Casimir Energy of the Universe and the Cosmological Constant [2]

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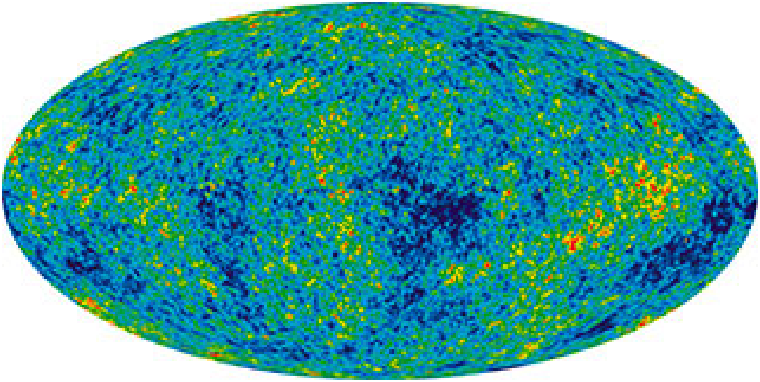
[2]Related ref. arXiv;131021(Proc. of APPC12), arXiv:1404.6627(Tribology Int. 93PA(2016)446, Elsevier)

Sec 1. Introduction: a.Cosmic Microwave Background Radiation 温度ゆらぎ

Figure: WMAP, Cosmic Microwave Background Radiation



Sec 1. Introduction b. History

Cosmic Microwave Background Radiation Observation Data is accumulating

- Dark Matter, Dark Energy ($\sim$ Cosmological Term)
- 'Micro' Theory of Gravity : Divergence Problem(Infra-red, Ultra-violet)
- Quntum Field Theory on dS_4 is not defined
  '01 E. Witten, inf-dim Hilbert space
  '03 J. Maldacena, Non-Gaussian ...
  '06 S. Weinberg , in-in formalism
  Schwinger-Keldysh formalism in '07 A.M. Polyakov
  '09- T. Tanaka & Y. Urakawa
  '11- H. Kitamoto & Y. Kitazawa

Sec 1. Introduction c. Noticeable Words and References

- A.M. Polyakov, '09
  Dark energy, like the black body radiation 150 years ago, hides secrets of fundamental physics
- E. Verlinde, '10
  Emergent Gravity
- A. Strominger et al, '11
  From Navier-Stokes to Einstein, arXiv:1101.2451
  From Petrov-Einstein to Navier-Stokes, arXiv:1104.5502

Sec 2. Background Field Formalism a.

B.S. DeWitt, 1967; G. 'tHooft, 1973; I.Y. Aref'eva, A.A. Slavnov & L.D. Faddeev, 1974

$\Phi(x)$: Scalar Field, $g_{\mu\nu}(x)$: Gravitational Field, $V(\Phi) = \frac{\sigma}{4!}\Phi^4, \ \sigma > 0$

$$S[\Phi; g_{\mu\nu}] = \int d^4x\sqrt{g}\left(\frac{-(R-2\lambda)}{16\pi G_N} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{m^2}{2}\Phi^2 - V(\Phi)\right) \qquad (1)$$

Background Expansion: $\Phi = \Phi_{cl} + \varphi$, NOT expand $g_{\mu\nu}$ (2)

Sec.2 Background Field Formalism b.

$$\mathrm{e}^{i\Gamma[\Phi_{cl};g_{\mu\nu}]} = \int \mathcal{D}\varphi \exp i \left\{ S[\Phi_{cl}+\varphi; g_{\mu\nu}] - \frac{\delta S[\Phi_{cl}; g_{\mu\nu}]}{\delta \Phi_{cl}} \varphi \right\} \Gamma[\Phi_{cl}; g_{\mu\nu}] \ ;$$

Φ_{cl} is perturbatively solved, at the tree level, as

$$\Phi_{cl}(x) = \Phi_0(x) + \int D(x-x') \ \sqrt{g} \frac{\delta V(\Phi_{cl})}{\delta \Phi_{cl}}\bigg|_{x'} d^4x' \quad ,$$

$$\sqrt{g}(\nabla^2 - m^2)\Phi_0 = 0 \quad , \quad \sqrt{g}(\nabla^2 - m^2)D(x-x') = \delta^4(x-x') \quad . \ (4)$$

$\Phi_0(x)$: asymptotic fields for n-point function of scattering matrix.

