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- Developments in String Theory and Quantum Field Theory

Negative anomalous dimensions in $\mathcal{N}$=4 SYM

Yusuke Kimura (OIQP)

1503.0621 [hep-th] with Ryo Suzuki

1. Introduction – brief review of N=4 SYM

Anomalous dimensions of $\mathcal{N}$=4 SYM (Conformal Field Theory)

Two-point functions

$$\lambda = g_{YM}^2 N$$

$$\langle O_\alpha(x) O_\beta(y) \rangle = \frac{c_\alpha \delta_{\alpha\beta}}{(x-y)^{2\Delta_\alpha}}$$

$$\Delta = \Delta_{(0)} + \lambda \Delta_{(1)} + \lambda^2 \Delta_{(2)} + \cdots$$

Scaling dimension

Dilatation generator ⊂ Conformal symmetry, $\mathrm{so}(4,2)$

$$\hat{D} O_\alpha(0) = \Delta_\alpha O_\alpha(0)$$

$$\hat{D} = \hat{D}_{(0)} + \lambda \hat{D}_{(1)} + \lambda^2 \hat{D}_{(2)} + \cdots$$

Via the radial quantisation, $R^4 \to S^3 \times R$

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

AdS/CFT correspondence

4D $N = 4$ $SU(N)$ SYM (CFT) $\Leftrightarrow$ string theory on $AdS_5 \times S^5$

$$\Delta(\lambda, N) = E(g_s, R_{AdS}/l_s)$$

$$\lambda = \left(\frac{R_{AdS}}{l_s}\right)^4 \qquad \frac{\lambda}{4\pi N} = g_s$$

$$(\lambda = g_{YM}^2 N)$$

Several ways of looking at the equation

- ✓ Check the duality
- ✓ Use to understand something new
  - String theory at small curvature $R_{AdS} \ll l_s$ is difficult
  - Gauge theory description is easier at $\lambda \ll 1$

Operator mixing problem

A_μ, $\Phi_a\ (a = 1, \cdots, 6)$, ψ_α

For the SO(6) sector, the 1-loop dilatation operator is given by

$$D_{1-loop} = \frac{1}{N} H$$

$$H = -\frac{1}{2} :tr[\Phi_m, \Phi_n][\partial_m, \partial_n]: - \frac{1}{4} :tr[\Phi_m, \partial_n][\Phi_m, \partial_n]:$$

$$(\partial_m)_{ij}(\Phi_n)_{kl} = \delta_{mn}\delta_{il}\delta_{jk}$$

Φ_m *are just matrices*

1) $\widehat{D}\chi_\alpha = M_{\alpha\beta}\chi_\beta$ for a general basis

2) Diagonalise $M_{\alpha\beta}$ to obtain $\widehat{D}O_\alpha = \Delta_\alpha O_\alpha$

Eigenvalue problem of the matrix model

Will study the spectrum of anomalous dimensions,
focusing on the sign of them

$$\Delta = \Delta_{(0)} + \lambda \Delta_{(1)}(1/N) + O(\lambda^2) + \cdots$$

1. In the planar limit, anomalous dimensions are all positive
2. But it is not the case when you include non-planar corrections

To understand physics of negative anomalous dimensions is to understand nonplanar corrections

Outline

- ✓ Brief review of N=4 SYM (CFT)
  - ✓ Anomalous dimensions
  - ✓ Dilatation operator
- ✓ Operator mixing problem
  - ✓ Planar vs Non-planar
- ✓ Negative anomalous dimensions [1503.0621, YK-R.Suzuki]

2. Operator mixing problem – planar vs non-planar

Operator mixing

$$tr[\Phi_m,\Phi_n][\partial_m,\partial_n] = 2tr(\Phi_m\Phi_n\partial_m\partial_n) - 2tr(\Phi_m\Phi_n\partial_n\partial_m)$$

$$tr[\Phi_m,\partial_n][\Phi_m,\partial_n] = 2tr(\Phi_m\partial_n\Phi_m\partial_n) - 2tr(\Phi_m\Phi_m\partial_n\partial_n)$$

$$tr(\Phi_m\Phi_n\partial_m\partial_n)tr(A\Phi_a)tr(B\Phi_b) = tr(BA\Phi_b\Phi_a + AB\Phi_a\Phi_b)$$
$$tr(\Phi_m\Phi_n\partial_m\partial_n)tr(A\Phi_a B\Phi_b) = tr(A)tr(B\Phi_b\Phi_a)+tr(B)tr(A\Phi_a\Phi_b)$$

