Instanton Effects in Orientifold ABJM Theory

Takao Suyama (Osaka City University)

Based on collaboration with S.Moriyama (Osaka CU)

Ref) arXiv:1511.01660

Grand potential of CSM

The grand potential $J_k(\mu)$ **turns out to be useful for discussing instanton effects.**

$$\sum_{n=-\infty}^{\infty} e^{J_k(\mu+2\pi i n)} = \sum_{N=0}^{\infty} |Z_k(N)| e^{\mu N}.$$

For ABJM theory,

$$J_k(\mu) = \underline{\frac{2}{3\pi^2 k}\mu_{\text{eff}}^3 + \left(\frac{k}{24}+\frac{1}{3k}\right)\mu_{\text{eff}}} + A(k) + J_k^{(\text{np})}(\mu_{\text{eff}}).$$

$\Rightarrow$ $1/N$ **corrections** **[Fuji,Hirano,Moriyama]**

$J_k^{(\text{np})}(\mu_{\text{eff}})$ **consists of instanton contributions. There is a relation to topological string theory on local** $\mathbb{P}^1 \times \mathbb{P}^1$**:** **[Hatsuda et al.]**

$$J_k^{(\text{np})}(\mu_{\text{eff}}) = \sum_{j_L, j_R} \sum_{\mathbf{d}=(d^1,d^2)} N^{\mathbf{d}}_{j_L,j_R}$$

$$\times \sum_{n=1}^{\infty} \left[\frac{s_R \sin 2\pi g_s n s_L}{n(2\sin \pi g_s n)^2 \sin 2\pi g_s n} e^{-n\mathbf{d}\cdot\mathbf{T}_{\text{eff}}} + \frac{\partial}{\partial g_s}\left(g_s \frac{-\sin\frac{\pi n}{g_s} s_L \sin\frac{\pi n}{g_s} s_R}{4\pi n^2 (\sin\frac{\pi n}{g_s})^3} e^{-\frac{n\mathbf{d}\cdot\mathbf{T}_{\text{eff}}}{g_s}} \right) \right].$$

Question: Is this a general story?

OSp theory

[Hosomichi et al.]

Consider OSp theory, a generalization of ABJM with

- $\mathcal{N} = 5$ **supersymmetry,**
- **gauge group** $\mathrm{O}(2N)_k \times \mathrm{USp}(2N)_{k/2}$**,** $\quad (k \in 2\mathbb{Z})$
- **2 bi-fundamental hypermultplets.**

OSp theory is an orientifold of ABJM theory, describing M2-branes on a background dual to an orientifold in Type IIB.

Despite the large SUSY, its non-perturbative effects have not been investigated so far.

$\Rightarrow$ **Interesting to determine** $J_k(\mu)$ **including instanton effects.**

$\Rightarrow$ **How general is the relation to topological string?**

We find [MS]

$$J_k^{\mathrm{OSp}[2]}(\mu) = J_k^{\mathrm{ABJM}}(\mu/2) - \log 2,$$

where $J_k^{[2]}(\mu)$ **is the duplication of** $J_k(\mu)$**.**

How to compute $J_k(\mu)$? (See [Hatsuda,Moriyama,Okuyama])

1. Fermi gas formalism [Marino,Putrov]

The grand partition function $\Xi_k(\mu)$ can be written as [MS]

$$\Xi_k(z) = \det\left(1 - z e^{-\frac{i}{2\hbar}\widehat{q}^2} \frac{\widehat{\Pi}_+}{2\cosh\frac{\widehat{p}}{2}} e^{\frac{i}{2\hbar}\widehat{q}^2} \frac{\widehat{\Pi}_+}{2\cosh\frac{\widehat{p}}{2}}\right) \times \langle\widetilde{0}| \cosh\frac{\widehat{q}}{2k}\left(1 - z e^{-\frac{i}{2\hbar}\widehat{q}^2} \frac{\widehat{\Pi}_+}{2\cosh\frac{\widehat{p}}{2}} e^{\frac{i}{2\hbar}\widehat{q}^2} \frac{\widehat{\Pi}_+}{2\cosh\frac{\widehat{p}}{2}}\right)^{-1} |0\rangle,$$

where $\widehat{\Pi}_+$ is a projector acting on eigenstates of $\widehat{q}$.

