Membrane interactions
and
a 3D analog of Riemann surfaces

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with
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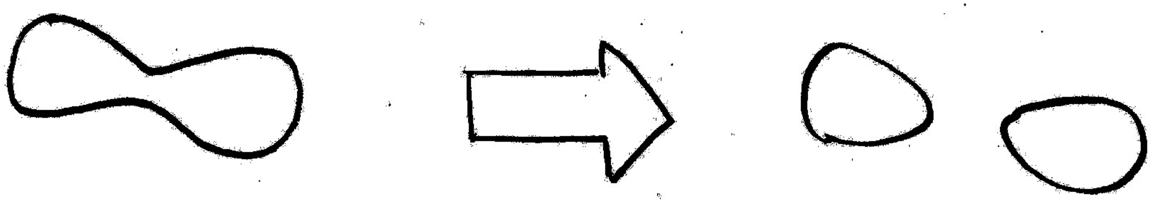
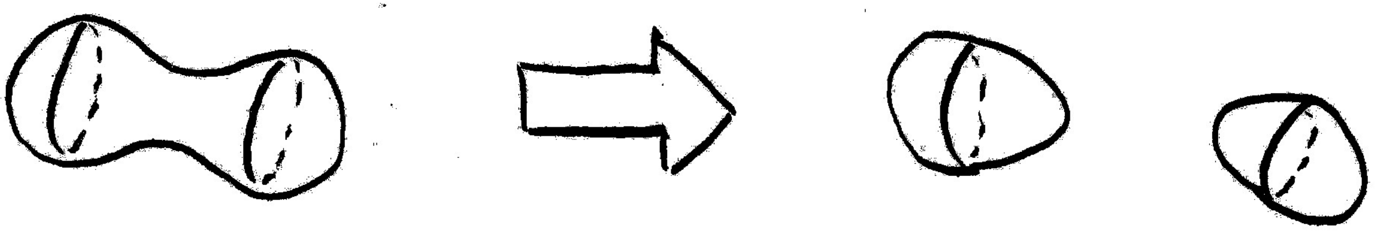
based on arxiv:1508.03367

- The formulation of M-theory is not established, though there is a good candidate: the matrix model (BFSS '96, deWit-Hoppe-Nicolai '89)

- Just like strings interact via splitting(-joining)



membranes in M-theory are expected to interact via splitting(-joining)



Does the matrix model capture splitting(-joining) processes of membranes? (cf. graviton exchange)

Quick Summary

- We studied BPS instanton eq of the pp-wave matrix model describing splitting(-joining) processes of membranes, e. g. 1 membrane ⇨ 2 membranes.

- Our main results are

☆ Under an approximation valid for the large matrix size, BPS instanton eq. can be mapped to 3D Laplace eq.

☆ Splitting of membranes (such as 1 membrane ⇨ 2 membranes) are described by 3D Laplace equation not on usual $\mathbb{R}^3$ but on a "Riemann space" : 2 copies of $\mathbb{R}^3$ stitched in a way analogous to Riemann surfaces.

Outline

1. Motivation for studying BPS instanton eq. in
   pp-wave matrix model
   ABJM duality, 3 pt function of monopole operators,
   splitting(-joining) interaction of membranes

2. When the matrix size is large, BPS instanton eq.
   can be mapped to 3D Laplace eq.

3. "Riemann space" description of splitting processes

4. Plots of splitting processes based on explicit solutions of 3D Laplace eq. on "Riemann space"

5. Summary

1.

Motivation for studying

BPS instanton eq. in

pp-wave matrix model

splitting(-joining) interaction of membranes

ABJM duality (AdS4/CFT3 duality)

3 pt function of monopole op.

Kovacs-Sato-Shimada'13

pp-wave matrix model $\sim$ approx. of bulk M-theory on $\mathrm{AdS}_4 \times S^7$ valid for state with large ang. mom. J

spherical membranes in matrix model $\Leftrightarrow$ monopole operators in boundary ABJM theory

(including near BPS fluctuations)