Sec.2 Background Field Formalism c. $x_{cl}(0)$, $x_{cl}(\beta)$

Aref'eva, Slavnov & Faddeev 1974
Harmonic Oscillator (Feynman's text '72)

Density Matrix

$$\rho(x_2, x_1; \beta) = \int \mathcal{D}x(\tau) \exp\left[-\frac{1}{\hbar}\int_0^\beta \left(\frac{\dot{x}^2}{2} + \frac{\omega^2}{2}x^2\right) d\tau\right]_{x(0)=x_1, x(\beta)=x_2}$$

Background Field Expansion: $x(\tau) = x_{cl}(\tau) + y(\tau)$

$$\rho(x_2, x_1; \beta) = \sqrt{\frac{1}{2\pi\hbar\beta}} \exp\left[-\frac{1}{\hbar}\int_0^\beta \left(\frac{\dot{x}_{cl}^2}{2} + \frac{\omega^2}{2}{x_{cl}}^2\right) d\tau\right] \quad . \tag{6}$$

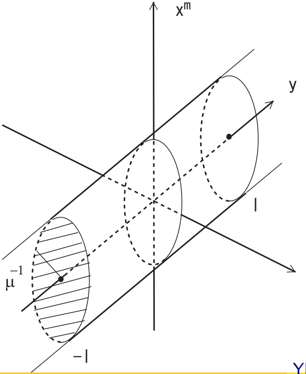
Transition probability is given by

$$\frac{\delta}{\delta x_{cl}(0)} \frac{\delta}{\delta x_{cl}(\beta)} \rho(x_2, x_1; \beta) \quad . \tag{7}$$

Sec 3. 5D Electromagnetism: a.Flat Geometry

5D Electromagnetism on the *flat* geometry
$S_{EM} = \int d^4xdy\sqrt{-G}\{-\frac{1}{4}F_{MN}F^{MN}\}, \ (G_{MN}) = \text{diag}(-1,1,1,1,1)$
The extra space is *periodic* (periodicity $2l$) and Z_2-parity

Figure: IR-regularized geometry of 5D flat space (8).



Sec 3. 5D EM.: b.Casimir Energy

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + dy^2,\ -\infty < x^\mu, y < \infty,\ y \to y + 2l,\ y \leftrightarrow -y \quad ,$$
$$(\eta_{\mu\nu}) = \mathrm{diag}(-1,1,1,1)\ , (X^M) = (X^\mu = x^\mu, X^5 = y) \equiv (x,y)\ ,$$
$$M, N = 0,1,2,3,5;\ \mu,\nu = 0,1,2,3. \ ($$

The Casimir energy $\quad e^{-l^4 E_{Cas}} \equiv \int \mathcal{D}A_M \exp\{iS_{EM}\}, \quad \tilde{p} \equiv \sqrt{p_\mu p^\mu}$

$$E_{Cas}(\Lambda, l) = \frac{2\pi^2}{(2\pi)^4}\int_{1/l}^{\Lambda} d\tilde{p} \int_{1/\Lambda}^{l} dy\ \tilde{p}^3 W(\tilde{p}, y) F(\tilde{p}, y), \quad F(\tilde{p}, y) \equiv$$
$$F^-(\tilde{p}, y) + 4F^+(\tilde{p}, y) = \int_{\tilde{p}}^{\Lambda} d\tilde{k} \frac{-3\cosh \tilde{k}(2y - l) - 5\cosh \tilde{k}l}{2\sinh(\tilde{k}l)}. \quad (9)$$

Λ the 4D-momentum cutoff; $W(\tilde{p}, y)$ the weight function

Sec 3. 5D EM.: b2.Heat Kernel, Propagator

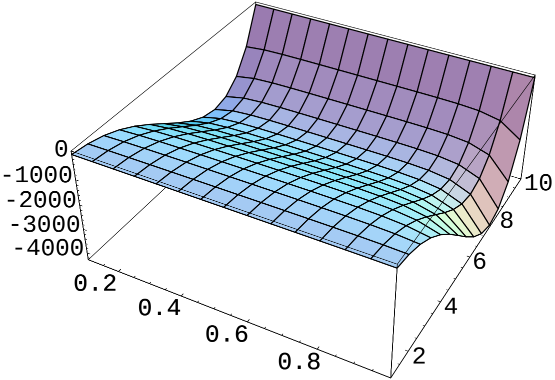
$$G_p^{\mp}(y,y') \equiv \int_0^\infty dt \;\; < y|e^{-(p^2-\partial_y^2)t}|y' > \Big|_{P=\mp} ,$$

$$(p^2-\partial_y^2)G_p^{\mp}(y,y') = \frac{1}{2}\{\hat{\delta}(y-y') \mp \hat{\delta}(y+y')\}$$

$$F^{\mp}(\tilde{p},y) \equiv \int_{p^2}^\infty dk^2 G_k^{\mp}(y,y) \quad . \qquad (10)$$