The definition of $\Xi_k(\mu)$

$$\Xi_k(z) = \sum_{N=0}^{\infty} z^N Z_k(N)$$

allows us to evaluate $Z_k(N)$ exactly for small k and N.

2. Ansatz for $J_k(\mu) = J_k^{(\mathrm{pert})}(\mu) + J_k^{(\mathrm{np})}(\mu)$:

$$J_k^{(\mathrm{pert})}(\mu) = \frac{C_k}{3}\mu^3 + B_k\mu + A_k, \qquad J_k^{(\mathrm{np})}(\mu) = \sum_a f_a(\mu)e^{-a\mu},$$

where $f_a(\mu)$ are polynomials of μ.

3. Numerical fitting

Choose parameters in $J_k(\mu)$ s.t.

$$|Z_k(N)| = \int_{-\infty i}^{+\infty i} \frac{d\mu}{2\pi i} e^{J_k(\mu) - \mu N}$$

is realized with (incredibly) high accuracy.

Our results

$Z_k(N)$ for for small k and N:

$$|Z_1(1)| = \frac{1}{4\sqrt{2}}, \quad |Z_1(2)| = \frac{-2+\pi}{64\pi}, \quad |Z_1(3)| = \frac{-2\sqrt{2}+(8-5\sqrt{2})\pi}{512\pi},$$

$$|Z_1(4)| = \frac{36-4\pi+(-25+16\sqrt{2})\pi^2}{8192\pi^2},$$

$$|Z_2(1)| = \frac{1}{16}, \quad |Z_2(2)| = \frac{-8+\pi^2}{512\pi^2}, \quad |Z_2(3)| = \frac{-8-32\pi+11\pi^2}{8192\pi^2},$$

$$|Z_2(4)| = \frac{192-560\pi^2-384\pi^3+177\pi^4}{1572864\pi^4},$$

$$|Z_3(1)| = \frac{3\sqrt{2}-2\sqrt{3}}{24}, \quad |Z_3(2)| = \frac{6+(3-2\sqrt{6})\pi}{192\pi},$$

$$|Z_3(3)| = \frac{54\sqrt{2}+60\sqrt{3}+(40-135\sqrt{2}+54\sqrt{3})\pi}{13824\Pi},$$

$$|Z_3(4)| = \frac{1188+36(9+10\sqrt{6})\pi+(-4721+240\sqrt{2}+864\sqrt{3}+972\sqrt{6})\pi^2}{663552\pi^2},$$

$$|Z_4(1)| = \frac{1}{16\pi}, \quad |Z_4(2)| = \frac{10-\pi^2}{1024\pi^2}, \quad |Z_4(3)| = \frac{78-121\pi^2+36\pi^3}{147456\pi^3},$$

$$|Z_4(4)| = \frac{876-4148\pi^2-2016\pi^3+1053\pi^4}{18874368\pi^4},$$

$$|Z_5(1)| = \frac{-5+2\sqrt{5+\sqrt{5}}}{20\sqrt{2}}, \quad |Z_5(2)| = \frac{-10+(5+4\sqrt{5}-4\sqrt{5+\sqrt{5}})\pi}{320\pi},$$

$$|Z_5(3)| = \frac{250+60\sqrt{5+\sqrt{5}}+(625+52\sqrt{2}-100\sqrt{5}-64\sqrt{10}-150\sqrt{5+\sqrt{5}})\pi}{32000\sqrt{2}\pi},$$

$$|Z_6(1)| = \frac{-3+2\sqrt{3}}{48}, \quad |Z_6(2)| = \frac{-216+(209-108\sqrt{3})\pi^2}{41472\pi^2},$$

$$|Z_6(3)| = \frac{(1944-4752\sqrt{3})+1440\pi+(-9171+5398\sqrt{3})\pi^2}{5971968\pi^2},$$