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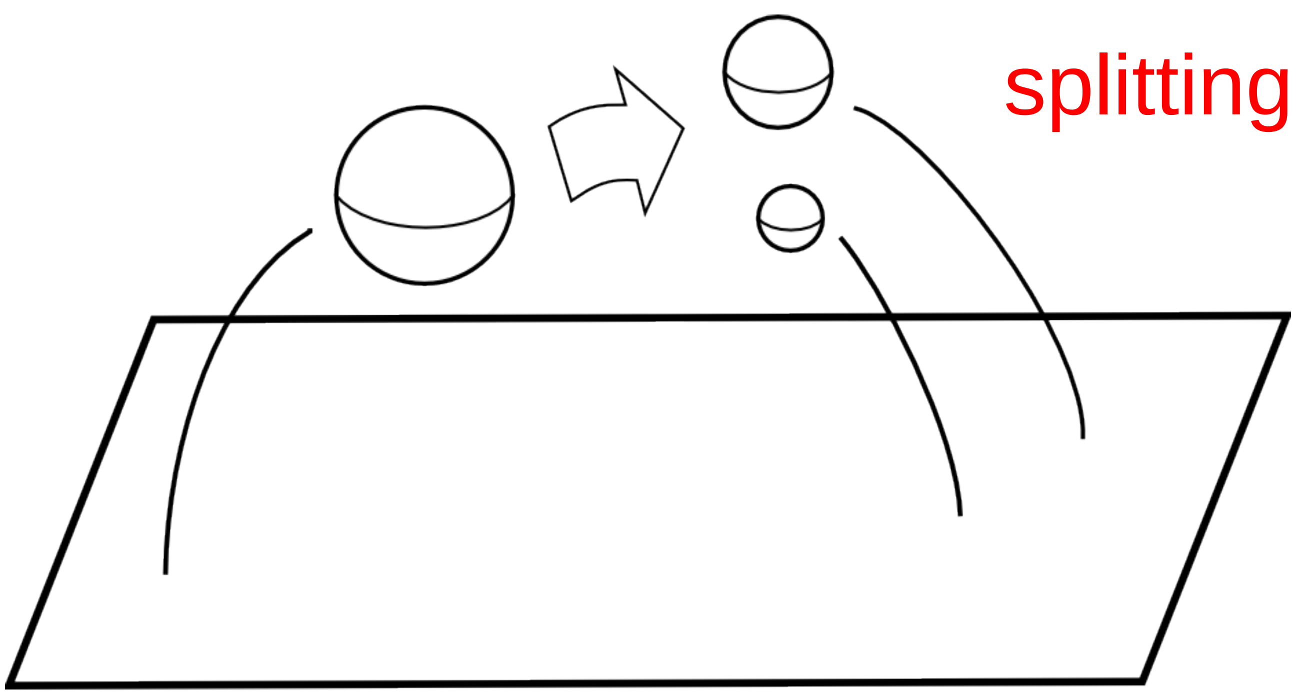
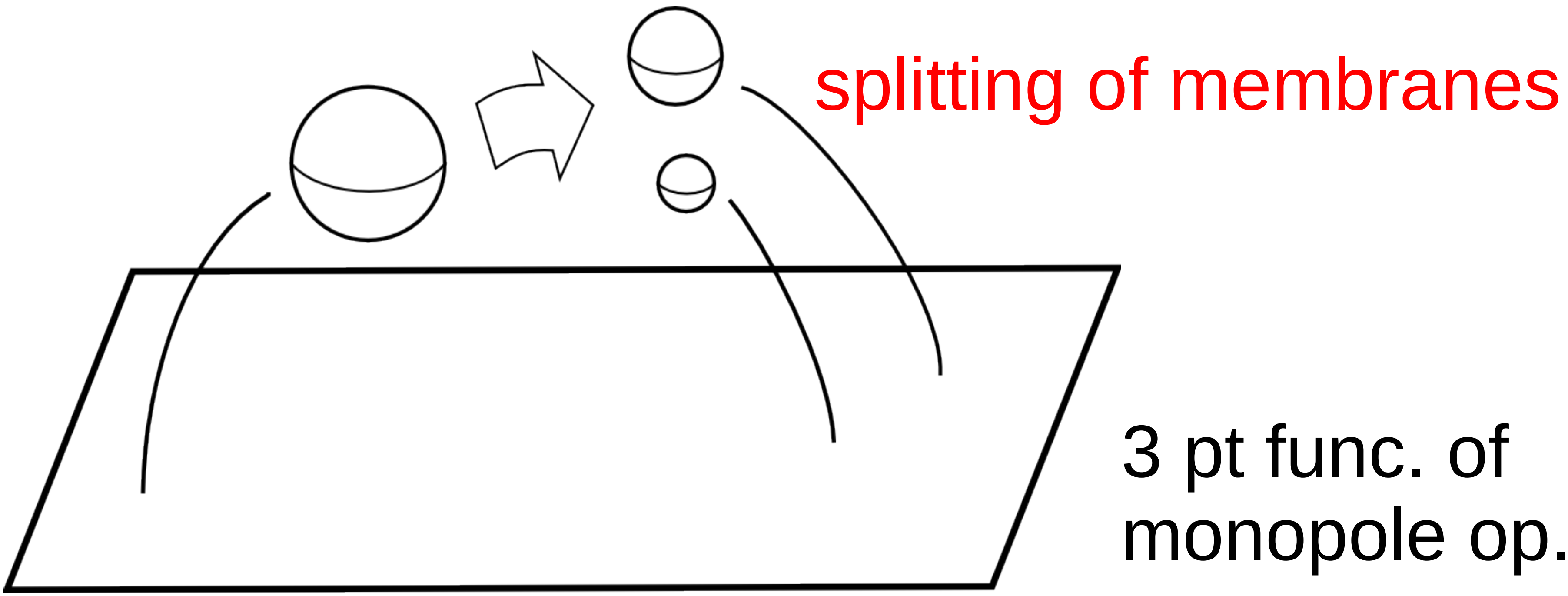


