O(D,D) Covariant Noether Currents and Global Charges in Double Field Theory

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based on arXiv:1507.07545, to appear in JHEP,
with J-H. Park (Sogang U.), S-J. Rey, W. Rim (SNU, IBS)

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Motivation

Double Field Theory (DFT)

<u>DFT</u>: manifestly O(*D*,*D*)-covariant formulation of Supergravity

Standard coords. x^μ in *D*-dims.

$\updownarrow$ T-dual

Dual coords. $\tilde{x}_\mu$ also have *D*-dims.

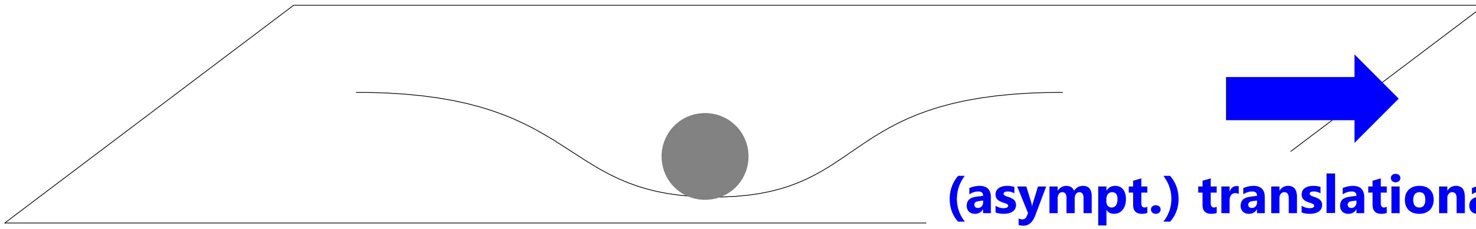
"Gravitational" theory in *2D*-dimemsions.

Formal aspects of DFT have been studied in detail, but the applications are not studied well.

⇒ We study Noether currents in DFT.

ADM momenta in General Relativity

Asymptotically Flat spacetime



(asympt.) translational sym.

$$x^\mu \to x^\mu + \xi^\mu$$

(asympt.) const.

The Noether charge has the form,

[Iyer, Wald, 1994]

$$Q[\xi] = \int (\mathrm{d}^{D-2}x)_{\mu\nu} \sqrt{-G} \left(K^{\mu\nu}[\xi] + 2\, \xi^{[\mu} B^{\nu]} \right) .$$

"Komar potential" $\quad K^{\mu\nu}[\xi] \equiv -2D^{[\mu}\xi^{\nu]}$

~~(general covariant)~~

ADM momenta : $\quad P_\mu^{\mathrm{ADM}} \equiv Q[\partial_\mu] .$

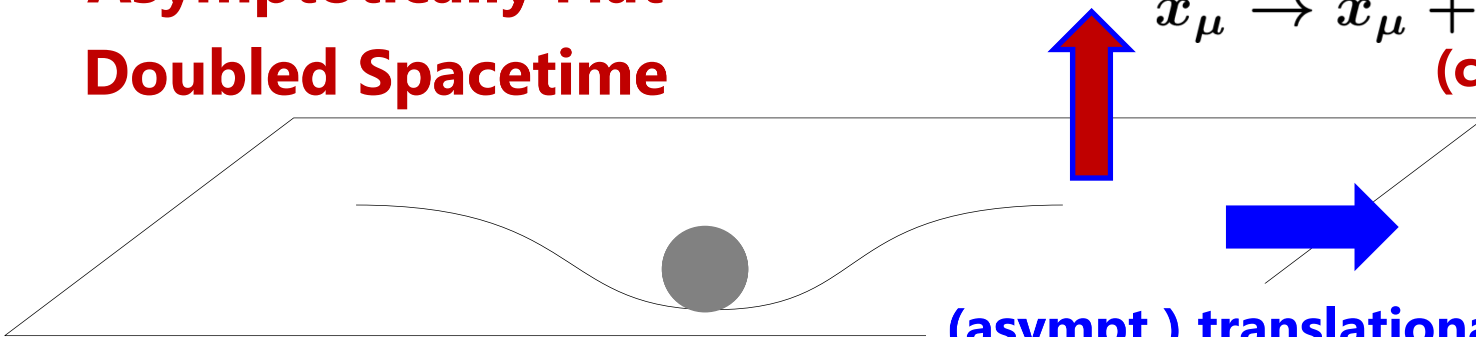
ADM momenta in DFT (1/2)

Asymptotically Flat
Doubled Spacetime

translational sym. !

$$\tilde{x}_\mu \to \tilde{x}_\mu + \tilde{\xi}_\mu$$

(const.)



