Large N behavior of M2-M5 brane system

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Based on arXiv:1511:xxxx

Introduction & result

[Aharony-Bergman-Jafferis-Maldacena'08]

N M2-branes → ABJM theory ← **known**

Low-energy limit

N M5-branes → 6d (2,0) SCFT ←**Mysterious!**

◆ Motivation

- To know the behavior of N M5-branes

◆ Approach

- Study the behavior of **mass-deformed ABJM**

⋓

M5 branes information

result

✓ Large N behavior of the free energy F of the **mass-deformed ABJM** theory

➡ $F \sim N^{\frac{3}{2}}, \quad (N \to \infty)$ (in small mass parameter region)

✓ We find the theory has the different type saddle-point solution, which are unknown types.

➡ **Phase transition** of F may occur.

Mass-deformed ABJM

[Gomis-Gomez Raamsdonk-Verinde'08]

[Hosomichi-Lee-Lee-Lee-Park'08]

- The action of Mass-defomed ABJM: $S_{\mathrm{mABJM}} = S_{\mathrm{ABJM}} + \underbrace{\zeta \int dx^3 \mathrm{Tr}\left(D + \widehat{D}\right)}_{\text{FI term deform}}$

S_{ABJM}

- $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ $\mathcal{N}=6$ SCF CS + matters
- Matter : $(N, \overline{N}) \times 2 \oplus (\overline{N}, N) \times 2$
  Interacting via quartic superpotential.

FI-term

- **Mass** terms of matter $\sim$ $(\frac{\zeta}{k})^2 YY^\dagger$, $\frac{\zeta}{k}\psi\psi^\dagger$
- Quartic terms of scalar $\sim$ $\frac{\zeta}{k} YY^\dagger YY^\dagger$ $\quad \frac{\zeta}{k} \backsim$Mass

Properties ➡

◆ Maximal supersymmetry $\mathcal{N}=6$

◆The vacua describe fuzzy M2-M5 brane system.

Matrix model and Large N approximation

- With localization method to the theory on S^3. [Kapustin-Willet-Yaakov '10] [Hama-Hosomichi-Lee '10]

Localization

$$Z(N) = \underbrace{\int \mathcal{D}X e^{-S_{mABJM}}}_{\text{Path integral}} \Rightarrow \underbrace{\prod_{i=1}^{N} \int d\lambda_i d\tilde{\lambda}_i e^{-f(\lambda,\tilde{\lambda})}}_{\text{Matrix Model}}$$

$$f(\lambda,\tilde{\lambda}) = \pi i k\left(\sum_{i\geq 1}\lambda_i^2 - \sum_{i\geq 1}\tilde{\lambda}_i^2\right) - 2\pi i\zeta\left(\sum_{i\geq 1}\lambda_i - \sum_{i\geq 1}\tilde{\lambda}_i\right) - \sum_{i>j}\log\sinh^2\pi(\lambda_i-\lambda_j) - \sum_{i>j}\log\sinh^2\pi(\tilde{\lambda}_i-\tilde{\lambda}_j) + \sum_{i,j\geq 1}\log\cosh^2\pi(\lambda_i-\tilde{\lambda}_j),$$

Saddle-point e.q.

$$\left.\frac{\partial f}{\partial \lambda_i}\right|_{\{\lambda_0,\tilde{\lambda}_0\}} = 0, \quad \left.\frac{\partial f}{\partial \tilde{\lambda}_0}\right|_{\{\lambda_0,\tilde{\lambda}_0\}} = 0$$

$$\xrightarrow{N \to \infty} \quad F = -\log Z(N) \sim f(\lambda_0, \tilde{\lambda}_0)$$

How to Solve Saddle-point e.q.

- $N \to \infty$, Taking continuous limit with our ansatz

- **Continuum Limit**

$$\sum_i (\cdots)_i \to N \int_I dx \rho(x)(\cdots)(x).$$

- **Our natural ansatz**

1. $\lambda^* = -\tilde{\lambda}$, $\lambda_i \to \lambda(x) = N^{\frac{1}{2}}(x + iy(x))$, $\tilde{\lambda}_i \to \tilde{\lambda}(x) = N^{\frac{1}{2}}(-x + iy(x))$, $x \in I$
2. $\rho(x) = \rho(x)_{\text{ev}} + \frac{1}{\sqrt{N}}\rho(x)_{\text{od}}$, $y(x) = y(x)_{\text{ev}} + \frac{1}{\sqrt{N}} y(x)_{\text{od}}$

Different scale! **Different scale!**

$$\left.\frac{\partial f}{\partial \lambda_i}\right|_{\{\lambda_0, \tilde{\lambda}_0\}} = 0, \quad \left.\frac{\partial f}{\partial \tilde{\lambda}_0}\right|_{\{\lambda_0, \tilde{\lambda}_0\}} = 0$$

$N \to \infty$

Continuous saddle-point e.q.

$$0 = -ikN^{\frac{1}{2}}(x + iy(x)) + i\zeta + N \int_I dx' \rho(x') \coth \pi N^{\frac{1}{2}}[(x - x') + i(y(x) - y(x'))]$$
$$-N \int_I dx' \rho(x') \tanh \pi N^{\frac{1}{2}}[(x + x') + i(y(x) - y(x'))].$$

