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Exotic Brane Junctions from F-theory

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arXiv:1602.08606

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revised version

- What is exotic brane?
- How can we use exotic branes?

What is exotic brane?

Exotic brane is

- ✔ from standard brane via string dualities in lower dim
- ✔ vortex-like (codim 2)

Obers and Pioline: hep-th/9809039
Eyras and Lozano: hep-th/9908094
de Boer and Shigemori: arXiv:1209.6056
etc..

Exotic b_n^c-brane has a tension

$$\frac{R_1 R_2 \cdots R_b \left(R_{b+1} \cdots R_{b+c}\right)^2}{g_s^n \, \ell_s^{b+2c+1}}$$

Performing string dualities, the tension is transformed :

$$\mathrm{T}_y \; : \qquad R_y \to \frac{\ell_s^2}{R_y}, \quad g_s \to \frac{\ell_s}{R_y} g_s$$

$$\mathrm{S} \; : \qquad g_s \to \frac{1}{g_s}, \quad \ell_s^2 \to g_s \, \ell_s^2$$

R_y : compact radius of y-direction

ℓ_s : string length

g_s : string coupling constant

Example duality chain of 5-branes :

$$\text{D5(12345)} \xrightarrow{\mathrm{S}} \text{NS5(12345)} \xrightarrow{\mathrm{T}_9} \text{KK5(12345,9)} \xrightarrow{\mathrm{T}_8} 5_2^2\text{(12345,89)} \xrightarrow{\mathrm{S}} 5_3^2\text{(12345,89)}$$

D5(12345): 5_1; NS5(12345): 5_2; KK5(12345,9): 5_2^1

Exotic brane is

- ✔ from standard brane via string dualities in lower dim
- ✔ vortex-like (codim 2)

Obers and Pioline: hep-th/9809039
Eyras and Lozano: hep-th/9908094
de Boer and Shigemori: arXiv:1209.6056
etc..

NOTE D7-brane is an object of codim 2 in 10D.

D7-brane physics has been studied for 20 years : F-theory

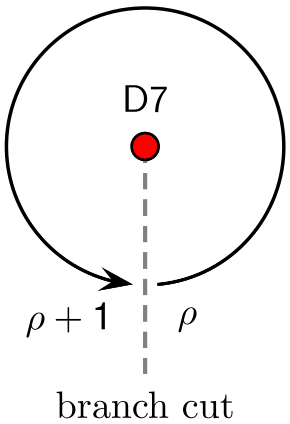
Vafa: hep-th/9602022
Sen: hep-th/9605150
etc..

D7-brane :

$$\rho(z) \equiv C^{(0)} + \mathrm{i}e^{-\phi} = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi} \log\left(\frac{\Lambda}{r}\right) \qquad (z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta})$$

When ρ moves around D7-brane counterclockwise $\theta \to \theta + 2\pi$,

it receives a magnetic "charge" of D7-brane (monodromy) : $\rho \to \rho + 1$



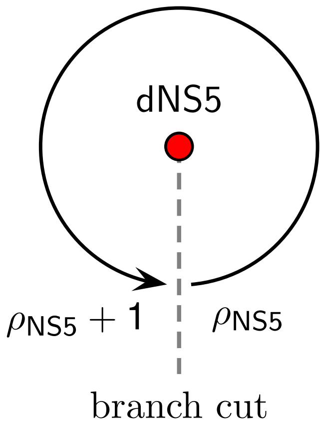
There exists a branch cut in z-plane.

defect NS5-brane : (transverse a, b directions are smeared)

$$\rho_{\text{NS5}}(z) \equiv B^{(2)}_{ab} + \mathrm{i}e^{+2\phi} = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{r}\right) \qquad (z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta})$$

