Yang-Baxter deformations of Minkowski spacetime

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1. Introduction

Yang-Baxter sigma models [Klimcik, 2002,2008]

A systematic way to study integrable deformations of 2D NLSMs.

An integrable sigma model ($AdS_5\times S^5$ spacetime) —YB deformations→ Integrable deformed sigma models (Lunin-Maldacena, gravity duals of NCYM)

Deformations are characterized by classical r-matrices → Solutions of the classical Yang-Baxter equation

To investigate physical and mathematical structures of YB deformations

flat space is simpler than $AdS_5\times S^5$ → (Some) deformed models are exactly solvable

2. Coset construction of 4D Minkowski

Problem : The Killing form on ISO(1,3)/SO(1,3) is degenerate.

→ 4D Minkowski is realized as a slice of Poincare AdS_5 .

$$\text{Poincaré AdS}_5=\frac{SO(2,4)}{SO(1,4)}$$

$$g=\exp\Big[p_0\,x^0+p_1\,x^1+p_2\,x^2+p_3\,x^3\Big]\exp\Big[\hat{d}\,\log z\Big]$$

$$ds^2=\mathrm{Tr}(A\overline{P}(A))=\frac{-(dx^0)^2+\sum_{i=1}^3(dx^i)^2+dz^2}{z^2}$$

$$\overline{P}(x)=\frac{1}{4}\Big[-\gamma_0\,\mathrm{Tr}(\gamma_0\,x)+\sum_{i=1}^{3}\gamma_i\,\mathrm{Tr}(\gamma_i\,x)+\gamma_5\,\mathrm{Tr}(\gamma_5\,x)\Big]$$

$$\overline{P}:\mathfrak{so}(2,4)\to\mathfrak{so}(2,4)/\mathfrak{so}(1,4)$$

$$A=g^{-1}dg$$

$$p_\mu\equiv\frac{1}{2}(\gamma_\mu-2n_{\mu5})\quad[p_\mu\,,p_\nu]=0$$

$$\{\,\gamma_\mu\,,\gamma_\nu\,\}=2\eta_{\mu\nu}$$

$$\gamma_5=i\gamma_1\gamma_2\gamma_3\gamma_0\qquad\hat{d}=\frac{\gamma_5}{2}$$

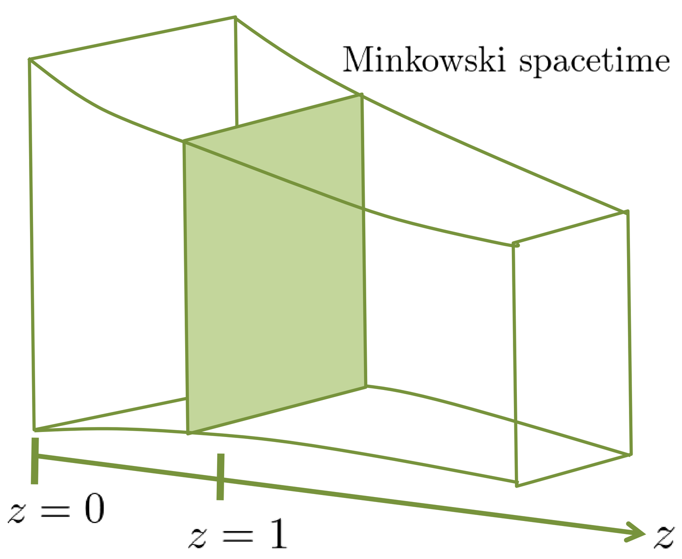
$$\frac{1}{4}[\,\gamma_\mu\,,\gamma_\nu\,]=n_{\mu\nu}\quad\frac{1}{4}[\,\gamma_\mu\,,\gamma_5\,]=n_{\mu5}$$

Minkowski spacetime

$$g=\exp\Big[p_0\,x^0+p_1\,x^1+p_2\,x^2+p_3\,x^3\Big]$$

$$ds^2=\mathrm{Tr}(AP(A))=-(dx^0)^2+\sum_{i=1}^{3}(dx^i)^2$$

$$P(x)=\frac{1}{4}\Big[-\gamma_0\,\mathrm{Tr}(\gamma_0x)+\sum_{i=1}^{3}\gamma_i\,\mathrm{Tr}(\gamma_ix)\Big]$$