Expanding Tanh & Coth:

$$\tanh(z) = \begin{cases} 1 - 2\sum_{n=1}^{\infty}(-1)^{n-1}e^{-2nz} & (\mathrm{Re}(z) \geq 0) \\ -1 + 2\sum_{n=1}^{\infty}(-1)^{n-1}e^{2nz} & (\mathrm{Re}(z) < 0) \end{cases}$$

$$\coth(z) = \begin{cases} 1 + 2\sum_{n=1}^{\infty} e^{-2nz} & (\mathrm{Re}(z) \geq 0) \\ -1 - 2\sum_{n=1}^{\infty} e^{2nz} & (\mathrm{Re}(z) < 0) \end{cases}$$

Integration by parts up to N^0

Leading saddle-point e.q.

$$0 = \zeta(1 - \dot{y}_{ev}^2) + \frac{1}{4}\frac{\rho(x)\ddot{y}_{ev}(x)}{1 + \dot{y}_{ev}^2(x)} \qquad 0 = \zeta \dot{y}_{ev} + \frac{1}{4}\left[\frac{\rho_e(x)}{1 + \dot{y}_e^2(x)}\right]'$$

$$0 = k\frac{d}{dx}[x y_{ev}(x)] + 2\int_I dx' \rho_{od}(x')\,\mathrm{sgn}(x - x') \qquad 0 = -kx - \frac{4\rho_{ev} y_{od}}{1 + \dot{y}_{ev}^2}$$

Solutions

Determined! **Leading saddle-point e.q.**

- $\rho(x), y(x)$

Determined!

- The integration interval $I = [-L, L]$ and integration constant

$$\int_I \rho(x)dx = 1 \quad \text{:Normalization condition}$$

$$N^{\frac{1}{2}} \int_I dx \rho(x) x = \frac{\zeta}{k} \quad :\int_I \rho(x)\mathrm{Im}\,(\text{saddle} - \text{point e.q.})$$

Two solutions!

$$y_{ev} = +b + \sqrt{x^2 + a^2}, \quad y_{od}(x) = -\frac{k}{16\zeta}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\rho_{ev}(x) = 4\zeta\frac{d}{dx}(x\sqrt{x^2 + a^2}), \quad \rho_{od}(x) = -\frac{k}{4}\frac{d^2}{dx^2}(x y_{ev}(x))$$

with $a^2 = \frac{k}{32\zeta}\left(1 - \frac{16\zeta^2}{k^2}\right)$, $b = \frac{\sqrt{k}}{4\sqrt{2\zeta}}\left(1 + \frac{16\zeta^2}{k^2}\right)$ $L = \frac{1}{\sqrt{2k}}$

$$\Rightarrow \boxed{f_{\mathrm{I}} = \frac{\pi\sqrt{2k}N^{\frac{3}{2}}}{3}\left(1 + \frac{16\zeta^2}{k^2}\right)} \xrightarrow{\zeta \to 0} \text{ABJM}$$

[Druckker-Marino-Putrov '10]
[Herzog-Klebanov-Pufu-Tesileanu '10]

$$y_{ev}(x) = -|x| + a, \quad y_{od} = -\frac{kx}{2}\frac{1}{8\zeta|x| + b}$$

$$\rho_{od}(x) = 8\zeta|x| + b, \quad \rho_{od}(x) = \frac{k}{2}\mathrm{sgn}(x)$$

with $a = \frac{2\sqrt{2\zeta}}{k}$, $b = \frac{k}{2\sqrt{2\zeta}}\left(1 - \frac{16\zeta^2}{k^2}\right)$, $L = \frac{\sqrt{2\zeta}}{k}$

$$\Rightarrow \boxed{f_{\mathrm{II}} = \frac{\pi\sqrt{2k}N^{\frac{3}{2}}}{3}\sqrt{\frac{k}{\zeta}}\left(\frac{3}{16} + \frac{14\zeta^2}{k^2} - \frac{16\zeta^4}{k^4}\right)} \xrightarrow{\zeta \to 0} \text{ABJM}$$

- The Leading behavior of F:

$$F = \pi N^{\frac{3}{2}}\Big[-4k\int_I dx[x\rho_{ev}(x)y_{od}(x) + x\rho_{od}(x)y_{ev}(x)] + 4\zeta\int_I dx[y_{ev}(x)\rho_{ev}(x)]$$
$$-4\int_I dx\int_I dx' \rho_{od}(x)\rho_{od}(x')|x - x'| + 2\int_I dx\frac{\rho_{ev}^2(x)}{1 + \dot{y}_{ev}^2}[\frac{1}{4} - 4y_{od}^2(x)]\Big]$$

Conclusion

Result

- Found the leading behavior : $\frac{\zeta}{k} \leq \frac{1}{4}$, $F \sim N^{3/2}$.
- Two solution: connected to ABJM and not $\Rightarrow$ Phase Transition

Future Problem

- Find large $\frac{\zeta}{k}$ saddle-point solution.
- What happen in the gravity side when phase transition occurs.