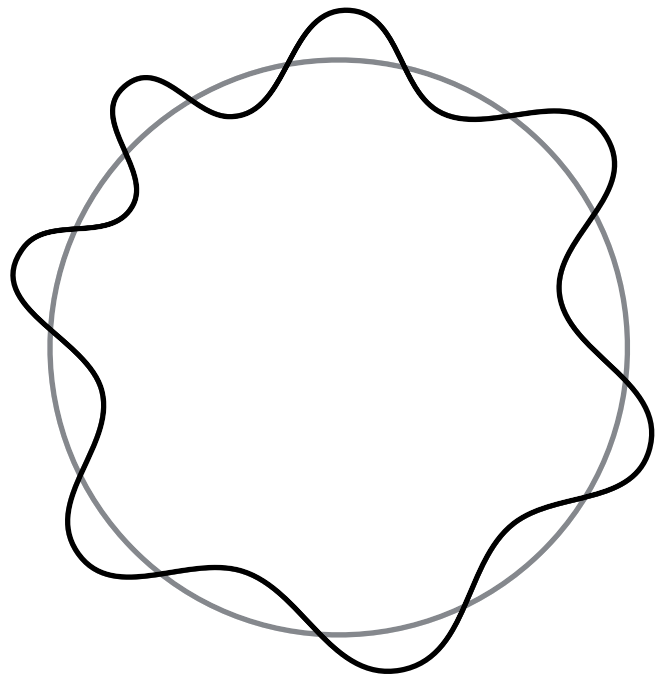
Double Field Theory & Non-Relativistic String Theory

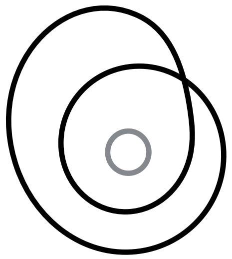
René Meyer
Stony Brook University

based on 1508.01121, with
Sung Moon Ko, Charles Melby-Thompson and Jeong-Hyuck Park

Double Field Theory?



$$R \leftrightarrow \frac{\alpha'}{R}$$



T-Duality

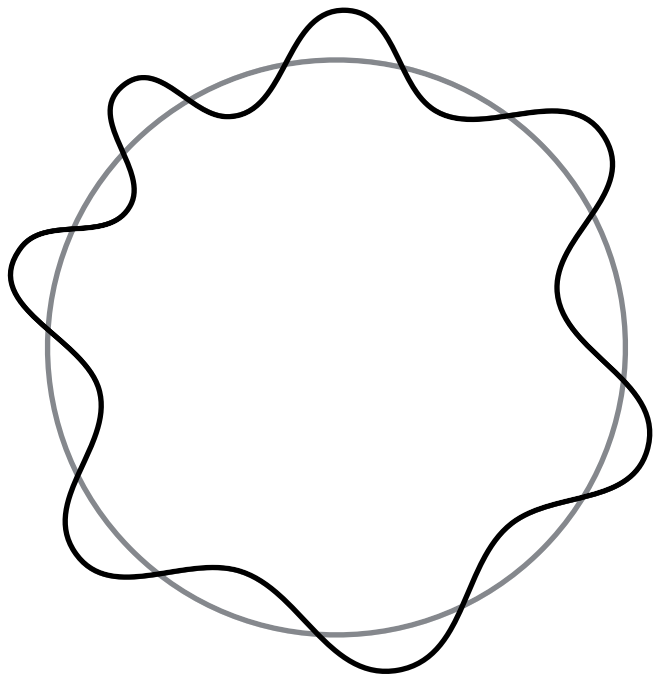
$G_{\mu\nu}$ $\quad\longleftrightarrow\quad$ $\tilde{G}_{\mu\nu}$

$B_{\mu\nu}$ $\quad\longleftrightarrow\quad$ $\tilde{B}_{\mu\nu}$

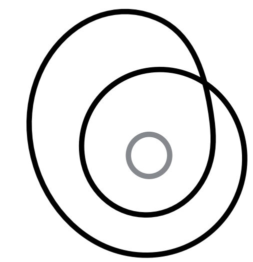
Φ $\quad\longleftrightarrow\quad$ $\tilde{\Phi}$