splitting of membranes

3-point func. of monopole op. (cf. AdS5/CFT4)

Long-term goal

Compare amplitude of splitting process of membranes in pp-wave matrix model and 3 pt func. of monopole op. in ABJM th

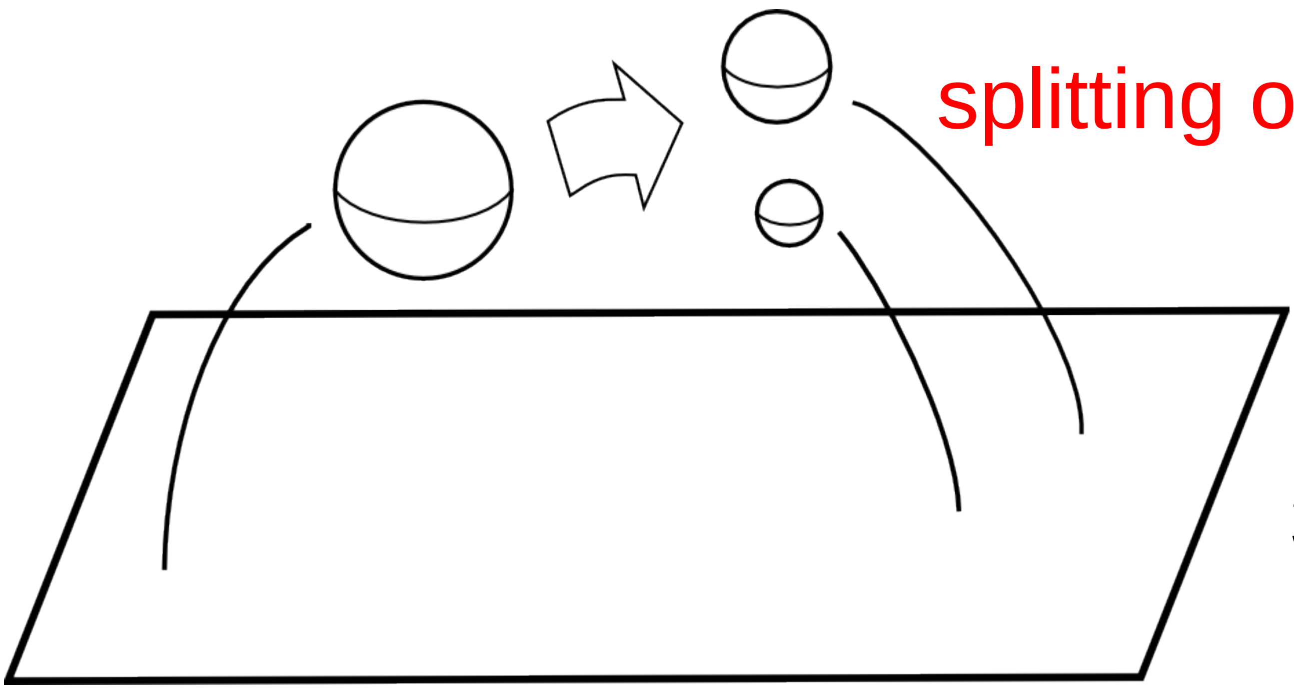


Long-term goal

Compare amplitude of splitting process of membranes in pp-wave matrix model and 3 pt func. of monopole op. in ABJM theory

Establish that the matrix model correctly captures splitting(-joining) interaction of membranes

splitting of membranes



3 pt func. of monopole op.