$$|Z_6(4)| = \frac{16860-136700\pi^2+190800\pi^3+207413\pi^4-81000\pi^5}{7549747200\pi^5},$$

$$|Z_8(1)| = \frac{-2+\pi}{64\pi}, \quad |Z_8(2)| = \frac{36-4\pi+(-25+16\sqrt{2})\pi^2}{8192\pi^2},$$

$$|Z_8(3)| = \frac{-600+324\pi+(2242+2016\sqrt{2})\pi^2+(-1431-144\sqrt{2})\pi^3}{4718592\pi^3},$$

$$|Z_{12}(1)| = \frac{9-\sqrt{3}\pi}{432\pi}, \quad |Z_{12}(2)| = \frac{702-12\sqrt{3}\pi+(-1561+864\sqrt{3})\pi^2}{248832\pi^2},$$

$$|Z_{12}(3)| = \frac{17982-2106\sqrt{3}\pi-27(4585+864\sqrt{3})\pi^2+(45684+3679\sqrt{3})\pi^3}{322486272\pi^3}.$$

Phase of $Z_k(N)$ is

$$Z_k(N) = (-i)^N |Z_k(N)|.$$

$J_k(\mu)$ for small k:

$$
\begin{aligned}
J_1^{(\mathrm{np})} &= \frac{1}{\sqrt{2}}e^{-\mu} - \frac{4}{3\sqrt{2}}e^{-3\mu} + \left[\frac{2\mu^2+\mu/2+1/8}{\pi^2}\right]e^{-4\mu} - \frac{16}{5\sqrt{2}}e^{-5\mu} + \frac{64}{7\sqrt{2}}e^{-7\mu} \\
&\quad + \left[-\frac{13\mu^2+\mu/8+9/64}{\pi^2}+2\right]e^{-8\mu} + \frac{256}{9\sqrt{2}}e^{-9\mu} - \frac{1024}{11\sqrt{2}}e^{-11\mu} \\
&\quad + \left[\frac{368\mu^2-76\mu/3+77/36}{3\pi^2}-32\right]e^{-12\mu} - \frac{4096}{13\sqrt{2}}e^{-13\mu} + \frac{16384}{15\sqrt{2}}e^{-15\mu} + \mathcal{O}(e^{-16\mu}), \\
J_2^{(\mathrm{np})} &= e^{-\mu} + \left[-\frac{2\mu^2+\mu+1/2}{\pi^2}\right]e^{-2\mu} + \frac{16}{3}e^{-3\mu} + \left[-\frac{13\mu^2+\mu/4+9/16}{\pi^2}+2\right]e^{-4\mu} \\
&\quad + \frac{256}{5}e^{-5\mu} + \left[-\frac{368\mu^2-152\mu/3+77/9}{3\pi^2}+32\right]e^{-6\mu} + \frac{4096}{7}e^{-7\mu} + \mathcal{O}(e^{-8\mu}), \\
J_3^{(\mathrm{np})} &= \frac{1}{\sqrt{2}}e^{-\mu} - \frac{1}{3}e^{-\frac{4}{3}\mu} + \frac{1}{2}e^{-\frac{8}{3}\mu} - \frac{4}{3\sqrt{2}}e^{-3\mu} + \left[\frac{2\mu^2+\mu/2+1/8}{3\pi^2}-\frac{8}{9}\right]e^{-4\mu} - \frac{16}{5\sqrt{2}}e^{-5\mu} \\
&\quad + \frac{25}{36}e^{-\frac{16}{3}\mu} - \frac{12}{5}e^{-\frac{20}{3}\mu} + \frac{64}{7\sqrt{2}}e^{-7\mu} + \left[-\frac{13\mu^2+\mu/8+9/64}{3\pi^2}+\frac{88}{9}\right]e^{-8\mu} + \frac{256}{9\sqrt{2}}e^{-9\mu} \\
&\quad - \frac{947}{189}e^{-\frac{28}{3}\mu} + \mathcal{O}(e^{-\frac{32}{3}\mu}),
\end{aligned}
$$