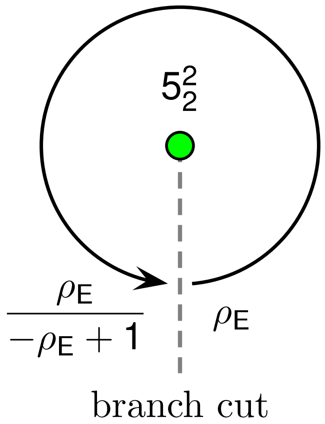
When ρ moves around NS5-brane counterclockwise $\theta \to \theta + 2\pi$,

it receives a magnetic "charge" of NS5-brane (monodromy) : $\rho_{\text{NS5}} \to \rho_{\text{NS5}} + 1$



~~In this case, we can identify $\rho_{\text{NS5}} + 1 \simeq \rho_{\text{NS5}}$ by B-field gauge transformation.~~

Exotic 5^2_2-brane : (T_{ab}-dualized from defect NS5-brane)



monodromy : $\dfrac{1}{\rho_E} \to \dfrac{1}{\rho_E} - 1$ where $\rho_E = -\dfrac{1}{\rho_{NS5}}$

We cannot identify this monodromy change

by
- B-field gauge transformation
- coordinate transformations

5^2_2-brane's property is from that of defect NS5-brane

via $SL(2,\mathbb{Z}) \in SO(2,2;\mathbb{Z})$

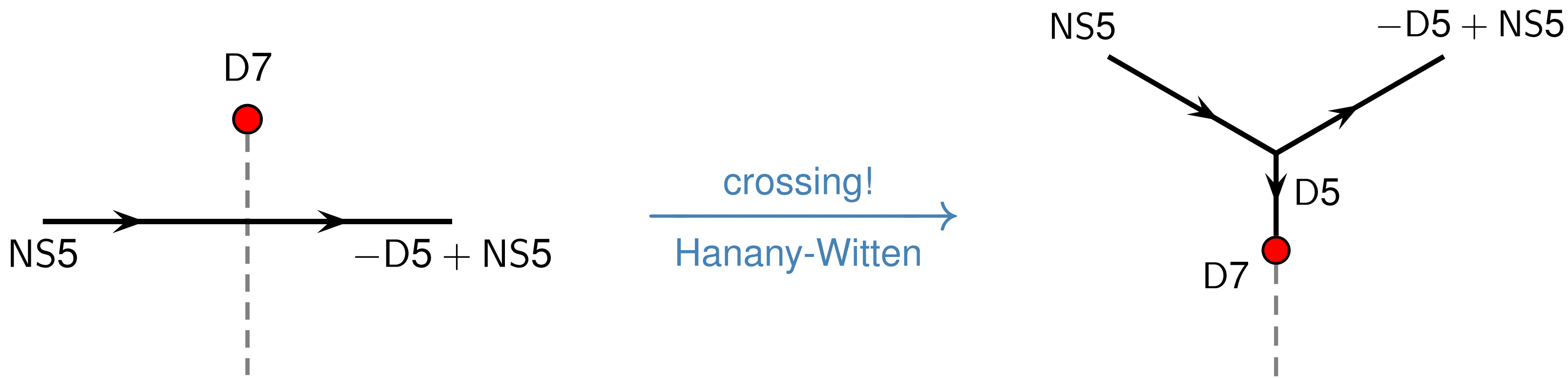
T_{ab}-duality

How can we use exotic branes?

Consider an NS5-brane crossing the branch cut of D7-brane from the left.

If D7-brane goes across NS5-brane,

a new 5-brane and a junction appear (Hanany-Witten effect).