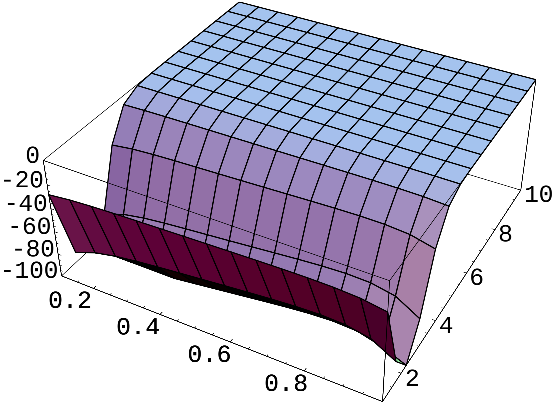
Sec 3. 5D EM: b'. $\tilde{p}^3 F(\tilde{p}, y)$ Graph

Figure: Graph of $\tilde{p}^3 F(\tilde{p}, y)$. $l = 1,\ \Lambda = 10,\ 0.1 \le y < 1, \quad 1 \le \tilde{p} \le 10.$



Sec 3. 5D EM: b" .$\tilde{p}^3 W_1(\tilde{p}, y) F(\tilde{p}, y)$ Graph

Figure: Graph of $\tilde{p}^3 W_1(\tilde{p}, y) F(\tilde{p}, y)$.
$l = 1,\ \Lambda = 10,\ 0.1 \leq y < 1, \quad 1 \leq \tilde{p} \leq 10.$



Sec 3. 5D EM: c.Casimir Energy

1) Un-weighted case: $W = 1$

Un-restricted integral region :

$$E_{Cas}(\Lambda, l) = \frac{1}{8\pi^2}\left[-0.1249 l\Lambda^5 - (1.41, 0.706, 0.353) \times 10^{-5}\ l\Lambda^5 \ln(l\Lambda)\right]$$

Randall-Schwartz integral region : $E_{Cas}^{RS} = \frac{1}{8\pi^2}[-0.0893\ \Lambda^4]$

2) Weighted case $E_{Cas}^{W} =$

$$\left\{\begin{array}{ll} -2.50\frac{\Lambda}{l^3} + (-0.142, 1.09, 1.13) \times 10^{-4}\frac{\Lambda\ln(l\Lambda)}{l^3} & \text{for}\quad W_1 \\ -6.03 \times 10^{-2}\frac{\Lambda}{l^3} & \text{for}\quad W_2 \\ -2.51\frac{\Lambda}{l^3} + (19.5, 11.6, 6.68) \times 10^{-4}\frac{\Lambda\ln(l\Lambda)}{l^3} & \text{for}\quad W_8 \end{array}\right. \qquad (12)$$

$W_1 = (1/N_1)\mathrm{e}^{-(1/2)l^2\tilde{p}^2-(1/2)y^2/l^2}$: elliptic
$W_2 = (1/N_2)\mathrm{e}^{-\tilde{p}y}$: hyperbolic
$W_8 = (1/N_8)\mathrm{e}^{-(l^2/2)(\tilde{p}^2+1/y^2)}$: reciprocal

Sec 3. 5D EM: d.Periodicity 2l renormalizes

The renormalization of the compactification size l.

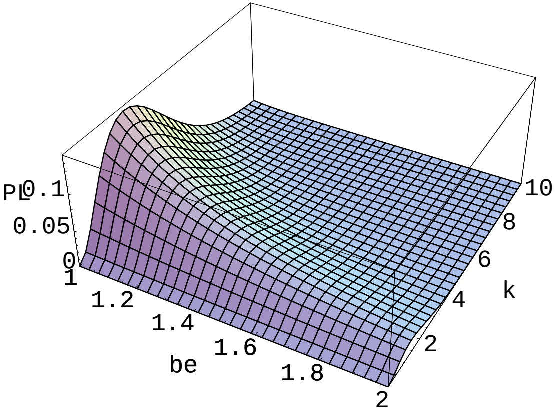
$$E^W_{Cas}/\Lambda l = -\frac{\alpha}{l^4}(1 - 4c\ln(l\Lambda)) = -\frac{\alpha}{l'^4} \quad , \tag{13}$$

The quantity Λl is the normalization factor.

$$\begin{aligned} l' &= l(1 + c\ \ln(l\Lambda)) \quad , \\ \text{Beta func.}:\ \beta &= \frac{\partial \ln(l'/l)}{\partial \ln \Lambda} = c \quad . \end{aligned} \tag{14}$$

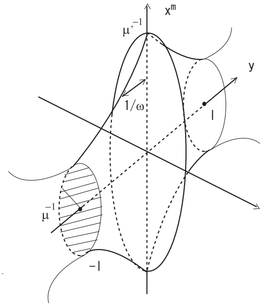
Sec 3. Notice: e.Casimir Energy of 4D EM

Figure: Graph of Planck's radiation formula.
$\mathcal{P}(\beta,k)=\frac{1}{(c\hbar)^3}\frac{1}{\pi^2}k^3/(\mathrm{e}^{\beta k}-1)\ \ (1\leq\beta\leq 2,\ 0.01\leq k\leq 10)$.