T. H. Buscher
Phys. Lett. B194B, 59 (1987)
Phys. Lett. B201, 466 (1988)

Double Field Theory?



$$R \leftrightarrow \frac{\alpha'}{R}$$



T-Duality

$$\widetilde{G}_{25,25} = \frac{1}{G_{25,25}}; \qquad e^{2\widetilde{\Phi}} = \frac{e^{2\Phi}}{G_{25,25}},$$

$$\widetilde{G}_{\mu 25} = \frac{B_{\mu 25}}{G_{25,25}}; \qquad \widetilde{B}_{\mu 25} = \frac{G_{\mu 25}}{G_{25,25}},$$

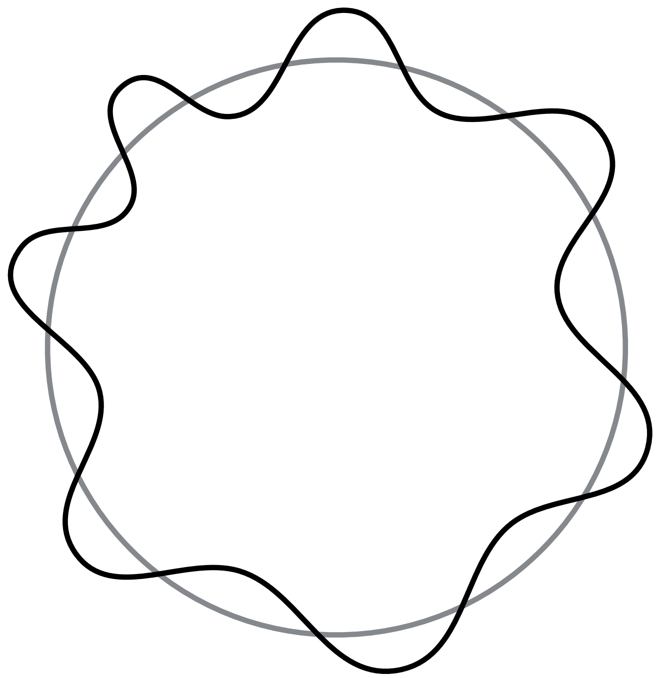
$$\widetilde{G}_{\mu\nu} = G_{\mu\nu} - \frac{G_{\mu 25}G_{\nu 25} - B_{\mu 25}B_{\nu 25}}{G_{25,25}},$$

$$\widetilde{B}_{\mu\nu} = B_{\mu\nu} - \frac{B_{\mu 25}G_{\nu 25} - G_{\mu 25}B_{\nu 25}}{G_{25,25}},$$

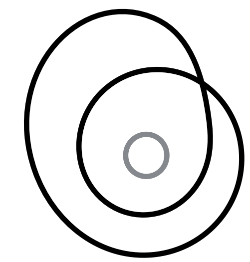
along X^{25} circle

T. H. Buscher
Phys. Lett. B194B, 59 (1987)
Phys. Lett. B201, 466 (1988)

Double Field Theory?



$$R \leftrightarrow \frac{\alpha'}{R}$$



T-Duality

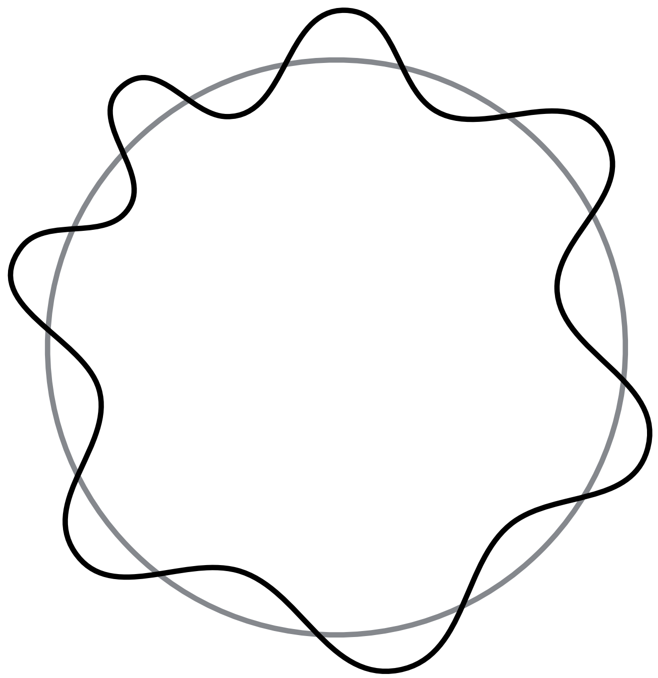
$$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}$$