Splitting process of membranes in pp-wave matrix model are realised as tunnelling between various vacua (or sectors)

BPS instantons are expected to give dominant contributions (Yee-Yi '03)

Properties of the moduli space of BPS instantons (such as dimension) were known (Bachas-Hoppe-Pioline '00)

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Desirable to have a better understanding of explicit instanton solutions in order to compute the tunnelling amplitude by integration over the instanton moduli space.

2. When the matrix size is large, BPS instanton eq. can be mapped to 3D Laplace eq.

BPS instanton eq. J x J matrices $Y^i \quad (i = 1, 2, 3)$

$$\frac{d}{dt} Y^i = -\frac{i}{2} \epsilon^{ijk} [Y^j, Y^k] + Y^i$$

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$$-i[Y^i, Y^j] \longrightarrow \{y^i, y^j\} = \partial_1 y^i \partial_2 y^j - \partial_2 y^i \partial_1 y^j$$

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Continuum ver. of BPS instanton eq, describing motions of a 2D surface (membrane) in 3D space

$$\frac{d}{dt} y^i = \frac{1}{2} \epsilon^{ijk} \{y^j, y^k\} + y^i$$

(can be thought as instanton eq. of pp-wave membrane theory)

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$$\frac{d}{ds}z^i = \frac{1}{2}\epsilon^{ijk}\{z^j, z^k\}$$

By interchanging dep. and indep. variables

$$z^i(s, \sigma^1, \sigma^2) \quad \longrightarrow \quad s(z^1, z^2, z^3)$$

it can be mapped to the 3D Laplace eq. (Ward '90, Hoppe '94)

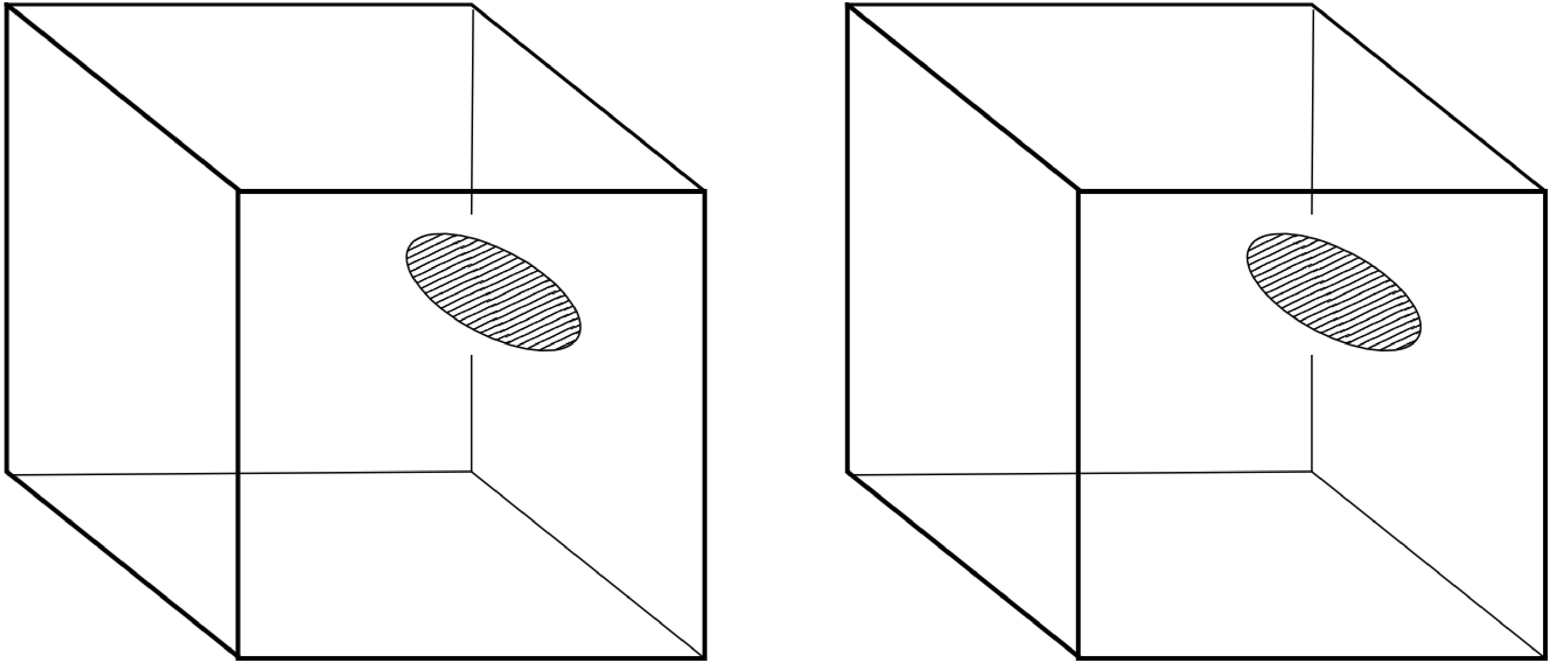
$$0 = \nabla^2 s = \left(\frac{\partial}{\partial z^i}\right)^2 s$$

“Evolution of equipotential surface defines membrane motion”

3. “Riemann space” description of splitting processes

1 membrane ⇨ 2 membrane processes are described by 3D Laplace eq on "Riemann space"

Prepare two $\mathbb{R}^3$, and stitch them on a "branch disk".