Sec 4. 5D Warped Model: a. Geometry

Figure: IR-regularized geometry of 5D warped space (15).



$$ds^2 = \frac{1}{\omega^2 z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) = \mathrm{e}^{-2\omega|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,\ |z| = \frac{1}{\omega}\mathrm{e}^{\omega|y|}.$$

Sec 4. 5D Warped Model.: b.Posi/Mom Propagator

$$G_p^{\mp}(z,z') = \mp\frac{\omega^3}{2}z^2 z'^2 \times$$
$$\frac{\{\mathbf{I}_0(\frac{\tilde{p}}{\omega})\mathbf{K}_0(\tilde{p}z) \mp \mathbf{K}_0(\frac{\tilde{p}}{\omega})\mathbf{I}_0(\tilde{p}z)\}\{\mathbf{I}_0(\frac{\tilde{p}}{T})\mathbf{K}_0(\tilde{p}z') \mp \mathbf{K}_0(\frac{\tilde{p}}{T})\mathbf{I}_0(\tilde{p}z')\}}{\mathbf{I}_0(\frac{\tilde{p}}{T})\mathbf{K}_0(\frac{\tilde{p}}{\omega}) - \mathbf{K}_0(\frac{\tilde{p}}{T})\mathbf{I}_0(\frac{\tilde{p}}{\omega})} \quad ,$$
$$\tilde{p} \equiv \sqrt{p^2} \quad , \quad p^2 \geq 0 \text{ (space-like)} \quad .(16)$$

Λ-regularized Casimir energy.

$$E_{Cas}^{\Lambda,\mp}(\omega, T) = \int \frac{d^4p}{(2\pi)^4}\bigg|_{\tilde{p}\leq\Lambda} \int_{1/\omega}^{1/T} dz\ F^{\mp}(\tilde{p}, z) \quad ,$$
$$F^{\mp}(\tilde{p},z) = \frac{2}{(\omega z)^3}\int_{\tilde{p}}^{\Lambda} \tilde{k}\ G_k^{\mp}(z,z)d\tilde{k} \equiv \int_{\tilde{p}}^{\Lambda} \mathcal{F}^{\mp}(\tilde{k},z)d\tilde{k} \quad , \quad (17)$$

Sec 4. 5D Warped Model: b'.E_{Cas}, Heat Kernel

$$
\begin{aligned}
\mathrm{e}^{-T^{-4}E_{Cas}} = \int \mathcal{D}\Phi_p(z) \exp\Bigg[i \int \frac{d^4p}{(2\pi)^4} 2 \int_{1/\omega}^{1/T} dz \\
\left\{ \frac{1}{2}\Phi_p(z)s(z)(s(z)^{-1}\hat{L}_z - p^2)\Phi_p(z) \right\} \Bigg]
\end{aligned}
$$

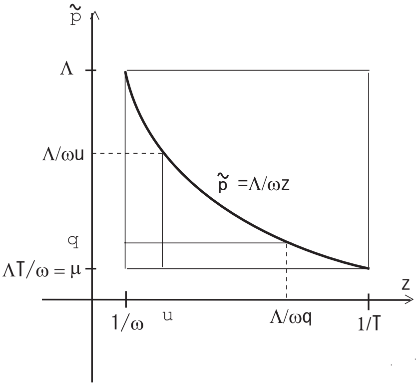
$$
T \equiv \omega \mathrm{e}^{-\omega l},\ \ s(z) = \frac{1}{(\omega z)^3} \quad , \quad \hat{L}_z \equiv \frac{d}{dz}\frac{1}{(\omega z)^3}\frac{d}{dz} - \frac{m^2}{(\omega z)^5}
$$

$$
\begin{aligned}
H_p^{\mp}(z,z';t) = (z|\mathrm{e}^{-(-s^{-1}\hat{L}_z + p^2)t}|z')|_{P=\mp}, \\
\{\frac{\partial}{\partial t} - (s^{-1}\hat{L}_z - p^2)\}H_p(z,z';t) = 0, \\
G_p^{\mp}(z,z') \equiv \int_0^{\infty} dt\ H_p^{\mp}(z,z';t) \quad . \qquad (18)
\end{aligned}
$$