Dilatation operator changes the number of traces by one
- Joining and splitting

When the two derivatives act on nearest neighbor matrices, we get a term whose trace structure is not changed

$$tr(\Phi_m\Phi_n\partial_m\partial_n)tr(A\Phi_a\Phi_b) = tr(A)tr(\Phi_b\Phi_a)+Ntr(A\Phi_a\Phi_b)$$

The two derivatives acting on non-nearest neighbor matrices, the trace structure changes

$$Htr(A\Phi_a\Phi_b) = N\left(tr(A\Phi_a\Phi_b) - tr(A\Phi_b\Phi_a) + \frac{1}{2}\delta_{ab}tr(A\Phi_m\Phi_m)\right)$$
$$+double\ traces$$

Nearest neighbour transpositions P and contractions C on flavor indices

$$P \cdot \Phi_a\Phi_b = \Phi_b\Phi_a \qquad C \cdot \Phi_a\Phi_b = \delta_{ab}\Phi_m\Phi_m$$

$$H = NH_P + H_{NP} \qquad H_P = 1 - P + \frac{1}{2}C$$

H_P : not changing the trace structure, but changing the flavour structure

H_{NP} : changing the trace structure and the flavour structure

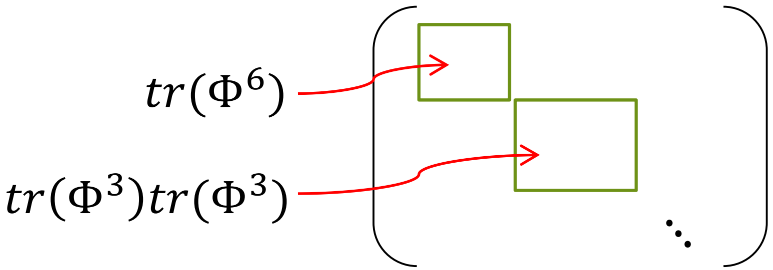
Planar limit

✓ Dilatation operator is $H_P = 1 - P + \frac{C}{2}$

- Mapped to the Hamiltonian of an integrable spin chain [02 Minahan-Zarembo]

✓ the mixing is only among operators with the same trace structure (i.e. trace structure has a meaning)

Block-diagonal mixing matrix ~



Non-planar – we do not have the above properties

(We can find a nice mixing pattern in non-planar situations in terms of Young diagrams)

Remark

$$H = NH_P + H_{NP}$$

Acting on a small operator $E = O(N) + O(1)$

Acting on a very large operator $E = O(NL) + O(L^2)$

The planar limit is $N \gg L$ $g_{eff} = L/N$

D-branes are considered to be described by large operators $L \sim O(N), N \gg 1$
- One can not use the planar limit

3. Negative anomalous dimensions

[1503.0621, YK-R.Suzuki]

Spectral problem in the so(6) singlet sector

- ✓ SO(6) singlet operators $tr(\Phi_a\Phi_b)tr(\Phi_a\Phi_b), tr(\Phi_a\Phi_b\Phi_a\Phi_a)$

  Singlets are mapped to singlets under dilatation

- Consider planar zero modes at one-loop - $H_P\psi_0 = 0$
  - Giving an interesting class of operators
  - Thanks to integrability, there is a large degeneracy in the planar spectrum.
- Turning on $1/N$ corrections, the planar zero modes will get anomalous dimensions of the form: $\gamma = 0 + \frac{1}{N}\gamma_1 + \frac{1}{N^2}\gamma_2 + \cdots$
  - ✓ Sign of γ_1, γ_2
  - ✓ Operator mixing among the planar zero modes

$L = 4$

There are 4 singlet operators

$t_1 = tr(\Phi_a\Phi_b)tr(\Phi_a\Phi_b), t_2 = tr(\Phi_a\Phi_a)tr(\Phi_b\Phi_b)$
$t_3 = tr(\Phi_a\Phi_b\Phi_a\Phi_a), t_4 = tr(\Phi_a\Phi_a\Phi_b\Phi_b)$