$$\widetilde{E}_{ij} = E^{ij}, \qquad \widetilde{E}_{aj} = E_{ak}E^{kj}, \qquad e^{2\widetilde{\Phi}} = e^{2\Phi}\det(E^{ij}),$$
$$\widetilde{E}_{ab} = E_{ab} - E_{ai}E^{ij}E_{jb},$$

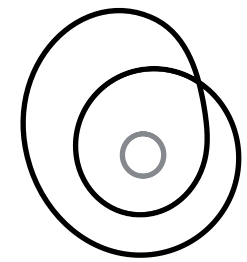
$O(D,D)$ T-duality transformation

T. H. Buscher
Phys. Lett. B194B, 59 (1987)
Phys. Lett. B201, 466 (1988)

Double Field Theory?



$$R \leftrightarrow \frac{\alpha'}{R}$$



T-Duality

$$\mathcal{H}_{AB} = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} \in O(D,D)$$

$$e^{-2d} = \sqrt{-G}e^{-2\Phi}$$

M. Duff, Tseytlin & Siegel

Double Field Theory!

Hull & Zwiebach, Siegel, Park et. al., Hohm, …

- Spacetime is formally doubled, $y^A = (\tilde{x}_\mu, x^\nu)$

  Field contents in DFT

  - $\mathcal{H}_{AB} = \mathcal{H}_{BA} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}, \qquad e^{-2d} = \sqrt{g}e^{-2\phi}$

  $\mathrm{O}(D,D)$ indices can be raise or lowered by metric

  - $\mathcal{J}_{AB} = \mathcal{H}_{AC}\mathcal{H}^C{}_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

- All the fields must live on a D-dimensional null hyperplane or 'section', subject to

$$\partial_A \partial^A = 2\frac{\partial^2}{\partial x^\mu \partial \tilde{x}_\mu} \equiv 0 \quad : \quad \text{section condition}$$

"You two should both be ashamed for saying I'm drunk ..."

- DFT Lagrangian constructed by Hull & Zwiebach (Hohm) reads

$$L_{\mathrm{DFT}} = e^{-2d}\left[\mathcal{H}^{AB}\left(4\partial_A\partial_B d - 4\partial_A d\partial_B d + \tfrac{1}{8}\partial_A\mathcal{H}^{CD}\partial_B\mathcal{H}_{CD} - \tfrac{1}{2}\partial_A\mathcal{H}^{CD}\partial_C\mathcal{H}_{BD}\right) + 4\partial_A\mathcal{H}^{AB}\partial_B d - \partial_A\partial_B\mathcal{H}^{AB}\right]$$

- Up to $\mathbf{O}(D,D)$ rotation, we may fix the section,

$$\frac{\partial}{\partial \tilde{x}_\mu} \equiv 0 . \qquad \Rightarrow L_{DFT} = \sqrt{-g}e^{-2\phi}\left(R + 4(\partial\phi)^2 - \tfrac{1}{12}H^2_{\mu\nu\rho}\right)$$

