YITP workshop November 11,2015

Defects in Open String Field Theory

Toshiko Kojita (Maskawa Institute, Kyoto Sangyo Univ)
with C.Maccaferri (Trino Univ) , T.Masuda (Nara women's Univ), M.Schnabl (AS CR)

World-sheet Defects

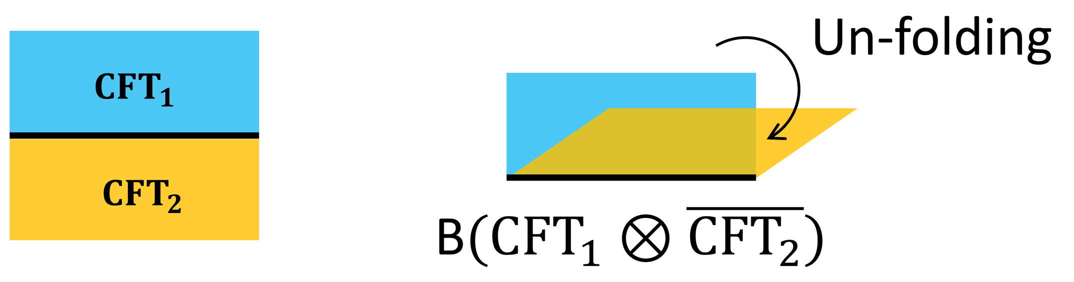


Fig.1

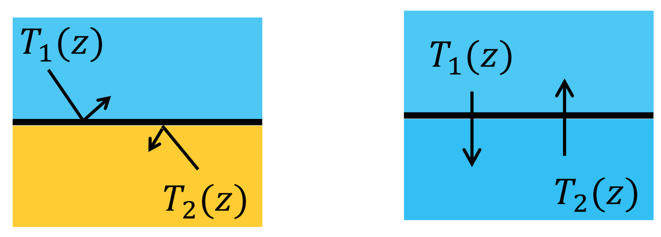


Fig.2

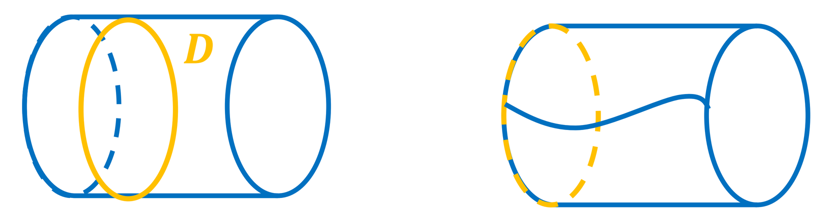


Fig.3

World-sheet defect is expressed as the line which can divide two the different CFT's. Let us consider the two different CFTs defined on the upper and lower half plane, respectively, which are joined together along a defect on the real line (Fig.1 l). If $T-\bar{T}$ is continuous across defect line, defect is conformal.

$$\lim_{y\to 0} T_1(x+iy) - \bar{T}_1(x+iy) = \lim_{y\to 0} T_2(x-iy) - \bar{T}_2(x-iy)$$

Defect is natural extension of conformal boundary (D-brane). The defect can be regarded as conformal boundary of CFT$_1$ ⊗ CFT$_2$ by using folding trick (Fig.1 r).

$T_1(x) = \bar{T}_1(x),\ T_2(x) = \bar{T}_2(x)$ "totally reflective"

$T_1(x) = T_2(x),\ \bar{T}_1(x) = \bar{T}_2(x)$ "totally transmissive"

To a topological (totally transmissive) defect one can associate an operator D which commutes with all the Virasoro operators. By the Schur's Lemma, D can be written as the linear combination of projectors P_j of Verma modules $[V_j] \times [\bar{V}_j]$.

$$[L_n, D] = [\bar{L}_n, D] = 0\ \ ({}^{\forall}n)\ \Rightarrow D_a = \sum_i n_a{}^i P_i$$

(※Diagonal minimal model)

['00 Petkova and Zuber]