Sec 4. 5D Warped Model: c.$(z, \tilde{p})$ integration region

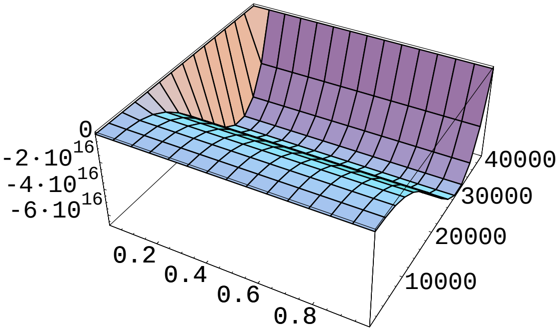
Λ: UV-regularization, $\mu \equiv \Lambda \frac{T}{\omega}$: IR-regularization

Figure: Space of $(z,\tilde{p})$ for the integration.



Sec 4. 5D Warped Model:: d. $-\frac{1}{2}\tilde{p}^3 F^-(\tilde{p}, z)$ graph

Figure: Behaviour of $(-1/2)\tilde{p}^3 F^-(\tilde{p}, z)$ (17).
$T = 1, \omega = 10^4, \Lambda = 4 \cdot 10^4$. $1.0001/\omega \le z < 0.9999/T$, $\Lambda T/\omega \le \tilde{p} \le \Lambda$.



Sec 5. Weight Func. and Casimir Ene.: a.Weight

$$E^{\mp}_{Cas}{}^{W}(\omega, T) \equiv \int \frac{d^4p}{(2\pi)^4} \int_{1/\omega}^{1/T} dz\ W(\tilde{p}, z) F^{\mp}(\tilde{p}, z) \quad ,$$

$$F^{\mp}(\tilde{p}, z) = s(z) \int_{p^2}^{\infty} \{G^{\mp}_k(z, z)\} dk^2 = \frac{2}{(\omega z)^3} \int_{\tilde{p}}^{\infty} \tilde{k}\ G^{\mp}_k(z, z) d\tilde{k} \quad ,$$

Examples of $W(\tilde{p}, z)$: $W(\tilde{p}, z) =$

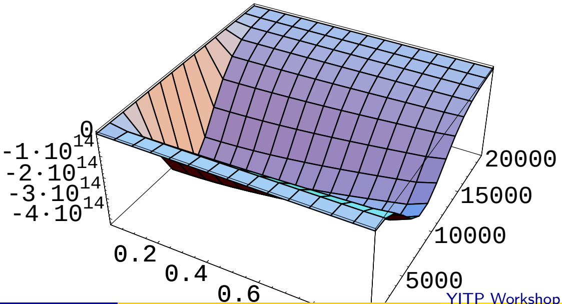
$$\begin{cases} (N_1)^{-1} \mathrm{e}^{-(1/2)\tilde{p}^2/\omega^2 - (1/2) z^2 T^2} \equiv W_1(\tilde{p}, z),\ N_1 = 1.711/8\pi^2 \\ (N_2)^{-1} \mathrm{e}^{-\tilde{p} z T/\omega} \equiv W_2(\tilde{p}, z),\ N_2 = 2\frac{\omega^3}{T^3}/8\pi^2 \\ (N_8)^{-1} \mathrm{e}^{-1/2(\tilde{p}^2/\omega^2 + 1/z^2 T^2)} \equiv W_8(\tilde{p}, z),\ N_8 = 0.4177/8\pi^2 \end{cases}$$

$$W_1 : \text{ elliptic},\ W_2 : \text{ hyperbolic},\ W_3 : \text{ reciprocal} \quad (19)$$

where $G^{\mp}_k(z, z)$ are defined in (16). N_i are normalization constants. We show the shape of the energy integrand $(-1/2)\tilde{p}^3 W_1(\tilde{p}, z) F^{-}(\tilde{p}, z)$ in Fig.9.