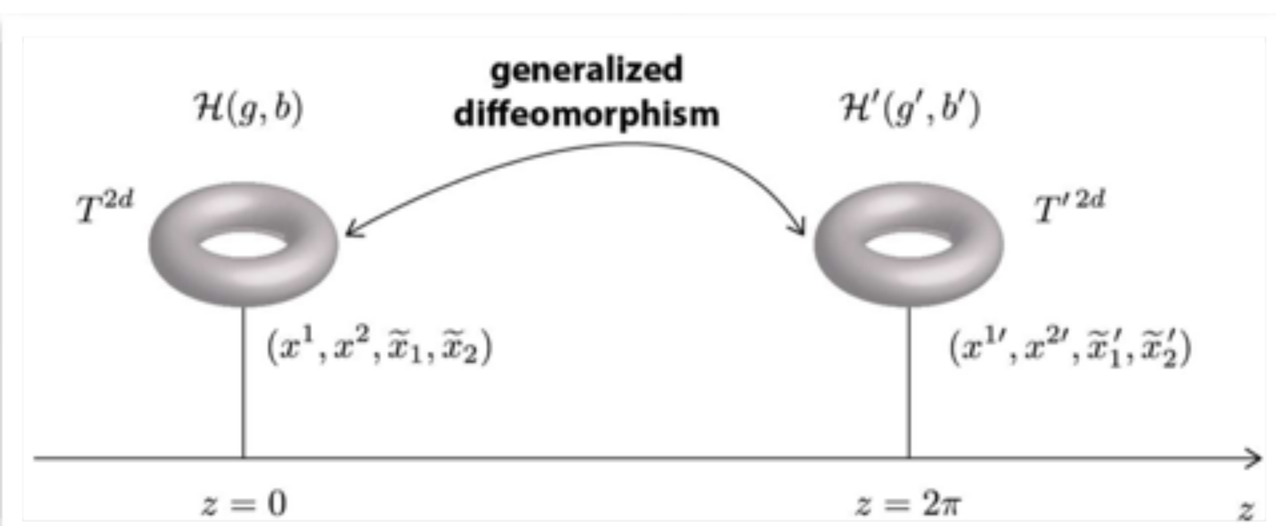
Geometry or No Geometry

Globally Geometric

$$\mathcal{H}_{AB} = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} \in O(D,D)$$

$$e^{-2d} = \sqrt{-G}e^{-2\Phi}$$

Locally Geometric & Globally Nongeometric



T-fold

Everywhere Nongeometric (Non-Riemannian)

$$\mathcal{H}_{AB} = \begin{pmatrix} 0 & N^{\mu}{}_{\lambda} \\ (N^t)_{\rho}{}^{\nu} & S_{\rho\lambda} \end{pmatrix}$$

$$N^2 = 1\,, \qquad S = S^t\,, \qquad SN = -(SN)^t$$

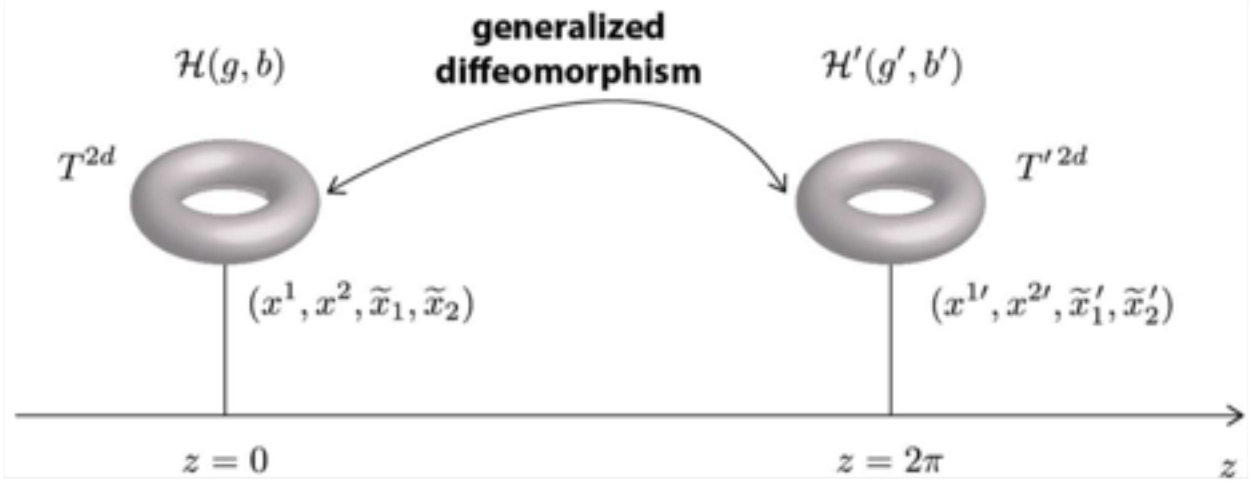
Geometry or No Geometry

Globally Geometric

$$\mathcal{H}_{AB} = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} \in O(D,D)$$

