}{3!}R^{ijk}(x)p_i p_j p_k$$

Poisson Courant Algebroids From QP-Manifolds

QP-formulation of the Poisson Courant algebroid on E

Graded manifold $\mathcal{M} = T^*[2]T[1]M$, **embedding map** $j : E \otimes TM \to \mathcal{M}$
Local coordinates (x^i, ξ_i, q^i, p_i) of (ghost-)degree $(0, 2, 1, 1)$
Symplectic form $\omega = \delta x^i \wedge \delta\xi_i + \delta q^i \wedge \delta p_j$ induces graded P. bracket $\{\cdot, \cdot\}$
Hamiltonian function

$$\Theta = \theta^{ij}(x)\xi_i p_j - \frac{1}{2}\frac{\partial\theta^{jk}}{\partial x^i}(x) q^i p_j p_k + \frac{1}{3!}R^{ijk}(x) p_i p_j p_k$$

Derived brackets recover operations on $\Gamma(E)$, for example:

$$[X + \alpha, Y + \beta]_D^\theta = j^*\{\{X^i(x)p_i + \alpha_i(x)q^i, \Theta\}, Y^j(x)p_j + \beta_j(x)q^j\}$$

Poisson Courant Algebroids From QP-Manifolds

QP-formulation of the Poisson Courant algebroid on E

Graded manifold $\mathcal{M} = T^*[2]T[1]M$, **embedding map** $j : E \otimes TM \to \mathcal{M}$
Local coordinates (x^i, ξ_i, q^i, p_i) of (ghost-)degree $(0, 2, 1, 1)$
Symplectic form $\omega = \delta x^i \wedge \delta\xi_i + \delta q^i \wedge \delta p_j$ induces graded P. bracket $\{\cdot, \cdot\}$
Hamiltonian function

$$\Theta = \theta^{ij}(x)\xi_i p_j - \frac{1}{2}\frac{\partial\theta^{jk}}{\partial x^i}(x) q^i p_j p_k + \frac{1}{3!} R^{ijk}(x) p_i p_j p_k$$

Derived brackets recover operations on $\Gamma(E)$, for example:

$$[X + \alpha, Y + \beta]_D^\theta = j^*\{\{X^i(x)p_i + \alpha_i(x)q^i, \Theta\}, Y^j(x)p_j + \beta_j(x)q^j\}$$

Classical master equation $\{\Theta, \Theta\} = 0$ gives structural restrictions

Poisson Courant Algebroids From QP-Manifolds

QP-formulation of the Poisson Courant algebroid on E

Graded manifold $\mathcal{M} = T^*[2]T[1]M$, **embedding map** $j : E \otimes TM \to \mathcal{M}$
Local coordinates (x^i, ξ_i, q^i, p_i) of (ghost-)degree $(0, 2, 1, 1)$
Symplectic form $\omega = \delta x^i \wedge \delta \xi_i + \delta q^i \wedge \delta p_j$ induces graded P. bracket $\{\cdot, \cdot\}$
Hamiltonian function

$$\Theta = \theta^{ij}(x)\xi_i p_j - \frac{1}{2}\frac{\partial \theta^{jk}}{\partial x^i}(x) q^i p_j p_k + \frac{1}{3!} R^{ijk}(x) p_i p_j p_k$$

Derived brackets recover operations on $\Gamma(E)$, for example:

$$[X + \alpha, Y + \beta]_D^\theta = j^*\{\{X^i(x)p_i + \alpha_i(x)q^i, \Theta\}, Y^j(x)p_j + \beta_j(x)q^j\}$$

Classical master equation $\{\Theta, \Theta\} = 0$ gives structural restrictions

Next step: A topological membrane is described by a Courant algebroid. Construct the topological membrane model with R-flux from this algebroid.