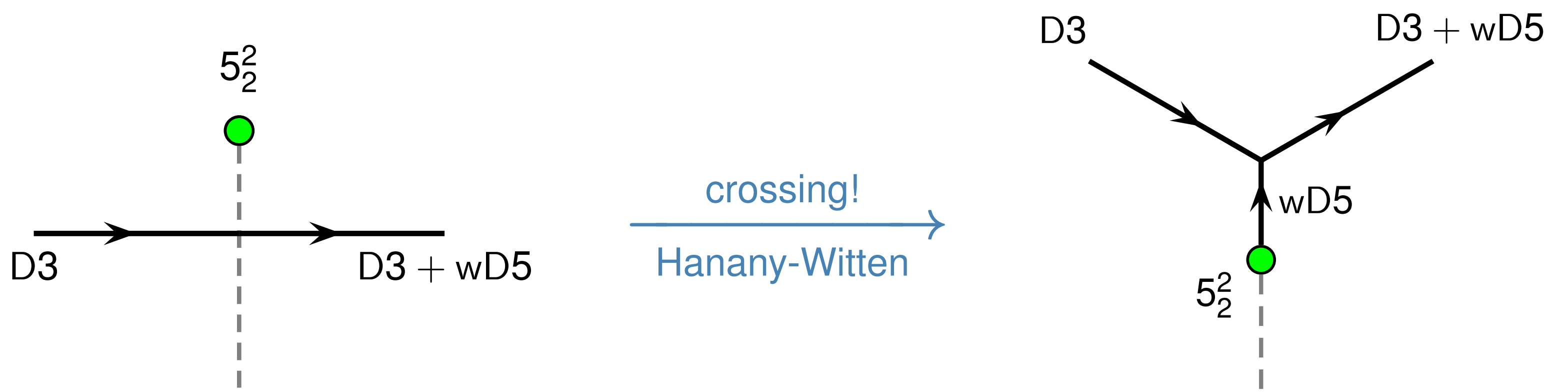
Note: D7-brane(1234567), D5(1234X), NS5(1234Y), X, Y $\in z$-plane

This is a brane junction in F-theory.

Gaberdiel and Zwiebach: hep-th/9709013
DeWolfe and Zwiebach: hep-th/9804210
etc..

Perform the T_{34}- and S-duality of the system of NS5-brane with D7-brane.

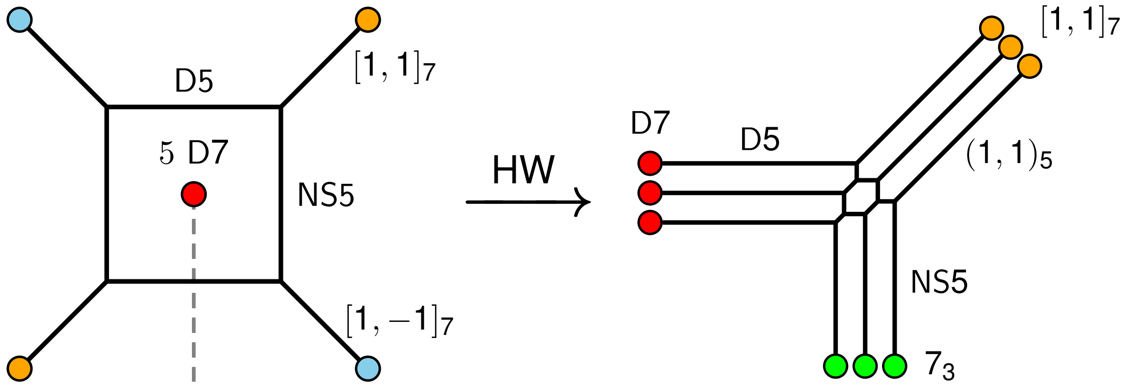
We obtain a config. that a new "3-brane" with exotic 5^2_2-brane :



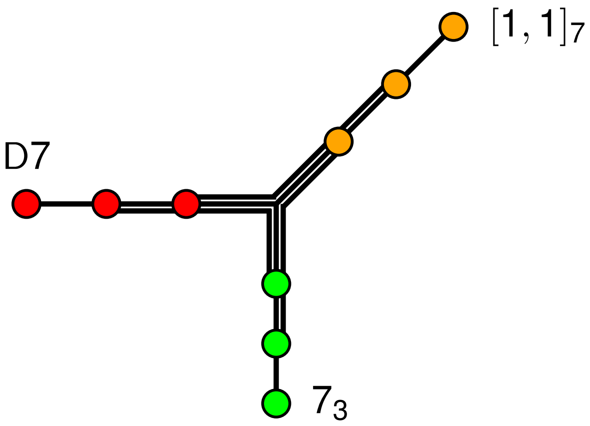
We find that D5(1234Y)-brane wrapped on T^2_{34} ($\equiv$ wD5)

is ending on 5^2_2-brane.