$$e^{-2d} = \sqrt{-G}e^{-2\Phi}$$

Locally Geometric & Globally Nongeometric



T-fold

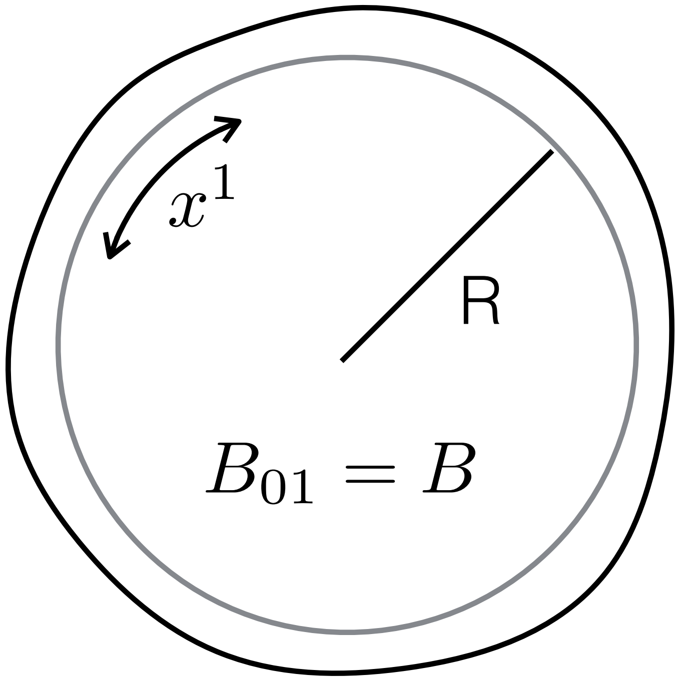
Everywhere Nongeometric (Non-Riemannian)

$$\mathcal{H}_{AB} = \begin{pmatrix} 0 & N^{\mu}{}_{\lambda} \\ (N^t)_{\rho}{}^{\nu} & S_{\rho\lambda} \end{pmatrix}$$

$$N^2 = 1\,, \qquad S = S^t\,, \qquad SN = -(SN)^t$$

Non-relativistic Closed String

J. Gomis & H. Ooguri hep-th/0009181



$$ds^2 = c^2 \underbrace{\left(-dt^2 + (dx^1)^2\right)}_{\eta_{\alpha\beta}\, dx^\alpha\, dx^\beta} + d\vec{x}^2$$

$B = c^2 - \mu$, with μ finite as $c \to \infty$.

$$E = \mu \frac{wR}{\alpha'} + \frac{\alpha' k^2}{2wR} + \frac{N + \tilde{N} - 2}{wR}$$

level-matching condition $wn = N - \tilde{N}$.

Lagrange multiplies β, γ, $\bar{\beta}$, $\bar{\gamma}$ survive

$$S = \frac{1}{2\pi\alpha'} \int d^2z \left(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + \frac{\mu}{2}\partial\gamma\bar{\partial}\bar{\gamma} + \partial X^i \bar{\partial} X^i\right)$$

DFT sigma model

K.-H. Lee & J.-H. Park hep-th/1307.8377

$$\mathcal{S} = \frac{1}{4\pi\alpha'}\int_{\Sigma} \mathrm{d}^2\sigma\, \mathcal{L}\,, \qquad \mathcal{L} = -\tfrac{1}{2}\sqrt{-h}h^{ab}D_aX^M D_bX^N \mathcal{H}_{MN}(X) - \epsilon^{ab}D_aX^M \mathcal{A}_{bM}$$

Doubled space-time: $x^A = (\tilde{x}_\mu, x^\nu)$

Section condition on fields and arbitrary products: $\partial_A\partial^A(\ldots) = 0$

Equivalent “coordinate gauge invariance”, e.g. $\Phi(x^A + \phi(x)\partial^A\varphi(x)) = \Phi(x^A)$