Construction of the Topological Membrane

Describe embedding $X \to \mathcal{M}$ of topological membrane into target space

Construction of the Topological Membrane

Describe embedding $X \to \mathcal{M}$ of topological membrane into target space

1. Target space: our QP-manifold $(\mathcal{M}, \omega, \Theta)$
2. Topological membrane: dg-manifold $(\mathcal{X} = T[1]X, D, \mu)$
   (X is 3-dim. membrane worldvolume)

Construction of the Topological Membrane

Describe embedding $X \to \mathcal{M}$ of topological membrane into target space

1. Target space: our QP-manifold $(\mathcal{M}, \omega, \Theta)$
2. Topological membrane: dg-manifold $(\mathcal{X} = T[1]X, D, \mu)$
   (X is 3-dim. membrane worldvolume)

Alexandrov-Kontsevich-Schwarz-Zaboronsky formulation gives QP-structure on $\mathrm{Map}(T[1]X, \mathcal{M})$ (mapping space)

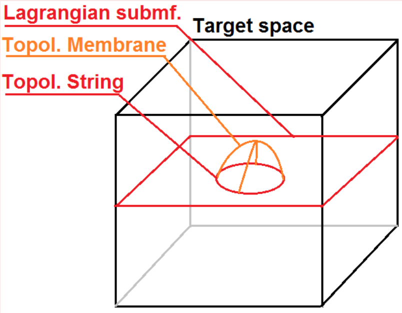
1. Graded symplectic structure $\boldsymbol{\omega} = \int_{\mathcal{X}} \mu \mathrm{ev}^* \omega$
2. Hamiltonian function S
3. Master equation $\{S, S\} = 0$ holds and leads to a **BV-formalism of a topological membrane**
4. Target space variables $\mapsto$ Superfields.
   Degree zero part is physical degree

Twisting the Topological Open Membrane

The **topological string model with R-flux** is the boundary of the topological open membrane. It is a twisted Poisson sigma model.

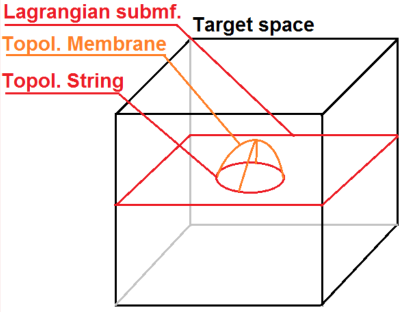
Twisting the Topological Open Membrane

The **topological string model with R-flux** is the boundary of the topological open membrane. It is a twisted Poisson sigma model.



Twisting the Topological Open Membrane

The **topological string model with R-flux** is the boundary of the topological open membrane. It is a twisted Poisson sigma model.



Twisting $(\mathcal{M}, \omega, \Theta)$

Twist of the topological open membrane ($\partial\mathcal{X} \neq \emptyset$) by canonical transformation generates a boundary term by changing the Lagrangian submanifold $\mathcal{L}$ with respect to ω

Construction of the R-flux string sigma model

Topological open membrane on $\mathrm{Map}(T[1]X, \mathcal{M})$

1. 3-dimensional membrane worldvolume X with non-zero boundary $\partial X \neq \emptyset$
2. **Symplectic structure** $\quad \boldsymbol{\omega} = \int_{\mathcal{X}} \mu \left(\delta \boldsymbol{x}^i \wedge \delta \boldsymbol{\xi}_i + \delta \boldsymbol{q}^i \wedge \delta \boldsymbol{p}_i\right)$

Hamiltonian function

$$S = \int_{\mathcal{X}} \mu \Big(-\boldsymbol{\xi}_i \boldsymbol{d}\boldsymbol{x}^i + \boldsymbol{p}_i \boldsymbol{d}\boldsymbol{q}^i + \theta^{ij}(\boldsymbol{x})\boldsymbol{\xi}_i \boldsymbol{p}_j - \frac{1}{2}\frac{\partial \theta^{jk}}{\partial x^i}(\boldsymbol{x})\boldsymbol{q}^i \boldsymbol{p}_j \boldsymbol{p}_k + \frac{1}{3!} R^{ijk}(\boldsymbol{x})\boldsymbol{p}_i \boldsymbol{p}_j \boldsymbol{p}_k\Big)$$