By modular transformation (Fig.3), coefficients n_a^i are related to the modular S-matrix. Consequently, defect obey analogous fusion rule $[\phi_i]\times[\phi_j] = N_{ij}^k[\phi_k]$, and we obtain a natural action on the boundary state.

$$D_a = \sum_i \frac{S_{ai}}{S_{0i}} P^i \quad \Rightarrow \quad \begin{cases} D_a D_c = \sum_e N_{dc}{}^e D_e \\ D_d \| \alpha \gg = \sum_\beta N_{d\alpha}^{\beta} \| \beta \gg \end{cases}$$

Cf) Analogy with boundary state

$$(L_n - \bar{L}_{-n})\|B_a \gg = 0 \quad \Leftrightarrow \quad \|B_a \gg = \sum_i n_{ai} |i\gg \quad \xrightarrow{\text{Modular inv}} \quad \|B_a \gg = \sum_i \frac{S_{ai}}{\sqrt{S_{0i}}} |i\gg$$

Open string defects

+ Motivation

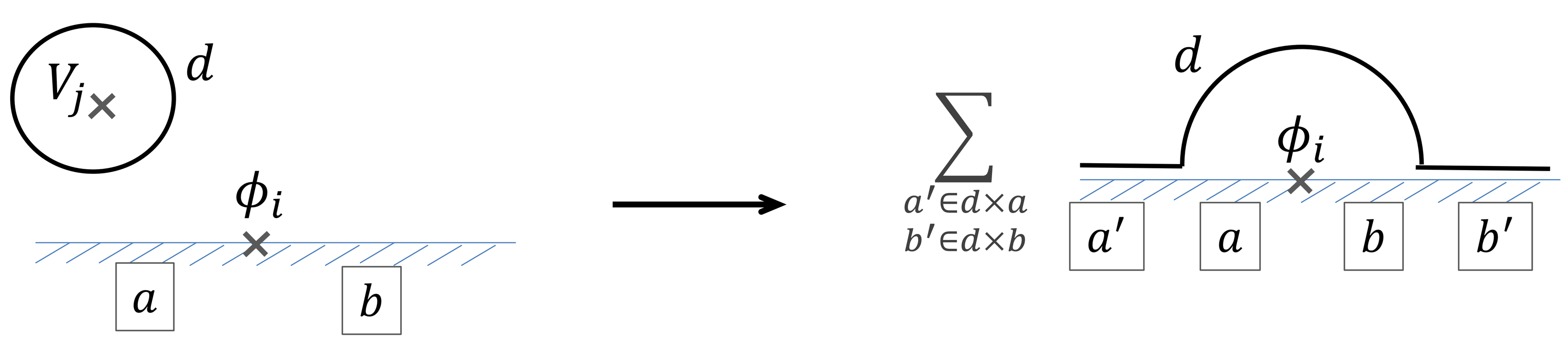
- The equation of motion of (bosonic) open string filed theory (SFT) is given by world-sheet BRST-charge Q_B and star product "$*$" which represents how to glue open strings in specific manner.
- The solution of EOM $\Psi_{a\to b}$ describes BCFT$_b$ in terms of BCFT$_a$. (~D-brane, boundary state)
- If operator $\mathcal{D}$ is independent from the ghost sector, and commute the BRST charge, and preserve the star algebra, we can obtain new solution $\mathcal{D}\Psi_{\rm sol}$.

EOM : $\boldsymbol{Q_B\Psi + \Psi * \Psi = 0.}$

$$\begin{cases} [Q_B, \mathcal{D}] = 0\ ,[b_n, \mathcal{D}] = [c_n, \mathcal{D}] = 0 \\ \Leftrightarrow\ \ [L_n, \mathcal{D}] = 0 \quad \text{"topological"} \\ \mathcal{D}(\Psi * \Phi) = \mathcal{D}\Psi * \mathcal{D}\Phi \end{cases}$$

→ **Defect maps solution to solution** $\mathcal{D}\Psi^{\rm sol}_{a\to b} = \Psi^{\rm sol}_{Da\to Db}$