Sec 5. Weight Func. and Casimir Ene.: b.$(-1/2)\tilde{p}^3 W_1(\tilde{p},z)F^-(\tilde{p},z)$ Graph

Figure: Behavior of $(-1/2)\tilde{p}^3 W_1(\tilde{p},z)F^-(\tilde{p},z)$(elliptic suppression).
$\Lambda = 20000,\ \omega = 5000,\ \ T = 1$.
$1.0001/\omega \leq z \leq 0.9999/T,\ \mu = \Lambda T/\omega \leq \tilde{p} \leq \Lambda.$



Sec 5. Weight Func. and Casimir Ene.: c. ${E^{\mp}_{Cas}}^W$

We can check the divergence (scaling) behavior of ${E^{\mp}_{Cas}}^W$ by *numerically* evaluating the $(\tilde{p}, z)$-integral (19) for the rectangle region of Fig.7.

$$-E^W_{Cas} = \begin{cases} \frac{\omega^4}{T}\Lambda \cdot 1.2 \left\{1 + 0.11 \ \ln\frac{\Lambda}{\omega} - 0.10 \ \ln\frac{\Lambda}{T}\right\} & \text{for} \quad W_1 \\ \frac{T^2}{\omega^2}\Lambda^4 \cdot 0.062 \left\{1 + 0.03 \ \ln\frac{\Lambda}{\omega} - 0.08 \ \ln\frac{\Lambda}{T}\right\} & \text{for} \quad W_2 \\ \frac{\omega^4}{T}\Lambda \cdot 1.6 \left\{1 + 0.09 \ \ln\frac{\Lambda}{\omega} - 0.10 \ \ln\frac{\Lambda}{T}\right\} & \text{for} \quad W_8 \end{cases} \quad (20)$$

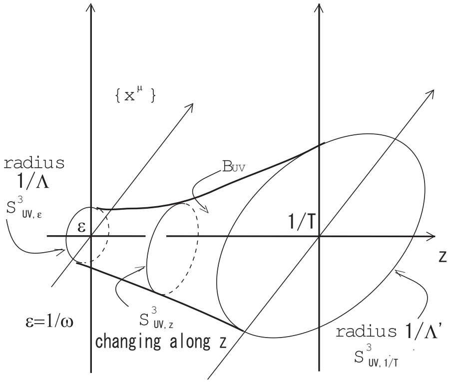
They give, after normalizing the factor Λ/T, only the log-divergence.

$$E^W_{Cas}/\Lambda T^{-1} = -\alpha\omega^4 \left(1 - 4c\ln(\Lambda/\omega) - 4c'\ln(\Lambda/T)\right) \quad , \quad (21)$$

This means the 5D Casimir energy is *finitely* obtained by the ordinary renormalization of the warp factor ω. In the above result of the warped case, the IR parameter l in the flat result (13) is replaced by the inverse of the warp factor ω.

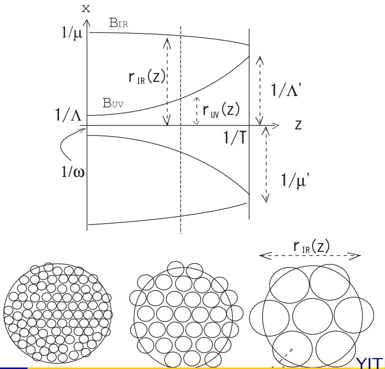
Sec 5. Weight Func. and Casimir Ene.: d.Regularization Surface

Figure: UV regularization surface in 5D coordinate space.



Sec 5. Weight Func. and Casimir Ene.: e.Regularization Surface

Figure: Regularization Surface B_{IR} and B_{UV} in the 5D coordinate space (x^μ, z). The three graphs at the bottom show the flow of coarse graining (renormalization).



Sec 6. Meaning of Weight: a.Casimir energy

We *propose* to *replace* the 5D space integral with the weight W, by the following path-integral. We newly define the Casimir energy in the higher-dimensional theory as follows.

$$\mathcal{E}_{Cas}(\omega, T, \Lambda) \equiv \int_{1/\Lambda}^{1/\mu} d\rho \int_{\tilde{p}(1/\omega)=\tilde{p}(1/T)=1/\rho} \prod_{a,z} \mathcal{D}p^a(z)$$

$$\left\{ \int_{1/\omega}^{1/T} F(\tilde{p}(z'), z')dz' \right\} \times \exp\left[-\frac{1}{2\alpha'} \int_{1/\omega}^{1/T} \frac{1}{\omega^4 z^4} \frac{1}{\tilde{p}^3} \sqrt{\frac{\tilde{p}'^2}{\tilde{p}^4} + 1}\ dz \right]$$

$$= \int_{1/\Lambda}^{1/\mu} d\rho \int_{r(1/\omega)=r(1/T)=\rho} \prod_{a,z} \mathcal{D}x^a(z)$$

$$\left\{ \int_{1/\omega}^{1/T} F(\frac{1}{r(z')}, z')dz' \right\} \times \exp\left[-\frac{1}{2\alpha'} \int_{1/\omega}^{1/T} \frac{1}{\omega^4 z^4} \sqrt{r'^2 + 1}\ r^3 dz \right] \quad ,(2$$