Construction of the R-flux string sigma model

Topological open membrane on Map$(T[1]X, \mathcal{M})$

1. 3-dimensional membrane worldvolume X with non-zero boundary $\partial X \neq \emptyset$
2. **Symplectic structure** $\boldsymbol{\omega} = \int_{\mathcal{X}} \mu \, (\delta \boldsymbol{x}^i \wedge \delta \boldsymbol{\xi}_i + \delta \boldsymbol{q}^i \wedge \delta \boldsymbol{p}_i)$

   Hamiltonian function
$$S = \int_{\mathcal{X}} \mu \Big(-\boldsymbol{\xi}_i \boldsymbol{d} \boldsymbol{x}^i + \boldsymbol{p}_i \boldsymbol{d} \boldsymbol{q}^i + \theta^{ij}(\boldsymbol{x}) \boldsymbol{\xi}_i \boldsymbol{p}_j - \frac{1}{2} \frac{\partial \theta^{jk}}{\partial x^i}(\boldsymbol{x}) \boldsymbol{q}^i \boldsymbol{p}_j \boldsymbol{p}_k + \frac{1}{3!} R^{ijk}(\boldsymbol{x}) \boldsymbol{p}_i \boldsymbol{p}_j \boldsymbol{p}_k \Big)$$
3. $\delta S|_{\partial \mathcal{X}} = 0$ determines boundary conditions

Construction of the R-flux string sigma model

Topological open membrane on $\mathrm{Map}(T[1]X, \mathcal{M})$

1. 3-dimensional membrane worldvolume X with non-zero boundary $\partial X \neq \emptyset$
2. **Symplectic structure** $\quad \boldsymbol{\omega} = \int_{\mathcal{X}} \mu \left(\delta \boldsymbol{x}^i \wedge \delta \boldsymbol{\xi}_i + \delta \boldsymbol{q}^i \wedge \delta \boldsymbol{p}_i \right)$

   Hamiltonian function
$$S = \int_{\mathcal{X}} \mu \Big(-\boldsymbol{\xi}_i \boldsymbol{d}\boldsymbol{x}^i + \boldsymbol{p}_i \boldsymbol{d}\boldsymbol{q}^i + \theta^{ij}(\boldsymbol{x}) \boldsymbol{\xi}_i \boldsymbol{p}_j - \frac{1}{2} \frac{\partial \theta^{jk}}{\partial x^i}(\boldsymbol{x}) \boldsymbol{q}^i \boldsymbol{p}_j \boldsymbol{p}_k + \frac{1}{3!} R^{ijk}(\boldsymbol{x}) \boldsymbol{p}_i \boldsymbol{p}_j \boldsymbol{p}_k \Big)$$
3. $\delta S|_{\partial \mathcal{X}} = 0$ determines boundary conditions
4. Twist by $\alpha = \frac{1}{2} B_{ij}(x) q^i q^j$ leads to twisted master equation $H = dB = \wedge^3 B^\flat R$ on the boundary

Construction of the R-flux String Sigma Model

Topological string with R-flux WZ term in two dimensions

For $B = \theta^{-1}$, the boundary model is a twisted AKSZ sigma model in two dimensions with WZ term

$$
\begin{aligned}
S \;=\; & \int_{\partial\mathcal{X}} \mu_{\partial\mathcal{X}} \; (\theta^{-1})_{ij}\boldsymbol{q}^i \boldsymbol{d}\boldsymbol{x}^j - \frac{1}{2} B_{ij}(\boldsymbol{x})\boldsymbol{q}^i\boldsymbol{q}^j \\
& + \int_{\mathcal{X}} \mu \; \frac{1}{3!} R^{ijk}(\boldsymbol{x})(\theta^{-1})_{il}(\theta^{-1})_{jm}(\theta^{-1})_{kn}\boldsymbol{d}\boldsymbol{x}^l \boldsymbol{d}\boldsymbol{x}^m \boldsymbol{d}\boldsymbol{x}^n
\end{aligned}
$$