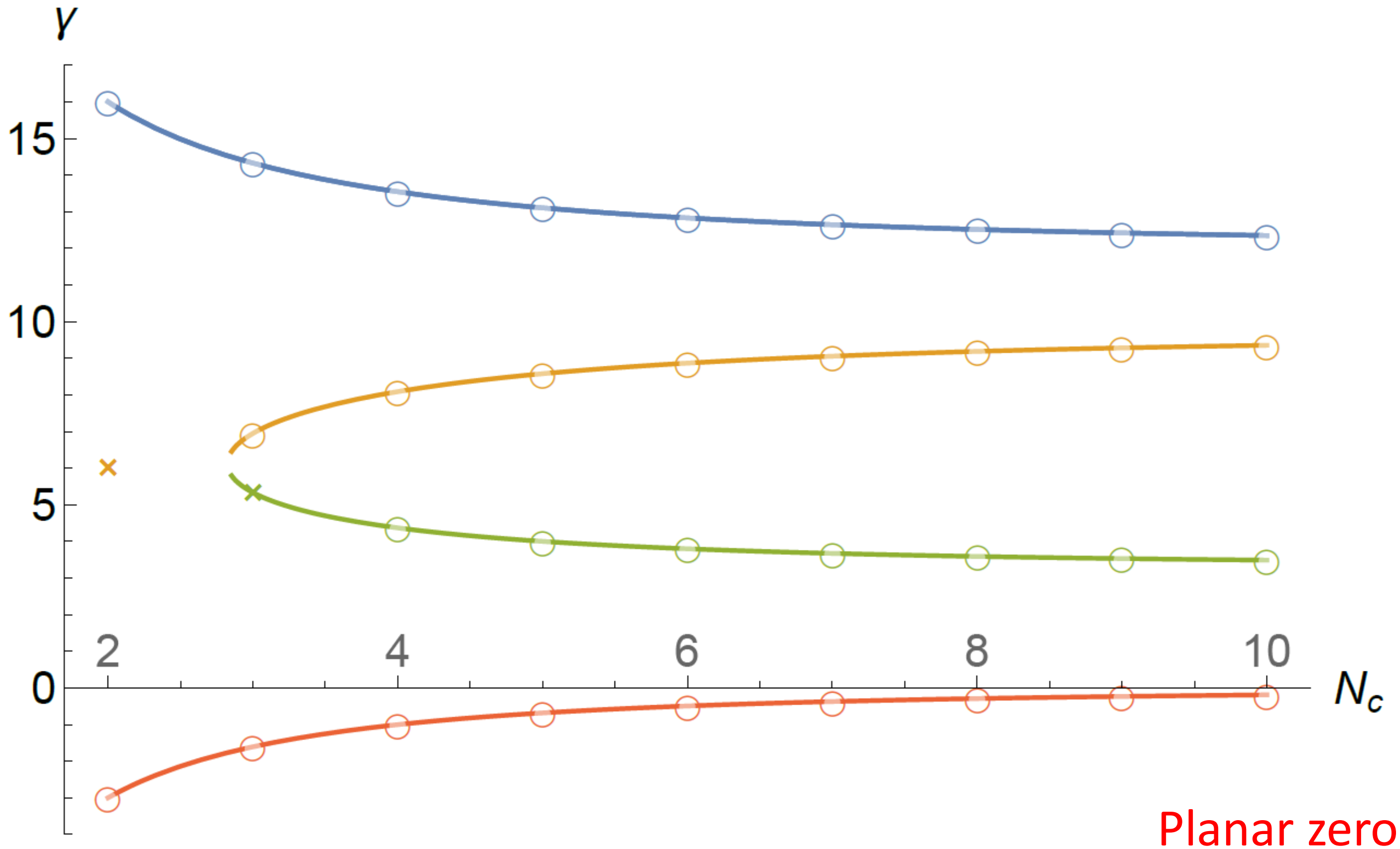
$$Ht_a = N\gamma_{ab}t_b$$

$$\gamma = \begin{pmatrix} 0 & 2 & -10/N & 10/N \\ 0 & 2 & -12/N & 12/N \\ -12/N & 2/N & 4 & -2 \\ -2/N & 7/N & -2 & 9 \end{pmatrix}$$

block-diagonal if $N \gg 1$, where the single-traces are orthogonal to the double-traces.