$$J_4^{(\mathrm{np})} = \left[\frac{2\mu^2+\mu+1/2}{2\pi^2}\right]e^{-2\mu} + \left[-\frac{13\mu^2+\mu/4+9/16}{2\pi^2}+2\right]e^{-4\mu}$$
$$+\left[\frac{184\mu^2-76\mu/3+77/18}{3\pi^2}-32\right]e^{-6\mu}+\mathcal{O}(e^{-8\mu}),$$

$$J_5^{(\mathrm{np})} = \frac{-5+3\sqrt{5}}{10}e^{-\frac{4}{5}\mu} - \frac{1}{\sqrt{2}}e^{-\mu} + \frac{5+7\sqrt{5}}{20}e^{-\frac{8}{5}\mu} + \frac{5-3\sqrt{5}}{15}e^{-\frac{12}{5}\mu} + \frac{4}{3\sqrt{2}}e^{-3\mu}$$
$$+\frac{5-31\sqrt{5}}{40}e^{-\frac{16}{5}\mu} + \left[\frac{2\mu^2+\mu/2+1/8}{5\pi^2}-\frac{1+5\sqrt{5}}{10}\right]e^{-4\mu} + \frac{485-91\sqrt{5}}{150}e^{-\frac{24}{5}\mu}$$
$$+\frac{16}{5\sqrt{2}}e^{-5\mu} + \frac{170+263\sqrt{5}}{175}e^{-\frac{28}{5}\mu} + \mathcal{O}(e^{-\frac{32}{5}\mu}),$$

$$J_6^{(\mathrm{np})} = \frac{1}{3}e^{-\frac{2}{3}\mu} - e^{-\mu} + \frac{1}{2}e^{-\frac{4}{3}\mu} + \left[-\frac{2\mu^2+\mu+1/2}{3\pi^2}+\frac{8}{9}\right]e^{-2\mu} + \frac{25}{36}e^{-\frac{8}{3}\mu} - \frac{16}{3}e^{-3\mu}$$
$$+\frac{12}{5}e^{-\frac{10}{3}\mu} + \left[-\frac{13\mu^2+\mu/4+9/16}{3\pi^2}+\frac{88}{9}\right]e^{-4\mu} + \frac{947}{189}e^{-\frac{14}{3}\mu} - \frac{256}{5}e^{-5\mu} + \mathcal{O}(e^{-\frac{16}{3}\mu}),$$

$$J_8^{(\mathrm{np})} = \frac{\sqrt{2}}{2}e^{-\frac{1}{2}\mu} - \frac{2\sqrt{2}}{3}e^{-\frac{3}{2}\mu} + \left[\frac{2\mu^2+\mu+1/2}{4\pi^2}\right]e^{-2\mu} - \frac{8\sqrt{2}}{5}e^{-\frac{5}{2}\mu} + \frac{32\sqrt{2}}{7}e^{-\frac{7}{2}\mu} + \mathcal{O}(e^{-4\mu}),$$

$$J_{12}^{(\mathrm{np})} = \sqrt{3}e^{-\frac{1}{3}\mu} - \frac{7}{6}e^{-\frac{2}{3}\mu} + \frac{9}{4}e^{-\frac{4}{3}\mu} - \frac{16\sqrt{3}}{5}e^{-\frac{5}{3}\mu} + \left[\frac{2\mu^2+\mu+1/2}{6\pi^2}+\frac{74}{9}\right]e^{-2\mu}$$
$$-\frac{185\sqrt{3}}{21}e^{-\frac{7}{3}\mu} + \mathcal{O}(e^{-\frac{8}{3}\mu}).$$