+ Open string defects



$$\mathcal{D}^d : \mathcal{H}^{(ab)}_{\rm open} \longrightarrow \sum_{\substack{a'\in d\times a \\ b'\in d\times b}} \mathcal{H}^{(a'b')}_{\rm open}$$

$$\mathcal{D}^d \phi_i^{(ab)} = \sum_{a'b'} X^{dab}_{ia'b'}\, \phi_i^{(a'b')}$$

Closed string defect D is projector which maps closed Hilbert space to closed Hilbert space $D_a : \mathcal{H}_{\rm close} \longrightarrow \mathcal{H}_{\rm close}$. On the other hand, open string defect is not a projector because defect changes the boundary condition s.t $D_d\|\alpha \gg = \sum_\beta N^\beta_{d\alpha}\|\beta \gg$. Therefore, defect action on boundary condition changing operator (bcco) ϕ_i may be written as linear combination (coefficients X^{dab}_{ipq}) of the state with suitable boundary conditions.

[cf. '03 Graham and Watts]

Requiring the defect to have the distribution property for the star product, we obtain the relation between the X^{dab}_{ipq} and boundary structure constant C^{abc}_{ijk} which determine the OPE of bccos. Using the relation $C^{(abc)k}_{ij} = F_{bk}\begin{bmatrix} a & c \\ i & j \end{bmatrix}$, and assuming the twist symmetry $X^{dab}_{ia'b'} = X^{dba}_{ib'a'}$, we can obtain the formula for the open string defect in terms of F-matrices.

$$\mathcal{D}^d\left(\phi_i^{(ab)}(x)\phi_j^{(bc)}(y)\right) = \left(\mathcal{D}^d\phi_i^{(ab)}(x)\right)\left(\mathcal{D}^d\phi_j^{(bc)}(y)\right)$$

$$\rightarrow\ X^{dab}_{ia'b'} = \frac{N(d,a,a')}{N(d,b,b')} F_{ba'}\begin{bmatrix} i & b' \\ a & d \end{bmatrix} = \sqrt{\frac{F_{ab'}\begin{bmatrix} i & a' \\ b & d \end{bmatrix}}{F_{ba'}\begin{bmatrix} i & b' \\ a & d \end{bmatrix}}}\, F_{ba'}\begin{bmatrix} i & b' \\ a & d \end{bmatrix}$$

Defect network and Fusion rule

+ defect fusion

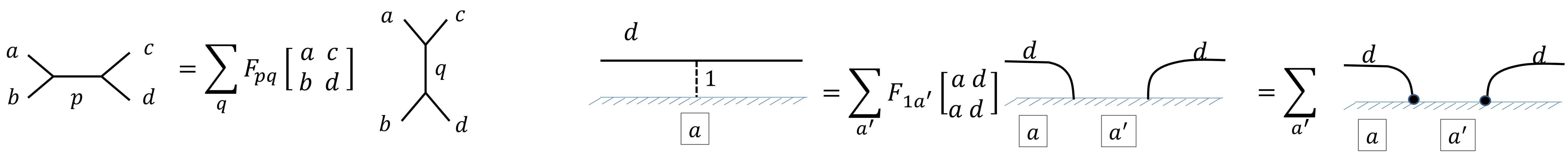
Although the closed string defect satisfies the fusion rule $D_d D_c = \sum_e N_{dc}^{e} D_e$, it is not true for open string defects. A nontrivial transformation U is required which relates the route of b.c $(a,b) \xrightarrow{c} (a',b') \xrightarrow{d} (a'',b'')$ and $(a,b) \xrightarrow{e} (a'',b'')$

$$\left(\mathcal{D}^d \mathcal{D}^c \phi_i\right)^{\{a\to a'\to a''\}\{b\to b'\to b''\}} = \left[U_{dc}\left(\sum_e N_e^{dc}\, \mathcal{D}^e \phi_i\right) U_{dc}^{-1}\right]^{\{a\to a'\to a''\}\{b\to b'\to b''\}} \longrightarrow \begin{aligned}&(U_{dc})^{\{a\to a'\to a''\}[e;\, a\to a'']}\\ &= (U_{dc}^{-1})^{[e;\, a\to a'']\{a\to a'\to a''\}} = \sqrt{F_{a'e}\begin{bmatrix} d & c\\ a'' & a\end{bmatrix}}\sqrt{F_{ea'}\begin{bmatrix} a'' & d\\ a & c\end{bmatrix}}\end{aligned}$$