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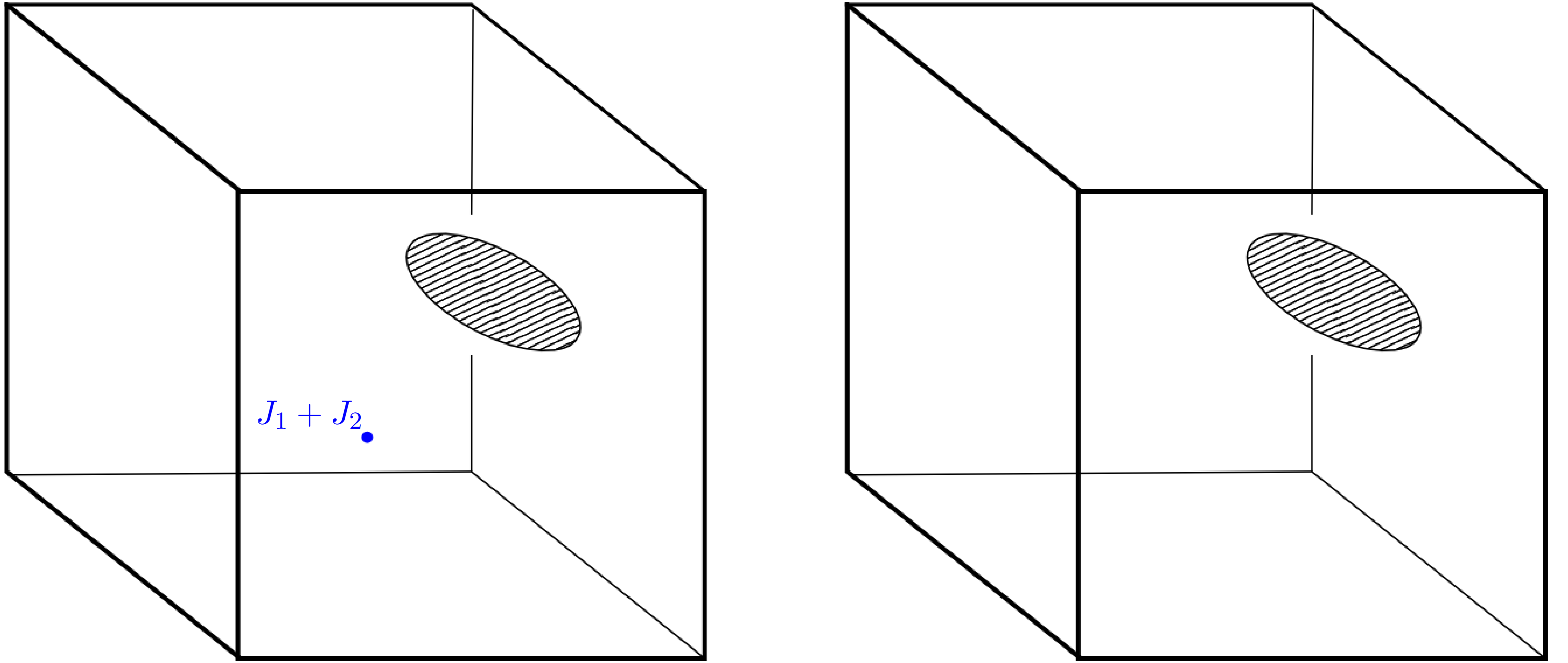