Sec 6. Meaning of Weight: b.Casimir Energy

where $\mu = \Lambda T/\omega$ and the limit $\Lambda T^{-1} \to \infty$ is taken. The string (surface) tension parameter $1/2\alpha'$ is introduced. (Note: Dimension of α' is [Length]^4.) The square-bracket ($[\cdots]$)-parts of (22) are $-\frac{1}{2\alpha'}$ Area $= -\frac{1}{2\alpha'}\int \sqrt{\det g_{ab}} d^4x$ (See (**??**)) where g_{ab} is the induced metric on the 4D surface. $F(\tilde{p}, z)$ is defined in (19) or (17) and shows the *field-quantization* of the bulk scalar (EM) fields.
The proposed definition, (22), clearly shows the 4D space-coordinates x^a or the 4D momentum-coordinates p^a are quantized (quantum-statistically, not field-theoretically) with the Euclidean time z and the "area Hamiltonian" $A = \int \sqrt{\det g_{ab}}\, d^4x$. Note that $F(\tilde{p}, z)$ or $F(1/r, z)$ appears, in (22), as the energy density operator in the quantum statistical system of $\{p^a(z)\}$ or $\{x^a(z)\}$.

Sec 7. Spring-Block Model (SBM) a.Model Figure

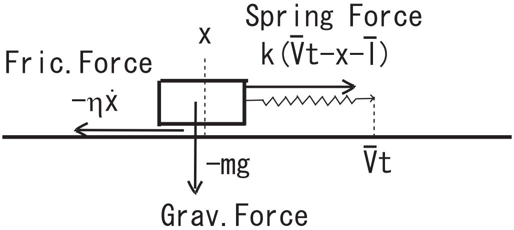


Figure: *The spring-block model, (**??**).*

Sec 7. SBM : b.1st Statistical Ensemble

Length

$$L_D = \int_0^\beta ds|_{on-path} = \int_0^\beta \sqrt{2V_1(y) + \dot{y}^2 + \dot{w}^2} dt$$

$$= h \sum_{n=0}^{\beta/h} \sqrt{2V_1(y_n) + \dot{y}_n^2 + \dot{w}_n^2},$$

$$\mathrm{e}^{-\beta F} = \int \prod_n dy_n dw_n \mathrm{e}^{-\frac{1}{\alpha}L_D} \ , \quad d\mu = \mathrm{e}^{-\frac{1}{\alpha}L_D} \prod_t \mathcal{D}y\mathcal{D}w, \qquad (23)$$

where the free energy F is defined.

Sec 7. SBM : c.(2nd) Metric in SBM

The second choice of the metric is the standard type(S.I.,2010):

$$(ds^2)_S \equiv \frac{1}{dt^2}[(ds^2)_D]^2 \quad - \text{on-path} \rightarrow (2V_1(y) + \dot{y}^2 + \dot{w}^2)^2 dt^2. \qquad (24)$$

Sec 7. SBM : d.2nd Statistical Ensemble

Length

$$L_S = \int_0^\beta ds|_{on-path} = \int_0^\beta (2V_1(y) + \dot{y}^2 + \dot{w}^2)dt = h\sum_{n=0}^{\beta/h}(2V_1(y_n) + \dot{y}_n^2 + \dot{w}_n^2),$$

$$d\mu = \mathrm{e}^{-\frac{1}{\alpha}L_S}\mathcal{D}y\mathcal{D}w,$$

$$\mathrm{e}^{-\beta F} = \int \prod_n dy_n dw_n \mathrm{e}^{-\frac{1}{\alpha}L_S} = (\text{const}) \int \prod_{n=0}^{\beta/h} dy_n \mathrm{e}^{-\frac{h}{\alpha}(2V_1(y_n)+\dot{y}_n^2)}, \quad (25)$$

where w_n is integrated out.

Sec 7. SBM : e. Analytic Solution of F

Taking the values:

$$\alpha = 1\ ,\ \beta = 1\ ,\ h = 1\ ,\ m = 1\ ,\ \eta = 1\ ,\\ \sqrt{k/\eta} = \omega_0 = 0.881374\ ,\ \sinh(\omega_0) = 1, \tag{26}$$

the free energy F is

$$F(\bar{V},\bar{\ell}) = -\frac{1}{2}\ln\frac{\omega_0}{2\pi} + (\sqrt{2}-2)\frac{\bar{V}^2}{\omega_0} + \sqrt{2}\bar{V}(1-\frac{1}{{\omega_0}^2})(\bar{\ell}-\bar{V})\\ +\bar{V}^2 + \frac{{\omega_0}^2}{3\bar{v}}\{(\bar{\ell}-\bar{V})^3 - \bar{\ell}^3\}. \tag{27}$$