Coordinate gauge field and covariant one forms:

$$Dx^M = dx^M - \mathcal{A}^M$$

$$\mathcal{A}^M = (A_\mu, \tilde{A}^\mu)$$

$$S = \frac{1}{2\pi\alpha'}\int d^2z \left(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + \frac{\mu}{2}\partial\gamma\bar{\partial}\bar{\gamma} + \partial X^i\bar{\partial}X^i\right)$$

$$\mathcal{H}_{MN} = \begin{pmatrix} 0 & 0 & \mathcal{E}^\alpha{}_\beta & 0 \\ 0 & \delta^{ij} & 0 & 0 \\ -\mathcal{E}_\alpha{}^\beta & 0 & 2\mu\eta_{\alpha\beta} & 0 \\ 0 & 0 & 0 & \delta_{ij} \end{pmatrix}$$

$$\gamma = X^1 + X^0 \qquad \bar{\gamma} = X^1 - X^0$$

$$A_+ = \beta \qquad A_- = -\bar{\beta}$$

$$\tilde{X}_\alpha = 0$$

$$(\mathcal{E}^\alpha{}_\beta) = \sigma^\alpha_\beta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \eta_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Fluctuations in DFT

S.-M. Ko, C. Melby-Thompson, RM, J.-H. Park, hep-th/1508.01121

$$\mathcal{L} = \tfrac{1}{8}e^{-2d}\left[(P^{AC}P^{BD} - \bar{P}^{AC}\bar{P}^{BD})S_{ABCD} - 2\Lambda\right]$$

$$P_{AB} = \frac{1}{2}(\mathcal{J}_{AB} + \mathcal{H}_{AB})$$

$$\bar{P}_{AB} = \frac{1}{2}(\mathcal{J}_{AB} - \mathcal{H}_{AB})$$

$$S_{ABCD} = \frac{1}{2}(R_{ABCD} + R_{CDAB} - \Gamma^{E}{}_{AB}\Gamma_{ECD})$$

$$\nabla_C T_{A_1\cdots A_n} = \partial_C T_{A_1\cdots A_n} - \omega\Gamma^{B}{}_{BC}T_{\omega A_1\cdots A_n} + \sum_{i=1}^{n}\Gamma_{CA_i}{}^{B}T_{\omega A_1\cdots A_{i-1}BA_{i+1}\cdots A_n}$$

$$\nabla_A d = -\tfrac{1}{2}e^{2d}\nabla_A(e^{-2d}) = \partial_A d + \tfrac{1}{2}\Gamma^{B}{}_{BA} = 0\,, \qquad \nabla_A P_{BC} = 0\,, \qquad \nabla_A \bar{P}_{BC} = 0$$

The DFT fluctuations δd and δP_{AB} satisfy the equations of motion:

$$(P^{AB} - \bar{P}^{AB})\nabla_A\partial_B\delta d - \tfrac{1}{2}\nabla_A\nabla_B\delta P^{AB} \equiv 0\,, \qquad (2.21)$$

$$P_A{}^C\bar{P}_B{}^D\nabla_C\partial_D\delta d + \tfrac{1}{4}(P_A{}^C\bar{\Delta}_B{}^D - \Delta_A{}^C\bar{P}_B{}^D)\delta P_{CD} \equiv 0\,. \qquad (2.22)$$

These two relations can be also derived from the following effective Lagrangian for the fluctuations around a given on-shell background,

$$\mathcal{L}_{\text{eff.}} := e^{-2d}\left[\tfrac{1}{2}(P-\bar{P})^{AB}\partial_A\delta d\,\partial_B\delta d - \tfrac{1}{2}\partial_A\delta d\,\nabla_B\delta P^{AB} + \tfrac{1}{8}\delta P^{AB}(\bar{\Delta}_A{}^C P_B{}^D - \Delta_A{}^C\bar{P}_B{}^D)\delta P_{CD}\right] \qquad (2.23)$$

Fluctuations around Gomis-Ooguri

S.-M. Ko, C. Melby-Thompson, RM, J.-H. Park, hep-th/1508.01121

Show: Kaluza-Klein Sector of Gomis-Ooguri string is empty: $\tilde{\partial}^\mu = 0$

$$\mathcal{H}_{AB} \mapsto \mathcal{H}_{AB} + h_{AB},\, d \mapsto d + \psi \qquad h_{AB} = h_{BA} \qquad \mathcal{H}_A{}^C \mathcal{H}_{CB} = J_{AB}$$