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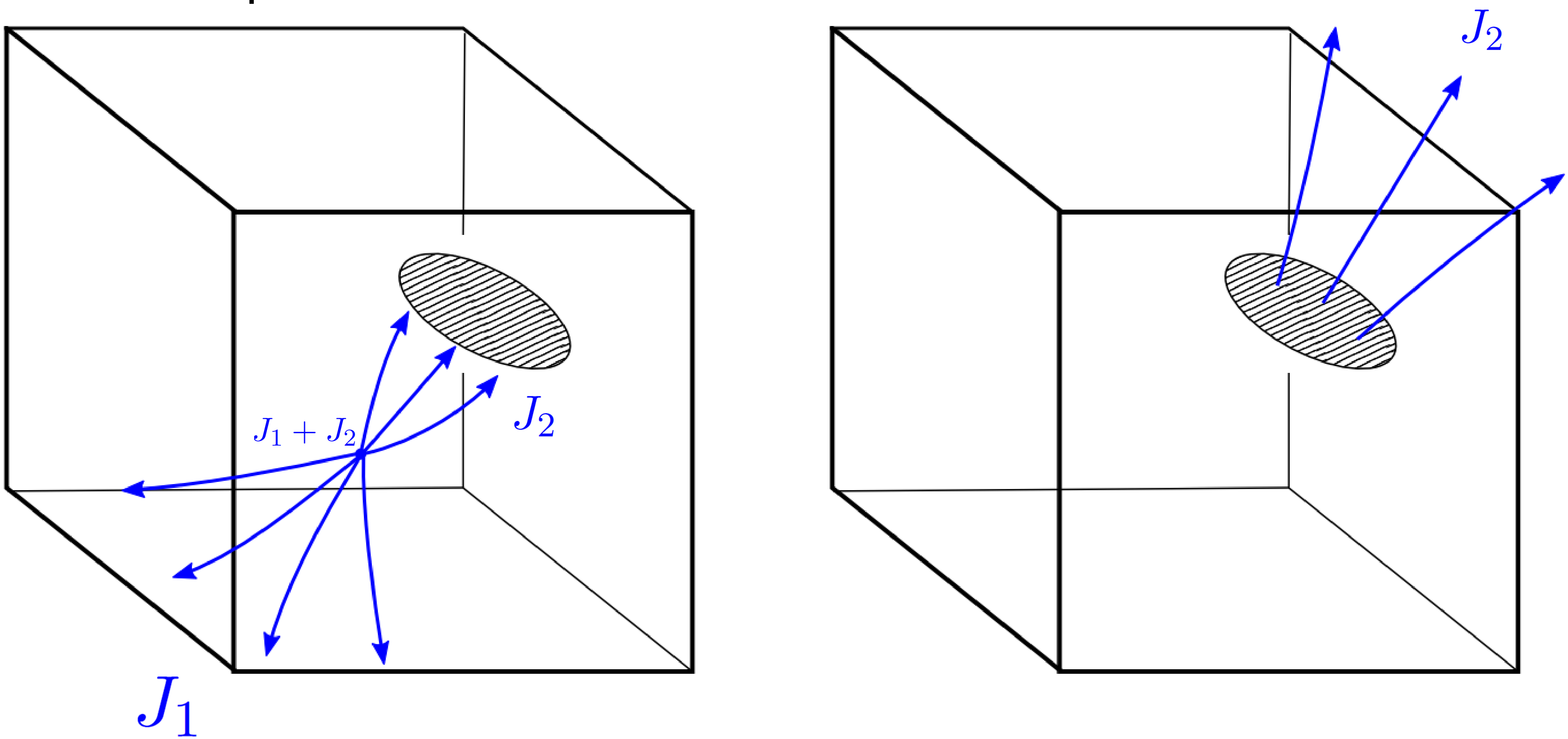