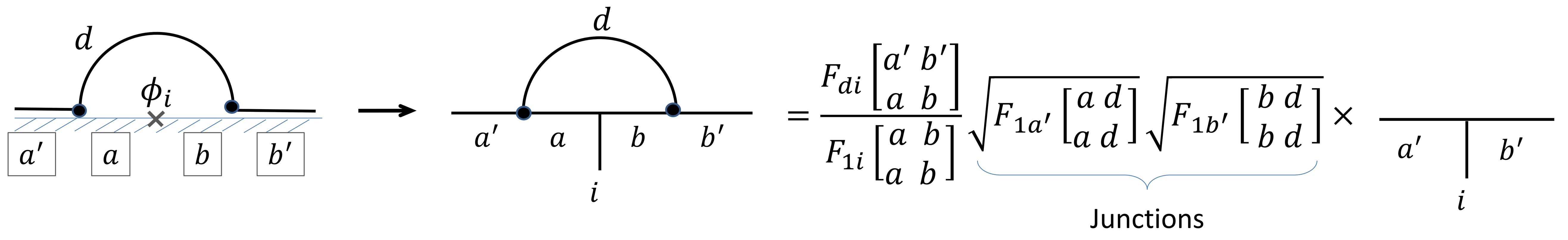
U-matrix rotate the degeneracy of intermediate b.c. Us and U^{-1}s cancel by considering all bccos in correlator.

+ defect network (Graphical understanding)

We assume the following defect network just like conformal block.
When defect attaches to the boundary, the numerical factor should be multiplied by the junction point. Thick dots express this factor.



This rule is consistent with defect action on bcco which was derived form algebraic construction.



Defect on open string field

	$(h,\bar{h})$	b.c	spectrum
1	$(0,0)$	$\|1\gg$	1
ε	$(1/2, 1/2)$	$\|\varepsilon\gg$	1
σ	$(1/16, 1/16)$	$\|\sigma\gg$	$1, \varepsilon$

+ mapping solution to solution

Ex) Critical Ising model

Ising sector of open string field on a sigma-brane can be written as $\Psi_{\sigma\to 1} = \psi_1^{(\sigma\sigma)} + \psi_\varepsilon^{(\sigma\sigma)}$

If $\Psi_{\sigma\to 1}$ is solution, EOM $Q_B\Psi + \Psi * \Psi = 0$ can be divided into two parts

$$\begin{cases} Q_B\psi_1^{(\sigma\sigma)} + \psi_1^{(\sigma\sigma)} * \psi_1^{(\sigma\sigma)} + \psi_\varepsilon^{(\sigma\sigma)} * \psi_\varepsilon^{(\sigma\sigma)} = 0 \\ Q_B\psi_\varepsilon^{(\sigma\sigma)} + \psi_1^{(\sigma\sigma)} * \psi_\varepsilon^{(\sigma\sigma)} + \psi_\varepsilon^{(\sigma\sigma)} * \psi_1^{(\sigma\sigma)} = 0 \end{cases} \xrightarrow{\text{implies there are other solutions}} \psi_1^{(\sigma\sigma)} - \psi_\varepsilon^{(\sigma\sigma)} \qquad \begin{pmatrix} \psi_1^{(\sigma\sigma)} & \pm\psi_\varepsilon^{(\sigma\sigma)} \\ \pm\psi_\varepsilon^{(\sigma\sigma)} & \psi_1^{(\sigma\sigma)} \end{pmatrix}$$

['14 Schnabl,Kudrna,Rapcak]