Construction of the R-flux String Sigma Model

Topological string with R-flux WZ term in two dimensions

For $B = \theta^{-1}$, the boundary model is a twisted AKSZ sigma model in two dimensions with WZ term

$$\begin{aligned} S &= \int_{\partial\mathcal{X}} \mu_{\partial\mathcal{X}} \ (\theta^{-1})_{ij} \boldsymbol{q}^i \boldsymbol{dx}^j - \frac{1}{2} B_{ij}(\boldsymbol{x}) \boldsymbol{q}^i \boldsymbol{q}^j \\ &\quad + \int_{\mathcal{X}} \mu \ \frac{1}{3!} R^{ijk}(\boldsymbol{x}) (\theta^{-1})_{il} (\theta^{-1})_{jm} (\theta^{-1})_{kn} \boldsymbol{dx}^l \boldsymbol{dx}^m \boldsymbol{dx}^n \end{aligned}$$

1. It is a Poisson sigma model deformed by an R-flux WZ term and equivalent to the standard H-twisted Poisson sigma model

Construction of the R-flux String Sigma Model

Topological string with R-flux WZ term in two dimensions

For $B = \theta^{-1}$, the boundary model is a twisted AKSZ sigma model in two dimensions with WZ term

$$
\begin{aligned}
S &= \int_{\partial\mathcal{X}} \mu_{\partial\mathcal{X}} \, (\theta^{-1})_{ij} \boldsymbol{q}^i \boldsymbol{dx}^j - \frac{1}{2} B_{ij}(\boldsymbol{x}) \boldsymbol{q}^i \boldsymbol{q}^j \\
&\quad + \int_{\mathcal{X}} \mu \, \frac{1}{3!} R^{ijk}(\boldsymbol{x}) (\theta^{-1})_{il} (\theta^{-1})_{jm} (\theta^{-1})_{kn} \boldsymbol{dx}^l \boldsymbol{dx}^m \boldsymbol{dx}^n
\end{aligned}
$$

1. It is a Poisson sigma model deformed by an R-flux WZ term and equivalent to the standard H-twisted Poisson sigma model
2. Through the existence of the Poisson tensor θ, this model realizes a lifting to a topological membrane theory, that is different from the lifting of the H-twisted Poisson sigma model

Duality between H-Flux and R-Flux Geometry

1. Standard Courant algebroid with H-flux and Poisson Courant algebroid with R-flux both are realized on $(T^*[2]T[1]M, \omega)$ with different Hamiltonian functions

$$\Theta_H = \xi_i q^i + \frac{1}{3!} H_{ijk} q^i q^j q^k$$

$$\Theta_R = \theta^{ij} \xi_i p_j - \frac{1}{2} \frac{\partial \theta^{jk}}{\partial x^i} q^i p_j p_k + \frac{1}{3!} R^{ijk} p_i p_j p_k$$

Duality between H-Flux and R-Flux Geometry

1. Standard Courant algebroid with H-flux and Poisson Courant algebroid with R-flux both are realized on $(T^*[2]T[1]M, \omega)$ with different Hamiltonian functions

$$\Theta_H = \xi_i q^i + \frac{1}{3!} H_{ijk} q^i q^j q^k$$

$$\Theta_R = \theta^{ij} \xi_i p_j - \frac{1}{2} \frac{\partial \theta^{jk}}{\partial x^i} q^i p_j p_k + \frac{1}{3!} R^{ijk} p_i p_j p_k$$

2. **Duality transformation between H-flux and R-flux**
Symplectomorphism $T : \Theta_H \mapsto \Theta_R = e^{\delta_b} e^{\delta_\beta} \Theta_H$ on $(T^*[2]T[1]M, \omega)$ where canonical transf. e^{δ_b} and e^{δ_β} generate b- and β-transform

Duality between H-Flux and R-Flux Geometry

1. Standard Courant algebroid with H-flux and Poisson Courant algebroid with R-flux both are realized on $(T^*[2]T[1]M, \omega)$ with different Hamiltonian functions

$$\Theta_H = \xi_i q^i + \frac{1}{3!} H_{ijk} q^i q^j q^k$$

$$\Theta_R = \theta^{ij} \xi_i p_j - \frac{1}{2} \frac{\partial \theta^{jk}}{\partial x^i} q^i p_j p_k + \frac{1}{3!} R^{ijk} p_i p_j p_k$$