Use Mathematica to compute eigenvalues

$L = 4$ γ (one-loop anomalous dimension) vs N



Planar zero mode

Negative mode

$L = 4$

$t_1 = tr(\Phi_a \Phi_b) tr(\Phi_a \Phi_b),\, t_2 = tr(\Phi_a \Phi_a) tr(\Phi_b \Phi_b)$
$t_3 = tr(\Phi_a \Phi_b \Phi_a \Phi_a),\, t_4 = tr(\Phi_a \Phi_a \Phi_b \Phi_b)$

The negative mode looks like

$$\psi = 6 T_{ab} T_{ab} + \frac{3}{4N}(14 t_3 - 4 t_4) + \frac{5}{168 N^2}(978 t_1 - 107 t_2) + \cdots$$

The leading term is given by the energy-momentum tensor, which is traceless and symmetric

$$T_{ab} = tr(\Phi_a \Phi_b) - \frac{1}{6} \delta_{ab} tr(\Phi_m \Phi_m)$$

$$P T_{ab} = T_{ab},\, C T_{ab} = 0$$

It is annihilated by the planar dilatation operator, $H_P = 1 - P + C/2$

$$H_P T_{ab} T_{ab} = (H_P T_{ab}) T_{ab} + T_{ab} (H_P T_{ab}) = 0$$

Planar zero modes

Number of planar zero modes and singlet operators

L	2	4	6	8	10	12
$\mathcal{Z}_L$	0	1	2	5	11	34
$\dim \mathcal{H}_L$	1	4	15	71	469	4477

$$H_P \psi_0 = 0 \qquad H_P = 1 - P + C/2$$

$$C_{i_1 i_2 \cdots i_l} = tr\left(\Phi_{(i_1}\Phi_{i_2}\cdots\Phi_{i_l)}\right)$$

1/2 BPS : symmetric and traceless

$$H_P C_{i_1 i_2 \cdots i_l} = 0$$

$L = 4 : C_{ij}C_{ij}$

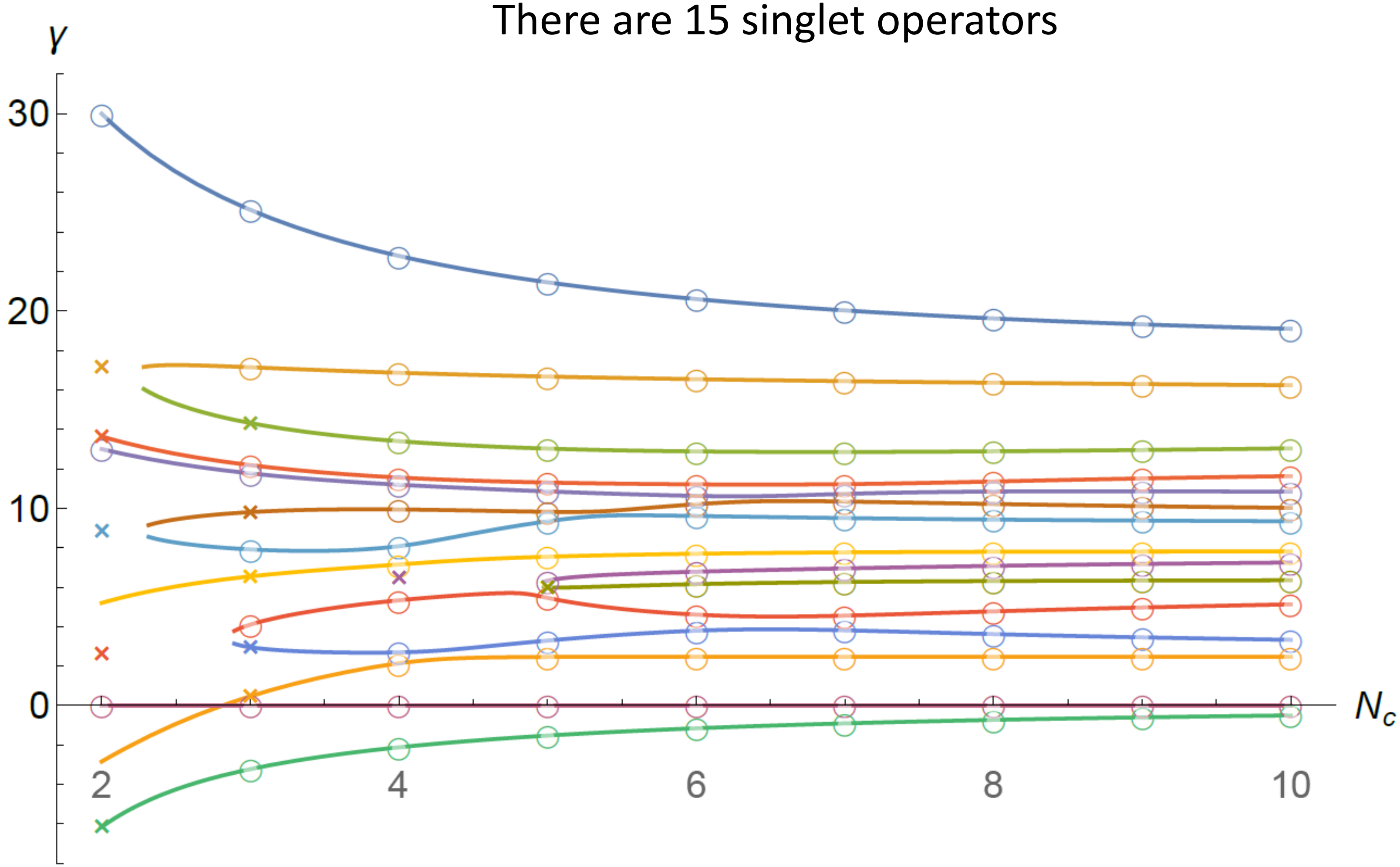
$L = 6 : C_{ijk}C_{ijk},\ C_{ij}C_{jk}C_{ki}$