(asympt.) translational sym.

$$x^\mu \to x^\mu + \xi^\mu$$

(const.)

■ Objective :

Find the expression for ADM momenta, like

$$Q[\xi] = \int (\mathrm{d}^{D-2}x)_{\mu\nu} \sqrt{-G} \left(K^{\mu\nu}[\xi] + 2\,\xi^{[\mu} B^{\nu]} \right) ,$$

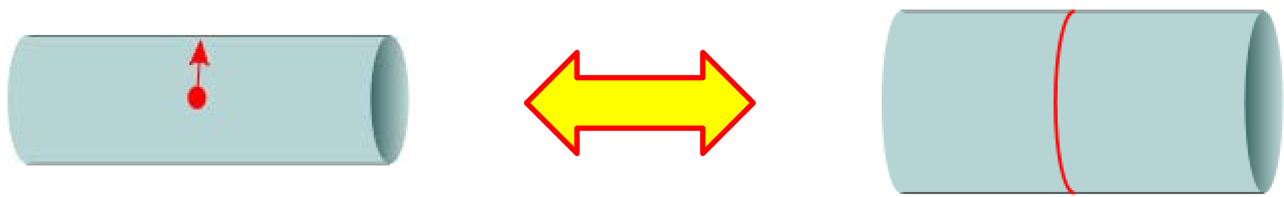
in the Double Field Theory.

ADM momenta in DFT (2/2)

DFT

x^μ **T-duality** $\tilde{x}_\mu$

$\tilde{x}_\mu$ $\longleftrightarrow$ x^μ



momenta **winding**

$$P_\mu^{\mathrm{ADM}} \equiv Q[\partial_\mu]$$

$$Q_{\mathbf{F1}} = \int \mathrm{e}^{-2\phi} * H^{(3)} \stackrel{?}{=} Q[\tilde{\partial}^\mu]\,.$$

■ **Expectation**:

ADM momenta & F1 charges will be unified by treating x^μ and $\tilde{x}_\mu$ on an equal footing.

Double Field Theory

C. Hull and B. Zwiebach, JHEP 0909, 099 (2009),
O. Hohm, C. Hull and B. Zwiebach, JHEP 1008 008 (2010),....,
I. Jeon, K. Lee, J-H. Park, JHEP 1104, 014 (2011), ...

Double Field Theory (1/3)

Manifestly O(*D*,*D*)-covariant formulation of Supergravity

***2D*-dimensional "Doubled Space"** $x^A = (\tilde{x}_\mu,\, x^\mu)$

$$\Longrightarrow \quad \partial_A = \left(\tilde{\partial}^\mu \equiv \frac{\partial}{\partial \tilde{x}_\mu},\; \partial_\mu \equiv \frac{\partial}{\partial x^\mu}\right)$$

Fund. fields:

Generalized metric: $\mathcal{H}_{AB} \equiv \begin{pmatrix} G^{-1} & -G^{-1}B \\ B\,G^{-1} & G - B\,G^{-1}B \end{pmatrix}$

T-duality inv. dilaton: $\mathrm{e}^{-2d} \equiv \sqrt{-G}\,\mathrm{e}^{-2\phi}$

DFT action:

$$S_{\rm DFT} = \int \mathrm{d}^{2D}x\, \underbrace{\mathrm{e}^{-2d}}_{\text{density}}\, \mathcal{R}\,.$$

[Hohm, Hull, Zwiebach, 2010]

$$\mathcal{R} \equiv 4\mathcal{H}^{AB}\,\partial_A\partial_B d - \partial_A\partial_B\mathcal{H}^{AB} - 4\mathcal{H}^{AB}\,\partial_A d\,\partial_B d + 4\partial_A\mathcal{H}^{AB}\,\partial_B d$$
$$+\frac{1}{8}\mathcal{H}^{AB}\,\partial_A\mathcal{H}^{CD}\,\partial_B\mathcal{H}_{CD} - \frac{1}{2}\mathcal{H}^{AB}\,\partial_A\mathcal{H}^{CD}\,\partial_C\mathcal{H}_{BD}\,.$$

Double Field Theory (2/3)

■ Gauge symmetry : generalized diffeo. $x^A \to x^A + V^A \,.$

Gauge parameter: $V^A(x,\tilde{x}) = \big(\tilde{v}_\mu(x,\tilde{x}),\, v^\mu(x,\tilde{x})\big)\,.$

$$\delta_V \mathcal{H}_{AB} = \hat{\pounds}_V \mathcal{H}_{AB}\,, \quad \delta_V\, \mathrm{e}^{-2d} = \hat{\pounds}_V\, \mathrm{e}^{-2d} = \partial_A(\mathrm{e}^{-2d}\, V^A)\,.$$