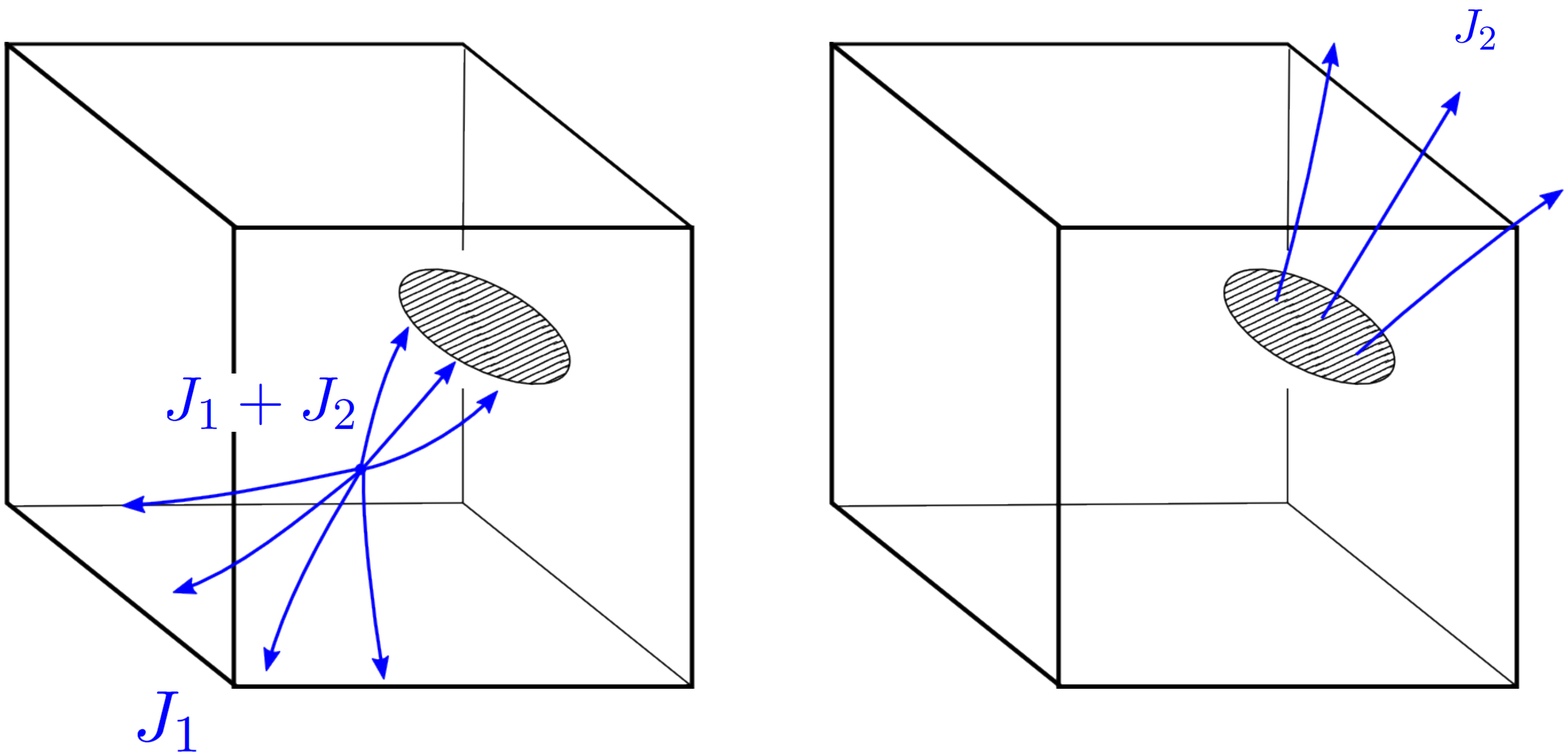
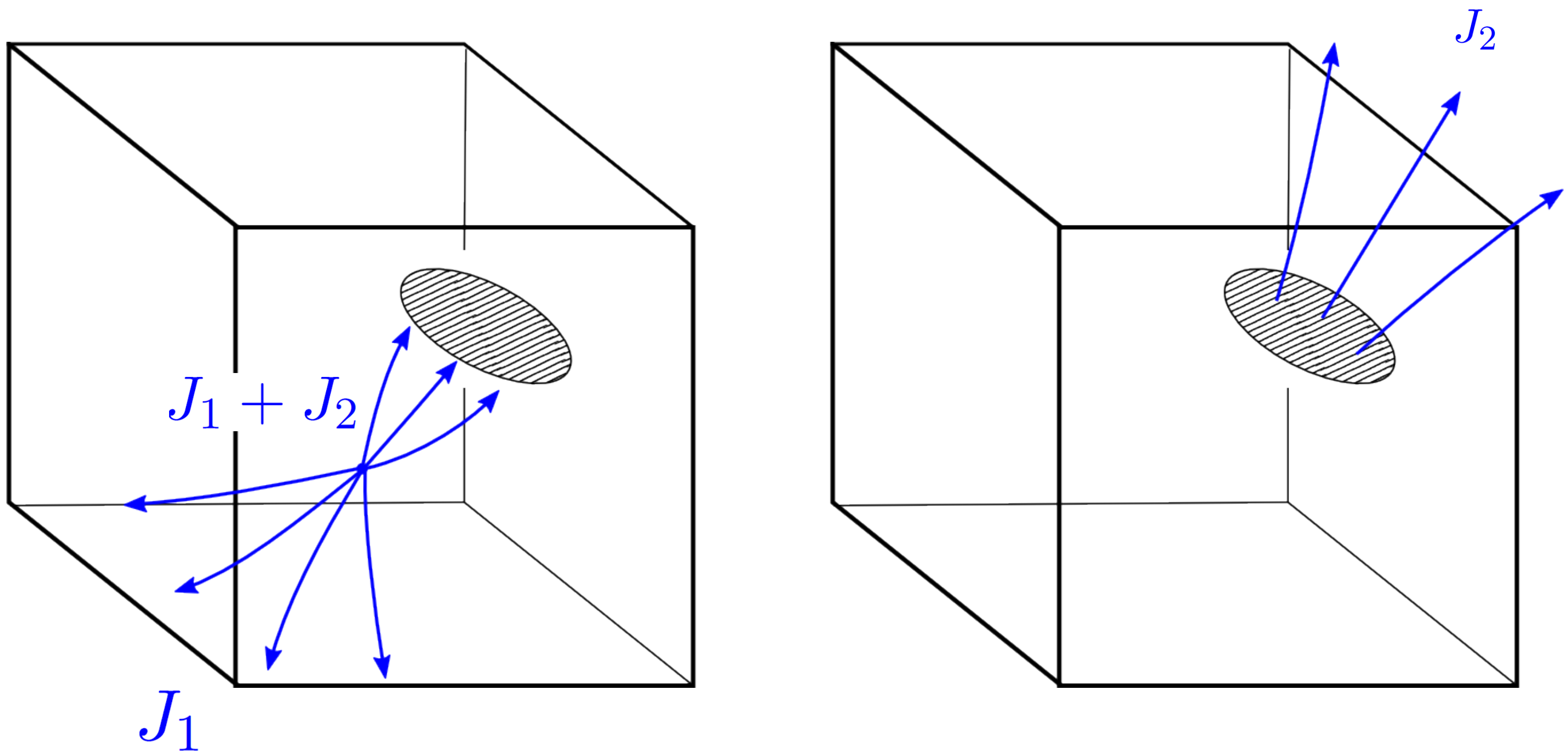