Lin. Gen. Diffeo.: $\delta h_{AB} = \hat{\mathcal{L}}_\xi \mathcal{H}_{AB} = \mathcal{H}_{AC}\partial_B \xi^C + \mathcal{H}_{CB}\partial_A \xi^C - \mathcal{H}_{AC}\partial^C \xi_B - \mathcal{H}_{CB}\partial^C \xi_A$

$$h_{AB} = \begin{pmatrix} h^{\alpha\beta} & h^{\alpha i} & h^\alpha{}_\beta & h^\alpha{}_j \\ h^{i\alpha} & h^{ij} & h^i{}_\alpha & h^i{}_j \\ h_\alpha{}^\beta & h_\alpha{}^j & h_{\alpha\beta} & h_{\alpha j} \\ h_i{}^\alpha & h_i{}^j & h_{i\alpha} & h_{ij} \end{pmatrix} = \begin{pmatrix} \hat{h}\eta^{\alpha\beta} & -\sigma^\alpha_\beta h^\beta{}_k g^{ki} & -\mu \hat{h} \mathcal{E}^\alpha{}_\beta & h^\alpha{}_j \\ & g^{im} h_{mn} g^{nj} & & -g^{ik} b_{kj} \\ & -2\mu \eta_{\alpha\gamma} h^\gamma{}_k g^{ki} & = 0 & = 0 \\ & & = 0 & h_{ij} \end{pmatrix}$$

Fourier expansion in plane waves: $h_{AB}(x) = h_{AB} e^{ip_+ x^+ + ip_- x^- + ik_i x^i}$

No propagating normalizable wave packet solution if there exists no solution with

$$k^2 \neq 0 \quad \text{and} \quad (p_- \neq 0 \quad \text{or} \quad p_+ \neq 0)$$

T-dual frame and Winding Modes

S.-M. Ko, C. Melby-Thompson, RM, J.-H. Park, hep-th/1508.01121

$$\mathcal{H}_{MN} = \begin{pmatrix} 0 & 0 & \mathcal{E}^{\alpha}{}_{\beta} & 0 \\ 0 & \delta^{ij} & 0 & 0 \\ -\mathcal{E}_{\alpha}{}^{\beta} & 0 & 2\mu\eta_{\alpha\beta} & 0 \\ 0 & 0 & 0 & \delta_{ij} \end{pmatrix}$$

T-duality along x1:

$$(\mathcal{O}_A{}^B) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$ds^2 = -2\mu dt^2 + 2dtdx + (dx^i)^2$$

$$E = \mu\frac{wR}{\alpha'} + \frac{\alpha' k^2}{2wR} + \frac{N + \tilde{N} - 2}{wR}$$

T-duality along x1: (lowest modes only)

$$E = \frac{\mu\tilde{n}R}{\alpha'} + \frac{\alpha' k^2}{2\tilde{n}R}.$$

T-dual section condition: $\tilde{\partial}^{\mu} = 0$ for $\mu \neq 1$ and $\partial_1 = 0$.

$$E = \mu p_\theta + \frac{k^2}{p_\theta}$$

The radius of the θ circle is given by $\tilde{R} = \alpha'/R$, and so the θ momentum is quantized in units of $1/\tilde{R}$,

$$p_\theta = \frac{\tilde{n}R}{\alpha'}.$$

KK Spectrum in dual frame = Gomis-Ooguri Winding Spectrum!

Galilean Invariance in DFT

S.-M. Ko, C. Melby-Thompson, RM, J.-H. Park, hep-th/1508.01121

Galilean invariant spectrum:

$$E = \mu \frac{wR}{\alpha'} + \frac{\alpha' k^2}{2wR} + \frac{N + \tilde{N} - 2}{wR}$$

Generators of Bargmann algebra (central extension of Galilean):