Gen. Lie derivative: $\hat{\pounds}_V W^A \equiv V^B\, \partial_B W^A - (\partial_B V^A - \partial^A V_B)\, W^B\,.$

■ Consistency of the theory

$$\boxed{\mathcal{J}^{AB}\,\partial_A * \,\partial_B * = 0\,.}$$ (*strong constraint*)

$$\mathcal{J}^{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ (Indices are raised or lowered)

$$\Longrightarrow \quad \frac{1}{2}\,\mathcal{J}^{AB}\,\partial_A * \,\partial_B * = \partial_\mu * \,\tilde{\partial}^\mu * = 0\,.$$

$$\Longrightarrow \quad \partial_\mu = 0 \quad \text{or} \quad \boxed{\tilde{\partial}^\mu = 0\,.}$$

In our work, we choose the canonical section.

Double Field Theory (3/3)

■ $\tilde{\partial}^{\mu} = 0$

DFT ⇨ **Conventional Supergravity (NS-NS sector)**

$$S_{\mathrm{DFT}} \to \frac{1}{2\kappa_{10}^2}\int\sqrt{-G}\,\mathrm{d}^d x\,\mathrm{e}^{-2\phi}\left(R + 4\,|\partial\phi|^2 - \frac{1}{12}\,|H|^2\right).$$

Gauge symmetry of DFT

$$\delta_V \mathcal{H}_{AB} = \hat{\pounds}_V \mathcal{H}_{AB}$$

$$\mathcal{H}_{AB} \equiv \begin{pmatrix} G^{-1} & -G^{-1}B \\ B\,G^{-1} & G - B\,G^{-1}B \end{pmatrix}$$

⇨

$$\begin{cases} \delta_V G_{\mu\nu} = \pounds_v G_{\mu\nu} \\ \delta_V B_{\mu\nu} = \pounds_v B_{\mu\nu} + 2\,\partial_{[\mu}\tilde{v}_{\nu]} \end{cases}$$

Diffeo. + B-field gauge transf.

$$V^A(x, \cancel{\tilde{x}}) = \left(\tilde{v}_\mu(x),\; v^\mu(x)\right)$$

B-field gauge transf.

***D*-dim Diffeo.**

Stringy Differential Geometry

[I. Jeon, K. Lee, J-H. Park, 2011;
Hohm, Zwiebach, 2012]

$$\delta_V W^A = \hat{\mathcal{L}}_V W^A$$

$$\delta_V \partial_B W^A = \hat{\mathcal{L}}_V \partial_B W^A + 2\partial_B \partial^{[A} V^{C]} W_C \,.$$

Covariant derivative : $\nabla_B W^A \equiv \partial_B W^A + \Gamma_B{}^A{}_C W^C \,.$

$$\delta_V \nabla_B W^A = \hat{\mathcal{L}}_V \nabla_B W^A \,.$$

$$R_{ABCD} \equiv \partial_A \Gamma_{BCD} - \partial_B \Gamma_{ACD} + \Gamma_{AC}{}^E \Gamma_{BCD} - \Gamma_{BC}{}^E \Gamma_{ACD} \,.$$

$$S_{ABCD} \equiv \frac{1}{2}\left(R_{ABCD} + R_{CDAB} - \Gamma^E{}_{AB} \Gamma_{ECD}\right) .$$

$\mathcal{S}_{AB} \equiv P_A{}^C \bar{P}_B{}^D S_{CED}{}^E \,.$ (tensor!) $\Longrightarrow$ $\mathcal{S} \equiv \mathcal{S}_A{}^A \,.$

Projection: $\left[P_A{}^B \equiv \frac{1}{2}(\delta_A^B + \mathcal{H}_A{}^B) \,, \quad \bar{P}_A{}^B \equiv \frac{1}{2}(\delta_A^B - \mathcal{H}_A{}^B) \right]$

$$\begin{aligned} \mathrm{e}^{-2d}\, \mathcal{S} &= \mathcal{L}_{\mathrm{DFT(Hohm\text{-}Hull\text{-}Zwiebach)}} + \partial_A (*)^A \\ &= \sqrt{-G}\, \mathrm{e}^{-2\phi} \left(R + 4|\partial\phi|^2 - \tfrac{1}{12}|H|^2 \right) \\ &\quad + \partial_\mu \left[4\, \mathrm{e}^{-2\phi}\, \partial_\nu (\sqrt{-G}\, G^{\mu\nu} \partial_\nu \phi) \right] . \end{aligned}$$