3. Yang-Baxter deformed Minkowski space

The action of Yang-Baxter sigma models of Minkowski spacetime (proposal)

$$S=-\frac{1}{2}\int_{-\infty}^{\infty}d\tau\int_0^{2\pi}d\sigma\,(\gamma^{\alpha\beta}-\epsilon^{\alpha\beta})\mathrm{Tr}\Big[A_\alpha P\circ\frac{1}{1-2\eta R_g\circ P}(A_\beta)\Big]$$

$$R_g(X)\equiv g^{-1}R(gXg^{-1})g\quad X\in\mathfrak{so}(2,4)$$

The R operator is defined by using a classical r-matrix:

$$R(X)\equiv\langle r_{12},1\otimes X\rangle=a\,\mathrm{Tr}(b\,X)-b\,\mathrm{Tr}(a\,X)$$

This is a solution of the CYBE $[R(X),R(Y)]-R\left([R(X),Y]+[X,R(Y)]\right)=0$

Classification of classical r-matrices

$r=a\wedge b=a\otimes b-b\otimes a$	$[a\,,b]=0$	$[a\,,b]\neq0$
(a) $r=$ Poincaré $\wedge$ Poincaré	$r\sim p_3\wedge n_{12}$	$r\sim(p_0-p_3)\wedge n_{03}$
(b) $r=$ Poincaré $\wedge$ non-Poincaré	$r\sim n_{12}\wedge\hat{d}$	$r\sim p_0\wedge\hat{d}$
(c) $r=$ non-Poincaré $\wedge$ non-Poincaré	$r\sim k_1\wedge k_2$	$r\sim k_0\wedge\hat{d}$

What we did is…

- Introduced Yang-Baxter deformations of Minkowski spacetime.
- Identified TsT-transformed backgrounds with classical r-matrices.
- Obtained T-dual of dS_4 and AdS_4 as Yang-Baxter deformations.

4. Twisted backgrounds

An abelian classical r-matrix

$$r=\frac{1}{2}p_3\wedge n_{12}$$

The associated metric and B-field are given by

$$ds^2=-(dx^0)^2+dr^2+\frac{r^2d\theta^2+(dx^3)^2}{1+\eta^2r^2}$$

$$B=\frac{\eta r^2}{1+\eta^2r^2}d\theta\wedge dx^3$$

Melvin background

This b.g. is also obtained as a TsT-trans. of Minkowski spacetime [Hashimoto-Thomas,0410123]

Further development

Lax pairs can be constructed [Kyono-JS-Yoshida, 1511.NNNNN]

5. Non-twisted backgrounds

A non-abelian r-matrix

$$r=\frac{1}{2}\hat{d}\wedge p_0\qquad[\,\hat{d}\,,\,p_0\,]=p_0$$

The deformed background is

$$ds^2=\frac{-(dx^0)^2+dr^2}{1-\eta^2r^2}+r^2\sin^2\theta d\phi^2+r^2d\theta^2$$

$$B=\frac{\eta r}{1-\eta^2r^2}dx^0\wedge dr\quad\text{(total derivative)}$$

$$x^1=r\cos\phi\,\sin\theta\,,x^2=r\sin\phi\sin\theta\,,x^3=r\cos\theta$$

$$x^0=t-\frac{1}{2\eta}\log(\eta^2r^2-1)$$

Perform a time-like T-duality and a coordinate transformation

$$ds^2=-(1-\eta^2r^2)dt^2+\frac{dr^2}{1-\eta^2r^2}+r^2(d\theta^2+\sin^2\theta d\phi^2)$$

This is the metric of dS_4 in the static coordinates.

6. A list of deformed backgrounds and classical r-matrices

TsT-transformed backgrounds (exactly solvable models)

$r=\frac{1}{2}p_3\wedge n_{12}$	Melvin twist	Melvin background
$r=\frac{1}{2\sqrt{2}}(p_0-p_3)\wedge n_{12}$	Null Melvin twist	pp-wave
$r=\frac{1}{2\sqrt{2}}p_2\wedge(n_{01}+n_{13})$	Melvin null twist	Hashimoto-Sethi
$r=\frac{1}{2}n_{12}\wedge n_{03}$	R Melvin R twist	Spradlin-Takayanagi-Volovich
$r=\frac{1}{2}p_\mu\wedge p_\nu$	Melvin shift twist	Locally flat spaces
$r=\frac{1}{2}(p_0+p_3)\wedge p_1$	Null Melvin shift twist	Locally flat spaces

Non-twisted backgrounds

$r=\frac{1}{2}\hat{d}\wedge p_0$	Non twist	T-dual of dS_4
$r=\frac{1}{2}\hat{d}\wedge p_1$	Non twist	T-dual of AdS_4
$r=\frac{1}{2\sqrt{2}}(\hat{d}-n_{03})\wedge(p_0-p_3)$	Non twist	pp-wave

κ-Poincare r-matrices $r=a^\mu\,n_{\mu\nu}\wedge p^\nu$ give same b.g.
[Borowiec-Kyono-Lukierski-JS-Yoshida, 1510.03083]