single membrane with ang. mom. $J_1 + J_2$, splitting into two membranes with J_1, J_2 .



Membrane interaction via splitting(joining) processes $\sim$ 3D Laplace equation on "Riemann space"

(cf. string interaction and Riemann surfaces)



Membrane interaction via splitting(joining) processes $\sim$ 3D Laplace equation on "Riemann space"

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In general # of copies of $\mathbb{R}^3$, #, shape, position of the branch disks, or branch loops bounding them are arbitrary. The moduli space of instantons are moduli space of branch loops.

4. Plots of splitting processes based on explicit solutions of 3D Laplace eq. on “Riemann space”

Some exact solutions of 3D Laplace eq. on "Riemann space" is obtained by Hobson in 1900 (following Sommerfeld 1896).

Hobson's solution corresponds to a branch disk bounded by a circular branch loop connecting 2 copies of $\mathbb{R}^3$. We will show the axially symmetric case.

1st example:

1 membrane ⟹ 2 membranes

2nd example:

1 membrane ⟹ 2 membranes

3rd example:

1 membrane ⟹ 2 membranes

4th example:

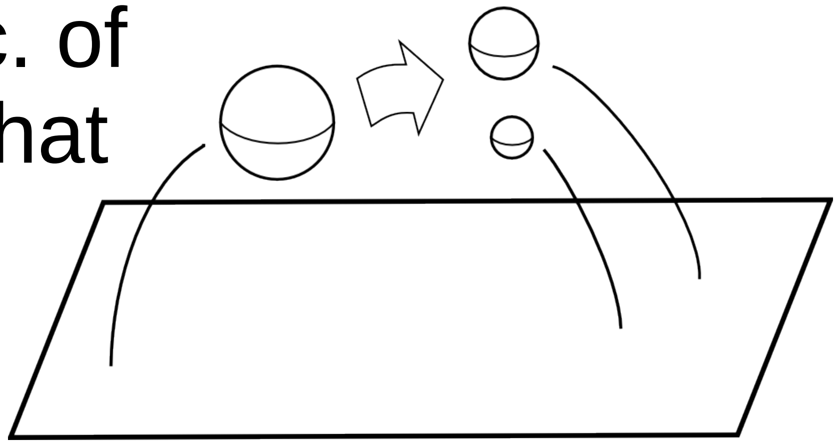
2 membrane $\Longrightarrow$ 2 membranes

(constructed by linear superposition of 2 Hobson's solutions)

5. Summary

Summary

1. Comparison to ABJM (3pt func. of monopole op.) may establish that the matrix model captures splitting(-joining) interaction of membranes via instantons.



2. BPS instanton eq. can be mapped to 3D Laplace eq. when matrix size is large
3. Splitting processes of membranes are described by 3D Laplace eq. on "Riemann space"