Entanglement entropy in the dS/CFT correspondence

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What I did

I. In Einstein gravity on dS we propose the holographic entanglement entropy as the analytic continuation of the extremal surface in Euclidean AdS.

II. We analyzed the free Sp(N) model dual to Vasiliev's higher spin gauge theory as a toy model even though dual conformal field theories for Einstein gravity on dS haven't been known yet.

III. In this Sp(N) model we confirmed the behaviour similar to our holographic result from Einstein gravity.

I. Introduction

AdS/CFT relates gravitational theories on AdS with non-gravitational theories.

➡ We can analyze **quantum** gravitational theories using non-gravitational theories.

Toward a quantum description of our Universe, it is natural to use AdS/CFT.

➡ We need dS/CFT instead of AdS/CFT since our Universe is approximately dS.

The holographic entanglement entropy (HEE) is a useful quantity to analyze gravitational theory.

In fact, a radial component of AdS metric and Einstein's equation are reproduced from HEE, for instance. [Nozaki-Ryu-Takayanagi, PRD 88 (2013) 2, 026012] [Lashkari-McDermott-Raamsdonk, JHEP 1404 (2014) 195]

HEE is a generalised quantity of the black hole entropy. [Lewkowycz-Maldacena, JHEP 1308 (2013) 090]

Black hole entropy formula holds even in dS and flat spacetime.

➡ HEE formula should hold in dS!!

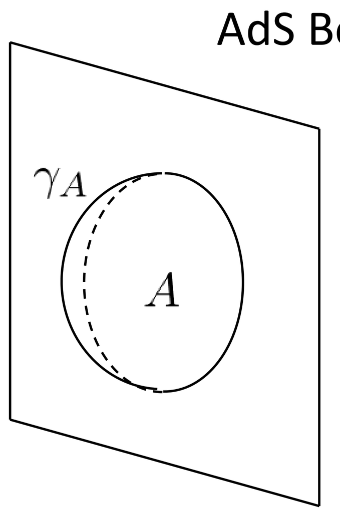
I investigate HEE in dS/CFT.

II. Proposal for HEE in Einstein gravity on dS

Ryu-Takayanagi formula [Ryu-Takayanagi, PRL 96 (2006) 181602]

$$S_A = \frac{\text{Area of } \gamma_A}{4G_\mathrm{N}}$$

γ_A : extremal surfaces in AdS



HEE for half plane

Poincare AdS metric : $ds^2 = \ell_{\mathrm{AdS}}^2 \dfrac{dz^2 + \sum_{i=1}^d dx_i^2}{z^2}$

Extremal surface : $0 \le z < \infty$ at $x_1 = 0$

$$\Rightarrow \quad S_A = \frac{V_{d-2}}{4G_\mathrm{N}} \int_\varepsilon^\infty dz \left(\frac{\ell_{\mathrm{AdS}}}{z}\right)^{d-1} = \frac{V_{d-2}\ell_{\mathrm{AdS}}^{d-1}}{4G_\mathrm{N}(d-2)} \cdot \frac{1}{\varepsilon^{d-2}}$$

cut-off

Proposal

Extremal surfaces in dS are given by the analytic continuation of the extremal surfaces in EAdS.

Performing double Wick rotation $z \to -i\eta\,, \quad \ell_{\mathrm{AdS}} \to -i\ell_{\mathrm{dS}}$

➡ Poincare dS metric : $ds^2 = \ell_{\mathrm{dS}}^2 \dfrac{-d\eta^2 + \sum_{i=1}^d dx_i^2}{\eta^2}$

Extremal surface : $0 \le \eta < i\infty$ at $x_1 = 0$

In general, extremal surfaces in dS extend in complex-valued coordinates.

$$\boxed{S_A = (-i)^{d-1} \frac{V_{d-2}\ell_{\mathrm{dS}}^{d-1}}{4G_\mathrm{N}(d-2)} \cdot \frac{1}{\varepsilon^{d-2}}}$$

We can generalise our proposal to a general set of asymptotically dS case.

III. Comparison with a toy model

The CFT dual to Einstein gravity on dS is not known yet.