2. **Duality transformation between H-flux and R-flux**
Symplectomorphism $T : \Theta_H \mapsto \Theta_R = e^{\delta_b} e^{\delta_\beta} \Theta_H$ on $(T^*[2]T[1]M, \omega)$ where canonical transf. e^{δ_b} and e^{δ_β} generate b- and β-transform

On the mapping space The duality transformation between H-flux and R-flux can be interpreted as the **change of boundary conditions of the topological membrane**

Courant Algebroids from Lie 3-Algebroids and Duality

1. Duality between H-flux and R-flux can be rephrased using the structure of a *Lie 3-algebroid*

1. Duality between H-flux and R-flux can be rephrased using the structure of a *Lie 3-algebroid*
2. This Lie 3-algebroid is constructed by a QP-manifold of degree 3, which contains a Hamiltonian function Θ of degree 4 and induces higher Dorfman brackets

Courant Algebroids from Lie 3-Algebroids and Duality

1. Duality between H-flux and R-flux can be rephrased using the structure of a *Lie 3-algebroid*
2. This Lie 3-algebroid is constructed by a QP-manifold of degree 3, which contains a Hamiltonian function Θ of degree 4 and induces higher Dorfman brackets
3. Twisted Lagrangian submanifolds within the Lie 3-algebroid lead to Courant algebroids realizing different flux configurations

Courant Algebroids from Lie 3-Algebroids and Duality

1. Duality between H-flux and R-flux can be rephrased using the structure of a *Lie 3-algebroid*
2. This Lie 3-algebroid is constructed by a QP-manifold of degree 3, which contains a Hamiltonian function Θ of degree 4 and induces higher Dorfman brackets
3. Twisted Lagrangian submanifolds within the Lie 3-algebroid lead to Courant algebroids realizing different flux configurations

Algebroid Duality

Symplectomorphism of QP manifolds $T : \mathcal{M}_1 \to \mathcal{M}_2$ that

- Preserves the QP structure
- Transformes Lagrangian submanifolds $T : \mathcal{L}_1 \to \mathcal{L}_2$

Summary and Outlook

1. Constructed a Courant algebroid on Poisson manifold, based on generalized geometry on the cotangent bundle, using QP-manifolds

Summary and Outlook

1. Constructed a Courant algebroid on Poisson manifold, based on generalized geometry on the cotangent bundle, using QP-manifolds
2. Developed a topological string model with R-flux, which is equivalent to the H-twisted Poisson sigma model, but lifts to a different topological membrane theory

Summary and Outlook

1. Constructed a Courant algebroid on Poisson manifold, based on generalized geometry on the cotangent bundle, using QP-manifolds
2. Developed a topological string model with R-flux, which is equivalent to the H-twisted Poisson sigma model, but lifts to a different topological membrane theory
3. Realized a duality transformation that relates the topological string model with R-flux to the topological string with H-flux using a symplectomorphism of QP-manifolds

Summary and Outlook

1. Constructed a Courant algebroid on Poisson manifold, based on generalized geometry on the cotangent bundle, using QP-manifolds
2. Developed a topological string model with R-flux, which is equivalent to the H-twisted Poisson sigma model, but lifts to a different topological membrane theory
3. Realized a duality transformation that relates the topological string model with R-flux to the topological string with H-flux using a symplectomorphism of QP-manifolds
4. Rephrased this duality transformation as an *algebroid duality* between substructures of a Lie 3-algebroid

Summary and Outlook

1. Constructed a Courant algebroid on Poisson manifold, based on generalized geometry on the cotangent bundle, using QP-manifolds
2. Developed a topological string model with R-flux, which is equivalent to the H-twisted Poisson sigma model, but lifts to a different topological membrane theory
3. Realized a duality transformation that relates the topological string model with R-flux to the topological string with H-flux using a symplectomorphism of QP-manifolds
4. Rephrased this duality transformation as an *algebroid duality* between substructures of a Lie 3-algebroid

1. We are aiming to use this framework to find a complete generalization of topological T-duality, that connects all (H, F, Q and R) fluxes