Sec 8. Discussion + Conclusion: a.Beta Function

$$E^W_{Cas}/\Lambda T^{-1} = -\alpha\omega^4 \left(1 - 4c\ln(\Lambda/\omega) - 4c'\ln(\Lambda/T)\right) = -\alpha\omega'^4 \quad ,$$
$$\omega' = \omega\sqrt[4]{1 - 4c\ln(\Lambda/\omega) - 4c'\ln(\Lambda/T)} \quad .(28)$$

we find the renormalization group function for the warp factor ω as

$$|c| \ll 1 \quad , \quad |c'| \ll 1 \quad , \quad \omega' = \omega(1 - c\ln(\Lambda/\omega) - c'\ln(\Lambda/T)) \quad ,$$
$$\beta(\beta\text{-function}) \equiv \frac{\partial}{\partial(\ln\Lambda)}\ln\frac{\omega'}{\omega} = -c - c' \quad .(29)$$

Sec 8. Discussion + Conclusion: b. c+c'

We should notice that, in the flat geometry case, the IR parameter (extra-space size) l is renormalized . In the present warped case, however, the corresponding parameter T is not renormalized, but the warp parameter ω is renormalized. Depending on the sign of $c + c'$, the 5D bulk curvature ω flows as follows. When $c + c' > 0$, the bulk curvature ω decreases (increases) as the the measurement energy scale Λ increases (decreases). When $c + c' < 0$, the flow goes in the opposite way.

Sec 8. Discussion + Conclusion: c.Cosm. Const.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = T^{matte}_{\mu\nu}$$

$$S = \int d^4x \sqrt{-g} \{ \frac{1}{G_N} (R + \lambda) \} + \int d^4x \sqrt{-g} \{ \mathcal{L}_{matter} \} \quad , \quad g = \det g_{\mu}$$

$$\frac{1}{G_N} \lambda_{obs} \sim \frac{1}{G_N {R_{cos}}^2} \sim m_\nu^4 \sim (10^{-3} eV)^4 \quad , \qquad (31)$$

where R_{cos} is the cosmological size (Hubble length), m_ν is the neutrino mass.

$$\frac{1}{G_N} \lambda_{th} \sim \frac{1}{{G_N}^2} = {M_{pl}}^4 \sim (10^{28} eV)^4 \quad . \qquad (32)$$

The famous huge discrepancy factor: $\lambda_{th}/\lambda_{obs} \sim 10^{124}$.

Sec 8. Discussion + Conclusion: d. Cosm. Const.

If we apply the present approach, we have the warp factor ω, and the result (28) strongly suggests the following choice:

$$\text{INPUT 1} \quad \Lambda = M_{pl} \quad ,$$

$$\text{INPUT 2(Newton's law exp.)} \ \omega \sim \frac{1}{\sqrt[4]{G_N R_{cos}{}^2}} = \sqrt{\frac{M_{pl}}{R_{cos}}} \sim m_\nu \sim 10^{-3}\text{eV}$$

$$\text{FACT} \quad S \sim \int d^4x\sqrt{-g}\frac{1}{G_N}\lambda_{obs} \sim R_{COS}^4\omega^4$$

$$\text{Result(28)requires} \quad \mathrm{e}^{-S} \leftrightarrow \mathrm{e}^{-E_{Cas}/T^4} = \exp\{-T^{-4}\Lambda T^{-1}\omega^4\}$$

$$\Longrightarrow \ T^5 = \frac{M_{pl}}{R_{cos}^4} \quad \text{OUTPUT} \quad . \qquad (34)$$

Sec 8. Discussion + Conclusion: e.Cosm. Const.

From the values: $M_{pl} = \frac{1}{\sqrt{G_N}} = 10^{28}\text{eV}$, $R_{cos} = 5 \times 10^{32}\text{eV}^{-1}$, $\omega \sim 10^{-3}\text{eV}$, we obtain

$$T = R_{cos}^{-1}(N_{DL})^{1/5} \sim 10^{-20}eV \quad , \quad \frac{\Lambda}{T} = (N_{DL})^{4/5} \sim 10^{50} \quad ,$$

$$\mu = M_{pl}N_{DL}^{-3/10} \sim 1GeV \sim m_N \quad , \quad N_{DL} = M_{pl}R_{cos} \sim 6 \times 10^{61} \quad ,(35)$$

We do not yet succeed in obtaining the right sign, but succeed in obtaining the finiteness and its gross absolute value of the cosmological constant. Now we understand that the smallness of the cosmological constant comes from the renormalization flow for the non asymptotic-free case ($c + c' < 0$ in (29)).