$L = 8 : C_{ijkl}C_{ijkl},\ C_{ijkl}C_{ij}C_{kl},\ C_{ijk}C_{ijl}C_{kl},\ C_{ij}C_{ji}C_{kl}C_{lk},\ C_{ij}C_{jk}C_{kl}C_{li}$

✓ *Can not construct single-trace planar zero modes*

$L = 6$

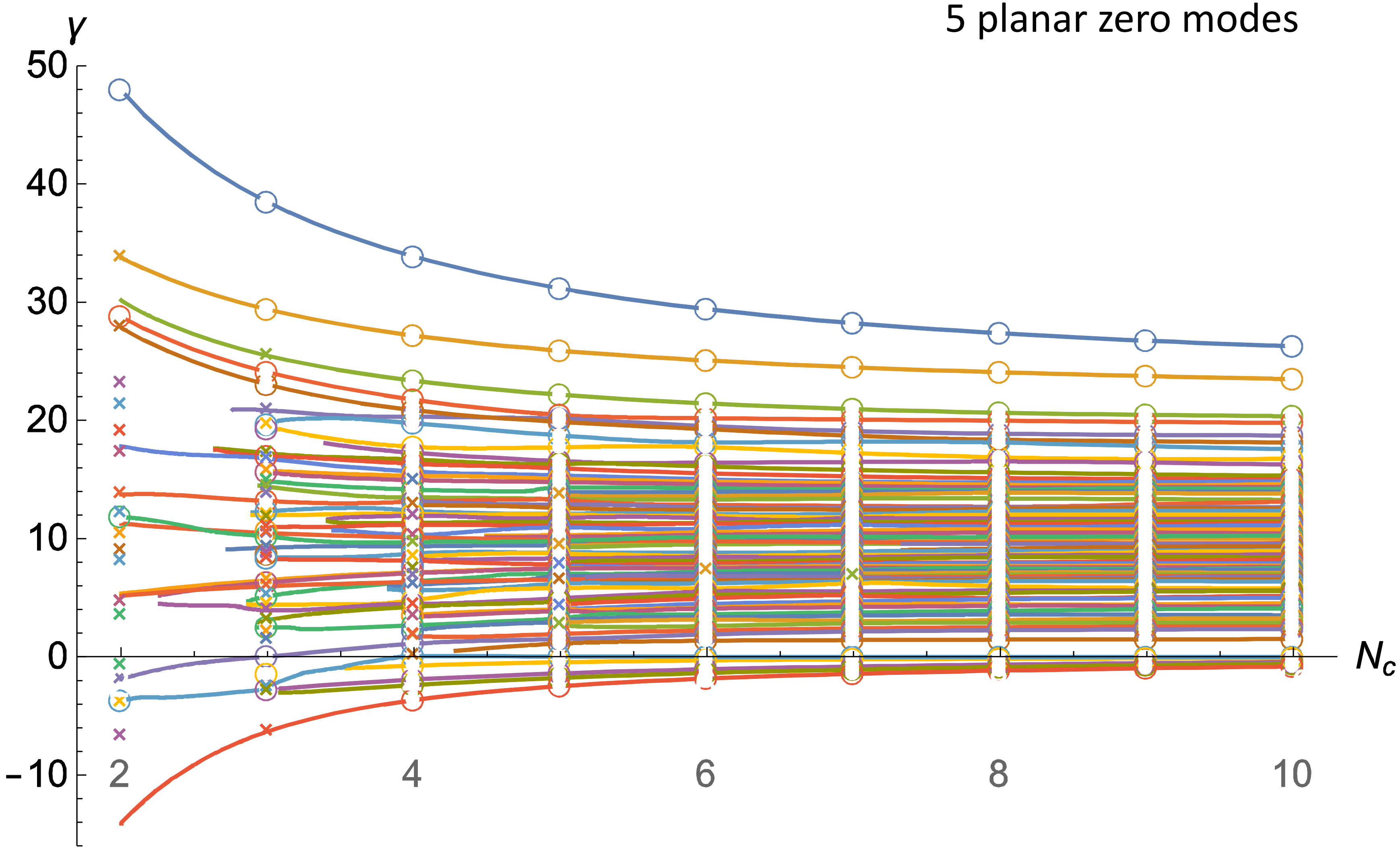
There are 15 singlet operators



There are 2 planar zero modes. One stays on the zero, and the other gets a negative anomalous dimension.

$L = 8$

71 singlets
5 planar zero modes



On the planar zero modes

Mathematica computation at $L = 4,6,8,10$ and analytic computation with some approximation

$$H\psi = N\gamma\psi \qquad H_0\psi_0 = 0$$

$$\psi = \psi_0 + \frac{\psi_1}{N} + \frac{\psi_2}{N^2} + \cdots \qquad \gamma = \gamma_0 + \frac{\gamma_1}{N} + \frac{\gamma_2}{N^2} + \cdots$$

- $\gamma_0 = 0, \gamma_1 = 0, \gamma_2 \leq 0$
  - Planar zero mode → negative mode
- ψ_0 is a linear combination of the planar zero modes with a fixed number of traces
  - $tr(\Phi^4)tr(\Phi^4)tr(\Phi^2)$ is orthogonal to $tr(\Phi^5)tr(\Phi^3)tr(\Phi^2)$ in the planar limit, but they mix by the $1/N$ effect.
  - the number of traces might be a good quantity

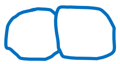
A possible interpretation of negative modes

Based on the standard correspondence of AdS/CFT

$$\Delta = \Delta_0 + \lambda \frac{\gamma_2}{N^2} + \cdots \qquad E = E_0 + \gamma_2 g_s^2 \frac{1}{\sqrt{\alpha'}} + \cdots \qquad \gamma_2 < 0$$

No interaction (planar zero mode)

Negative mode



The negative modes would describe multi-particle (multi-string) states with non-zero binding energy. [02 Arutuynov, Penati, Petkou, Santambrogio, Sokatchev]

The number of states might be related to the number of traces in ψ_0

Summary

- ✓ Studied the non-planar mixing
- ✓ so(6) singlet sector
  - ✓ Spectrum explicitly up to $L = 10$
  - ✓ Planar zero modes → negative modes
- ✓ $\gamma \sim \gamma_2/N^2$ is consistent with an analysis of 4-pt functions

- Anomalous dimensions are always positive in the planar limit
- Also positive in $su(2)$ sector, $so(6)$ non-singlet sector even with non-planar corrections
- Another sector with negative anomalous dimensions: large spin operators like $\phi\partial^S\phi$, $s \gg 1$