➡ analyze the free Sp(N) model as a toy model

holographic dual of Vasiliev's higher-spin theory on dS

Action

$$I = \int d^d x\, \Omega_{ab} \partial\chi^a \cdot \partial\chi^b \quad \text{with} \quad \Omega_{ab} = \begin{pmatrix} 0 & 1_{N/2\times N/2} \\ -1_{N/2\times N/2} & 0 \end{pmatrix}$$

anti-commuting scalar fields

N is even integer

Introducing $\eta^a = \chi^a + i\chi^{a+\frac{N}{2}}$ and $\bar{\eta}^a = -i\chi^a - \chi^{a+\frac{N}{2}}$ $\left(a = 1, \cdots, \frac{N}{2}\right)$

$$\Rightarrow \quad I = \int d^d x\, \partial\bar{\eta}^a \cdot \partial\eta^a$$

Entanglement entropy for half plane

$$\boxed{\begin{aligned} S_A &= -\lim_{n\to 1} \frac{\partial}{\partial(1/n)} \left(\log Z_{\mathbb{R}^2/Z_N \times \mathbb{R}^{d-2}} - \frac{1}{n}\log Z_{\mathbb{R}^d}\right) \\ &= -\frac{NV_{d-2}}{6(d-2)(4\pi)^{\frac{d-2}{2}}} \cdot \frac{1}{\varepsilon^{d-2}} \end{aligned}}$$

Since fields anti-commute, the EE is minus that of standard field theories.

Replica trick (Review)

Entanglement entropy is defined as a von Neumann entropy:

$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A = -\lim_{n\to 1} \frac{\partial}{\partial n} \mathrm{tr}_A \rho_A^n = -\lim_{n\to 1} \frac{\partial}{\partial n} \log \mathrm{tr}_A \rho_A^n$$

($\rho_A = \mathrm{tr}_B \rho$ ← total density matrix)

This calculation is difficult. Instead we calculate $\mathrm{tr}\rho_A^n$.

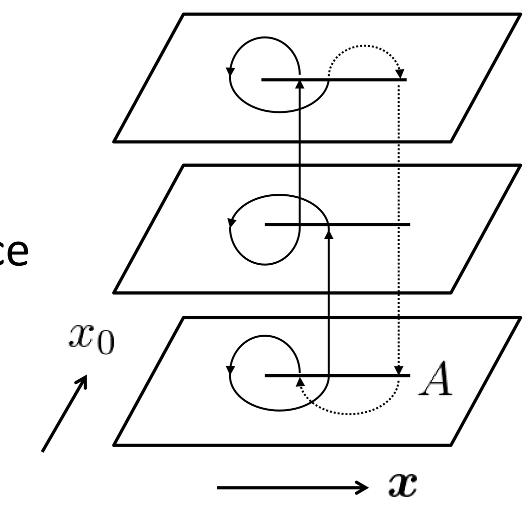
Path integral representation of ρ_A

$$[\rho_A]_{\phi_-\phi_+} = \frac{1}{Z} \int \prod_x d\phi(x)\, e^{-S[\phi]} \prod_{\boldsymbol{x}\in A} \delta[\phi(-0,\boldsymbol{x}) - \phi_-(\boldsymbol{x})]\delta[\phi(+0,\boldsymbol{x}) - \phi_+(\boldsymbol{x})]$$

$$\Rightarrow \quad \mathrm{tr}\rho_A^n = \frac{1}{Z^n} \int \prod_{x\in\Sigma_n} d\phi(x)\, e^{-S[\phi]}$$

n-sheeted Riemann surface

Partition function on Σ_n



IV. Conclusion & Discussion

HEE behaves as $S_A \propto (-i)^{d-1}$ in dS_d+1.

EE behaves as $S_A \propto -S_A^{\text{standard field theories}}$.

➡ dS_d+1/CFT_d make senses only when $d \in 4\mathbb{Z} - 1$.

The most interesting case, dS_4/CFT_3, is included.

The most simple case, dS_3/CFT_2, is excluded.

This is consistent with results in subsection 5.2 in [Maldacena, JHEP 0305 (2003) 013].

Our proposal has been checked only in the simple case, half plane.

However, our proposal holds in any entanglement surfaces.