Generalization of
Einstein-Hilbert action

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Noether current (standard procedure)

DFT action:

$$\mathcal{L}_{\text{NS-NS}} = \mathrm{e}^{-2d}\,\mathcal{S}\,.$$

$$\delta\mathcal{L}_{\text{NS-NS}} = 2\,\mathrm{e}^{-2d}\big(-\mathcal{S}\,\delta d + \mathcal{S}_{AB}\,\delta\mathcal{H}^{AB}\big) + \partial_A\big(\mathrm{e}^{-2d}\,\Theta^A\big)\,.$$

EOM

$$\mathcal{S} = 0 \qquad\qquad \mathcal{S}_{AB} = 0$$ (traceless part)

Pre-symplectic potential

$$\boxed{\Theta^A = \mathcal{H}^{AB}\partial_B\delta d - \nabla_B\delta\mathcal{H}^{AB}\,.}$$

In particular, under a global sym. $x^A \to x^A + X^A \quad (\delta = \hat{\pounds}_X)$

$$\delta_X\mathcal{L}_{\text{NS-NS}} \overset{\text{EOM}}{\approx} \partial_A\big(e^{-2d}\,\Theta^A|_{\delta\to\hat{\pounds}_X}\big)$$
$$= \partial_A\big(\mathrm{e}^{-2d}\,\mathcal{S}\,X^A\big)\,.$$

$$H^A[X] \equiv \Theta^A|_{\delta\to\hat{\pounds}_X} - \mathcal{S}\,X^A\,. \quad\Rightarrow\quad \partial_A\big(\mathrm{e}^{-2d}\,H^A[X]\big) \overset{\text{EOM}}{\approx} 0\,.$$

(Noether current)

$$\left(J^A[X] \equiv -2\,\underbrace{G^{AB}}_{\approx\, 0}X_B + H^A[X]\,. \quad\Rightarrow\quad \partial_A\big(\mathrm{e}^{-2d}\,J^A[X]\big) = 0\,.\right)$$

(“Einstein tensor”)

Boundary term

To make the Dirichlet problem well-defined,

we add a boundary term to the DFT action:

Our action: $$\widehat{\mathcal{L}}_{\text{NS-NS}} = \mathrm{e}^{-2d}\,\mathcal{S} - \partial_A(\mathrm{e}^{-2d}\,B^A)\,.$$

$$\left(B^A \equiv 4\,\mathcal{H}^{AB}\,\partial_B d - \partial_A\mathcal{H}^{AB}\right)$$

$$\delta\widehat{\mathcal{L}}_{\text{NS-NS}} = (\text{e.o.m.}) + \partial_A\left([*]^A\,\delta d + [*]^{AB}\,\delta\mathcal{H}_{AB}\right)\,.$$

No $\partial_A\delta$!

(Identically conserved) Noether current

$$\widehat{J}^A[X] \equiv -2\,G^{AB}\,X_B + \Theta^A|_{\delta\to\hat{\pounds}_X} - (\mathcal{S} - \nabla_B B^B)\,X^A$$
$$= \partial_B\left(\mathrm{e}^{-2d}\left[K^{AB}[X] + 2X^{[A}\,B^{B]}\right]\right)\,.$$

O(*D,D*)-covariant generalization of the Komar potential

$$\boxed{K^{AB}[X] \equiv 4\,(\bar{P}^{[A}{}_C\,P^{B]}{}_D - P^{[A}{}_C\,\bar{P}^{B]}{}_D)\,\nabla^C X^D\,.}$$

Noether Charge

$$\widehat{J}^{A}[X] = \partial_{B}\left(\mathrm{e}^{-2d}\left[K^{AB}[X] + 2X^{[A}B^{B]}\right]\right).$$

■ **Noether charge :**

$$Q[X] = \int_{\Sigma_t} (\mathrm{d}^{D-1}x)_{\mu}\,\widehat{J}^{\mu}[X]$$

$$= \int_{\partial\Sigma_t} (\mathrm{d}^{D-2}x)_{\mu\nu}\sqrt{-G}\,e^{-2\phi}\left(K^{\mu\nu}[X] + 2X^{[\mu}B^{\nu]}\right).$$

$$X^{A} = \begin{pmatrix} \zeta_{\mu} + B_{\mu\nu}\,\xi^{\nu} \\ \xi^{\mu} \end{pmatrix}. \quad \Rightarrow \quad \boxed{K^{\mu\nu}[X] = -2\,D^{[\mu}\xi^{\nu]} - H^{\mu\nu\rho}\,\zeta_{\rho}\,.}$$

[see also C. Blair, arXiv:1507.07541, "Conserved currents of double field theory"]

July 28th

$$\boxed{Q[\tilde{\partial}^{z}] = \int_{S^{D-3}_{\infty}} e^{-2\phi}\ *_{D}\,H^{(3)}\,.}$$

Namely, the dual components of the ADM momenta are F1 charges !