The general structure is

$$J_k(\mu) = \frac{1}{3\pi^2 k}\mu^3 + \left(\frac{5}{12k} + \frac{k}{48}\right)\mu + A_k + \sum_{m=1}^{\infty} d_m(k) e^{-\frac{4m}{k}\mu}$$
$$+ \sum_{l=1}^{\infty}\Big(a_{2l}(k)\mu^2 + b_{2l}(k)\mu + c_{2l}\Big)e^{-2l\mu} + \underbrace{\sum_{l=1}^{\infty} c_{2l+1} e^{-(2l+1)\mu}}_{\text{absent for ABJM}}.$$

absent for ABJM

"Odd instanton" terms can be resumed as

$$J^{(\text{odd MB})}_{k\equiv 1,3 \bmod 8}(\mu) = -J^{(\text{odd MB})}_{k\equiv 5,7 \bmod 8}(\mu) = \frac{1}{8}\log\frac{1+2\sqrt{2}e^{-\mu}+4e^{-2\mu}}{1-2\sqrt{2}e^{-\mu}+4e^{-2\mu}},$$
$$J^{(\text{odd MB})}_{k\equiv 2 \bmod 8}(\mu) = -J^{(\text{odd MB})}_{k\equiv 6 \bmod 8}(\mu) = \frac{1}{8}\log\frac{1+4e^{-\mu}}{1-4e^{-\mu}},$$
$$J^{(\text{odd MB})}_{k\equiv 0 \bmod 4}(\mu) = 0.$$

Relation to ABJM

Some observations:

- The perturbative part satisfies

$$J_k^{\mathrm{OSp(pert)}}\left(\mu+\frac{\pi i}{2}\right)+J_k^{\mathrm{OSp(pert)}}\left(\mu-\frac{\pi i}{2}\right)=J_k^{\mathrm{ABJM(pert)}}(\mu)-\log 2.$$

- By this μ-shift, odd instanton terms cancel:

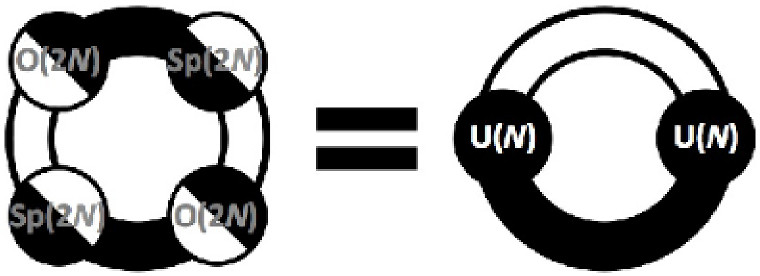
$$e^{-(2l+1)(\mu+\frac{\pi i}{2})}+e^{-(2l+1)(\mu-\frac{\pi i}{2})} = 0.$$

$\Rightarrow$ We find

$$\boxed{J_k^{\mathrm{OSp}[2]}(\mu)=J_k^{\mathrm{ABJM}}(\mu/2)-\log 2,}$$

including non-perturbative terms.

$J_k^{\mathrm{OSp}[2]}(\mu)$ is the grand potential of the duplicate quiver:



Explicitly,

$$J_k^{[2]}(\mu) = J_k^{[1]}\Big(\frac{\mu-\pi i}{2}\Big) + J_k^{[1]}\Big(\frac{\mu+\pi i}{2}\Big) + \log\Big[1+\sum_{n\neq 0} e^{J_k^{[1]}(\frac{\mu-\pi i}{2}+2\pi i n)+J_k^{[1]}(\frac{\mu+\pi i}{2}-2\pi i n)-J_k^{[1]}(\frac{\mu-\pi i}{2})-J_k^{[1]}(\frac{\mu+\pi i}{2})}\Big].$$

This is originally for orbifold theories. **[Honda,Moriyama]**

OSp theory is related to ABJM theory
(and topological string theory)
through the duplication of the quiver diagram.

Summary

- We have determined the grand potential of OSp theory including non-perturbative terms.
- The results are similar to ABJM theory, but interesting differences (e.g. odd instantons, phase factors).
- We have found a simple relation to ABJM theory in terms of the grand potential.

Future directions

- General constraints on the grand potential of CSM.
  (modular invariance?)
- WKB analysis of OSp theory.
- Deeper understanding of the relation to ABJM theory.
- Several deformations.
- Wilson loops.
- etc.