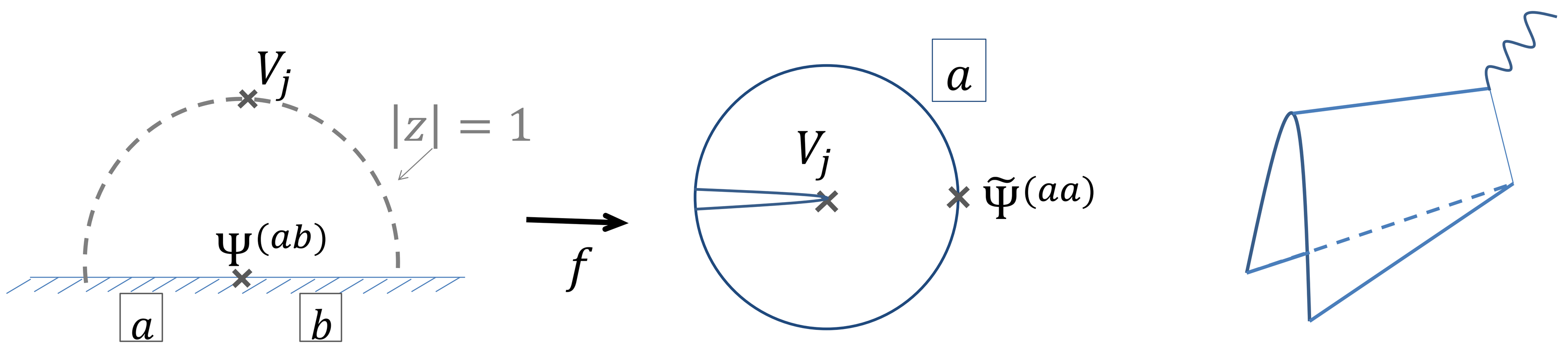
These are obtained by defect action on the solution

$$\mathcal{D}^\varepsilon\left(\psi_1^{(\sigma\sigma)} + \psi_\varepsilon^{(\sigma\sigma)}\right) = \psi_1^{(\sigma\sigma)} - \psi_\varepsilon^{(\sigma\sigma)} = \Psi_{\mathcal{D}^\varepsilon(\sigma\to 1)} = \Psi_{\sigma\to\varepsilon} \qquad \mathcal{D}^\sigma\mathcal{D}^\sigma\left(\psi_1^{(\sigma\sigma)} + \psi_\varepsilon^{(\sigma\sigma)}\right) = \begin{pmatrix} \psi_1^{(\sigma\sigma)} & \psi_\varepsilon^{(\sigma\sigma)} \\ \psi_\varepsilon^{(\sigma\sigma)} & \psi_1^{(\sigma\sigma)} \end{pmatrix} = \Psi_{2\sigma\to 1+\varepsilon}$$

+ gauge invariant observable

$$\boxed{W(\mathcal{V},\Psi) \equiv \langle \mathcal{V}(i)\; w\circ\Psi(0)\rangle_{\text{UHP}} = \langle \mathcal{V}(0)\; f\circ\Psi(1)\rangle_{\text{disk}}}$$

where $w(z) = \frac{2z}{1-z^2}$, $\xi(w) = \frac{1+iw}{1-iw}$, $f(z) = \xi\circ w(z)$,
and $\tilde{\Psi} = f\circ\Psi$



- Inserting the $\mathcal{V}$ on the midpoint of the open string, gluing the left and right halves together. This vertex express the conversion between open string and closed string.
- Because $\mathcal{V}$ is $(0,0)$ primary, and $\{Q_B, \mathcal{V}\} = 0$, $W(\mathcal{V},\Psi)$ is gauge invariant quantity ['01 Hashimoto,Itzhaki, '01 Gaiotto et al.]
- On-shell closed string tadpole on the disk whose boundary condition is given by the solution $\langle \mathcal{V}|c_0^-|B_\Psi\rangle$ (conjecture) ['08 Ellwood]

$\boxed{\text{Geometrical proof of } Tr_{V_j}[\mathcal{D}^d\Psi] = Tr_{\mathcal{D}^d V_j}[\Psi]}$

We can map in the same manner, and deform the defect to circle V_j.
Since F-matrix coming from network cancels with junctions, we obtain the RHS.