Extension

■ DFT + Cosmological const. + YM

$$\mathcal{L}_{\rm DFT} = e^{-2d}\left[\mathcal{S} - 2\Lambda + g_{\rm YM}^{-2}\,{\rm Tr}\left(P^{AC}\,\bar{P}^{BD}\,\mathcal{F}_{AB}\,\mathcal{F}_{CD}\right)\right] - \partial_A\left(e^{-2d}\,B^A\right).$$

[I. Jeon, K. Lee, J-H. Park, 2011]

gauge field: V_A

■ (Identically) conserved current

$$e^{-2d}\,\hat{J}^A = \partial_B\left[e^{-2d}\left(K^{AB}[X] + 2X^{[A}B^{B]}\right) + 12\,g_{\rm YM}^{-2}\,{\rm Tr}\left\{e^{-2d}(P\mathcal{F}\bar{P})^{[AB}\,V^{C]}\,X_C\right\}\right].$$

Applications

1. **Pure Einstein gravity** $\left(B_{\mu\nu} = 0,\ \phi = 0\right)$
   (Our formula reproduces the well-known ADM energy.)

2. **Null wave background**

$$\begin{aligned} P_A^{(\mathrm{P})} &= (\tilde{P}^t,\ \tilde{P}^z,\ \tilde{P}^m\,;\ P_t,\ P_z,\ P_m) \\ &= \ n_D\,(0,\ldots,0\,;\,+1,-1,0,\ldots,0)\,. \end{aligned}$$

3. **F1 background**

$$\begin{aligned} P_A^{(\mathrm{F1})} &= \qquad (\tilde{P}^t,\ \tilde{P}^z,\ \tilde{P}^m\,;\ P_t,\ P_z,\ P_m) \\ &= \tilde{n}_D\,(0,-1,0,\ldots,0\,;\,+1,0,\ldots,0)\,. \end{aligned}$$

4. **Non-Riemannian background** **(previous talk)**

5. **Other examples**
   (R-N black hole, linear-dilaton background,…)

Non-Riemannian background

F1-background :

$$\mathrm{d}s^2 = H^{-1}(r)(-\mathrm{d}t^2 + \mathrm{d}z^2) + \delta_{mn}\mathrm{d}y^m\mathrm{d}y^n\,,$$
$$B_{tz} = H^{-1}(r)\,,\quad \phi = 0 \quad \left(H(r) \equiv 1 + \tfrac{\gamma_D}{r^{D-4}}\right).$$

$$\mathcal{H}_{AB} \equiv \begin{pmatrix} G^{-1} & -G^{-1}B \\ B\,G^{-1} & G - B\,G^{-1}B \end{pmatrix}$$

T-dualities along the $(t,\ z)$ -directions

Non-Riemannian BG.

[K. Lee, J-H Park, '13]

$$\mathcal{H}_{AB} \equiv \begin{pmatrix} G^{-1} & -G^{-1}B \\ B\,G^{-1} & G - B\,G^{-1}B \end{pmatrix} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$

In the conventional (super)gravity, we cannot calculate ADM momenta

$$\begin{aligned} P_A^{(\text{N-R})} &= (\tilde{P}^t,\, \tilde{P}^z,\, \tilde{P}^m\,;\, P_t,\, P_z,\, P_m) \\ &= \ (0,0,0,\ldots,0\,;\,0,0,0,\ldots,0)\,. \end{aligned}$$

Summary

- **DFT is a manifestly O(*D*,*D*)-covariant formulation of supergravity.**
- **From the strong constraint,**
  the background fields are independent of $\tilde{x}_\mu$.
  (i.e., there are isometries in the dual directions)
- **We calculated O(*D*,*D*)-covariant Noether currents associated with the translational symmetries in asymptotically flat doubled spacetimes.**
- **We showed that the Noether charges associated with translations along the dual directions coincide with the string winding charges.**

$$Q[\tilde{\partial}^z] = \int_{S^{D-3}_\infty} e^{-2\phi} *_D H^{(3)} .$$

Future directions

- **Extension to the Exceptional Field Theory**
  (U-duality covariant generalization),

  In EFT, there are many additional dual coordinates.
  ⇨ We can unify All brane charges
  as the *generalized* ADM momenta

- **Black hole thermodynamics**
  Hawking temperature,
  Bekenstein-Hawking entropy in DFT/EFT ?