$H = -\partial_t$

$P_i = -\partial_i$

$M_{ij} = -(x^i \partial_j - x^j \partial_i)$

Circle U(1): $Q = -\partial_1$

U(1) particle number: $N = -\tilde{\partial}^1$

Galilean Boosts: $B_i = -t\partial_i - x^i \tilde{\partial}^1$,

Close into the Bargmann Algebra under the C-Bracket:

$$[\xi, \eta]_C = \xi^A(\partial_A \eta^B)\partial_B - \eta^A(\partial_A \xi^B)\partial_B - \frac{1}{2}\xi_A(\partial^B \eta^A)\partial_B + \frac{1}{2}\eta_A(\partial^B \xi^A)\partial_B$$

$$[B_i, H] = P_i \qquad [B_i, P_j] = \delta_{ij} N \qquad [M_{ij}, P_k] = \delta_{ik} P_j - \delta_{jk} P_i$$

$$[M_{ij}, B_k] = \delta_{ik} B_j - \delta_{jk} B_i \qquad [M_{ij}, M_{k\ell}] = \delta_{ik} M_{j\ell} - \delta_{i\ell} M_{kj} - \delta_{jk} M_{i\ell} + \delta_{j\ell} M_{ik} \,.$$

Doubled Schrödinger Geometry

S.-M. Ko, C. Melby-Thompson, RM, J.-H. Park, hep-th/1508.01121

Non relativistic scale invariance: $t \mapsto \lambda^z t\,, \qquad \vec{x} \mapsto \lambda \vec{x}\,.$

The infinitesimal generator D of these transformations has the following commutators (C-brackets) with the Bargmann generators:

$$[D,H] = zH\,, \qquad [D,P_i] = P_i\,, \qquad [D,B_i] = -(z-1)B_i\,, \qquad [D,N] = -(2-z)N\,, \qquad [D,M_{ij}] = 0$$

Finally, for the special case $z = 2$ an additional generator C with commutators

$$[C,H] = D\,, \qquad [C,D] = C\,, \qquad [C,P_i] = B_i\,, \qquad [C,B_i] = [C,M_{ij}] = [C,N] = 0\,,$$

Schrödinger invariant DFT geometry: (no solution)

$$\mathcal{H}_{AB} = \begin{pmatrix} 0 & \sigma^{\alpha}{}_{\beta}(u) \\ \sigma_{\alpha}{}^{\beta}(u) & \mathcal{H}_{\alpha\beta} \end{pmatrix}, \qquad \mathcal{H}_{IJ} = \begin{pmatrix} u^2\,\delta^{ij} & 0 \\ 0 & u^{-2}\,\delta_{ij} \end{pmatrix}, \qquad \mathcal{H}_{AI} = 0\,,$$

$$\mathcal{H}_{\alpha\beta} = \begin{pmatrix} -\frac{1}{u^{2z}} & 0 \\ 0 & u^{4-2z} \end{pmatrix}, \qquad \sigma^{\alpha}{}_{\beta}(u) = (\sigma_{\beta}{}^{\alpha}(u))^T = \begin{pmatrix} 0 & -u^2 \\ -\frac{1}{u^2} & 0 \end{pmatrix}.$$

$$H = -\partial_t\,,$$

$$P_m = -\partial_m\,,$$

$$N = -\tilde{\partial}^1\,,$$

$$D = -zt\partial_t - x^m\partial_m - u\partial_u - (z-2)x^1\partial_1$$

$$B_m = -t\partial_m - x^m\tilde{\partial}^1\,,$$

$$M_{mn} = -(x^m\partial_n - x^n\partial_m)\,.$$

$$C = -t^2\partial_t - tx^m\partial_m - tu\,\partial_u - \frac{1}{2}(x^2+u^2)\tilde{\partial}^1.$$

Conclusions

S.-M. Ko, C. Melby-Thompson, RM, J.-H. Park, hep-th/1508.01121

- DFT: T-duality invariant low energy effective action for massless string modes
- Geometric as well as non-geometric string backgrounds
- Non-relativistic Closed String (NRCS) J. Gomis & H. Ooguri hep-th/0009181
  Concrete, healthy example of a non-relativistic, Galilean invariant string vacuum
- NRCS is a locally non-geometric background of DFT sigma model
  K.-H. Lee & J.-H. Park hep-th/1307.8377
- DFT correctly captures the (empty) KK and non relativistic winding spectrum
- New Schrödinger invariant background
- New non-geometric string vacua for
  Cosmology?
  (Non)relativistic Holography?
  …