5D theory on 5-branes with 5 D7-branes :



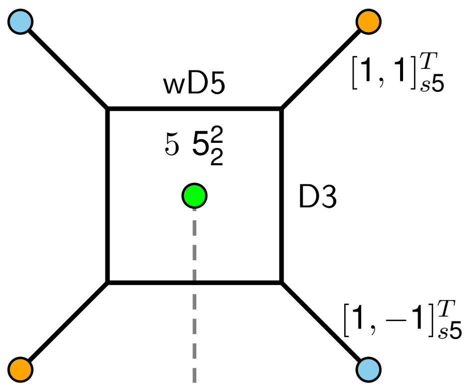
IIB	0	1	2	3	4	5	6	7	8	9
5 D7	–	–	–	–	–	–	–	–		
D5	–	–	–	–	–				–	
NS5	–	–	–	–	–					–
$(1,1)_5$	–	–	–	–	–				angle	

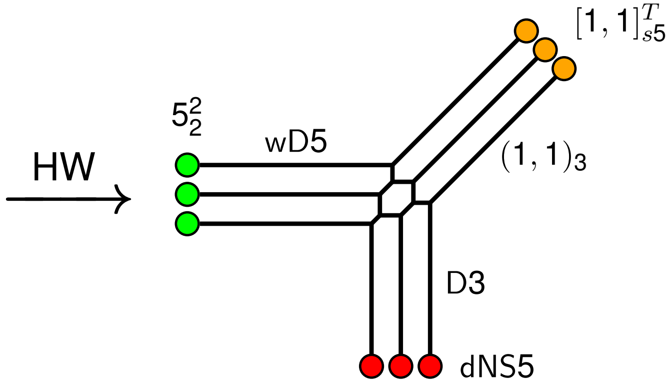


5D T_3 theory

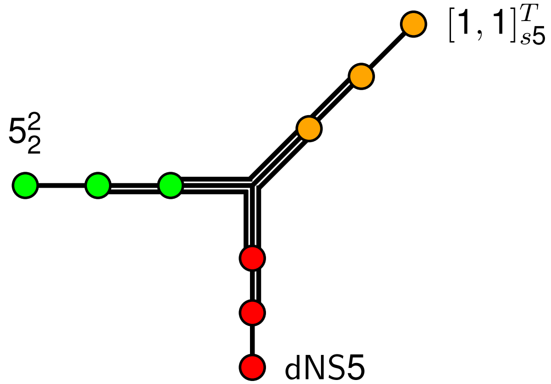
Aharony and Hanany: hep-th/9704170
DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179
Gaiotto and Witten: arXiv:0804.2902, 0807.3720
Benini, Benvenuti and Tachikawa: arXiv:0906.0359

3D theory on "3-branes" with 5 exotic 5^2_2-branes :





IIB	0	1	2	③	④	5	6	7	8	9
$5\ 5^2_2$	—	—	—	$\bullet^2$	$\bullet^2$	—	—	—		
wD5	—	—	—	—	—				—	
D3	—	—	—							—
$(1,1)_3$	—	—	—						angle	



mirror of 3D T_3 theory

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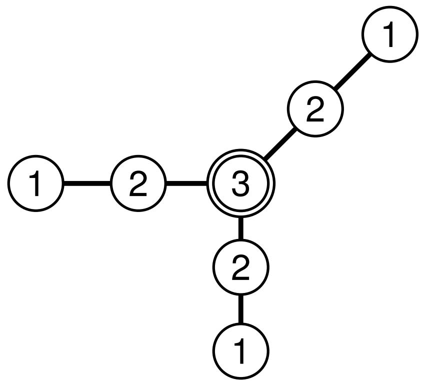
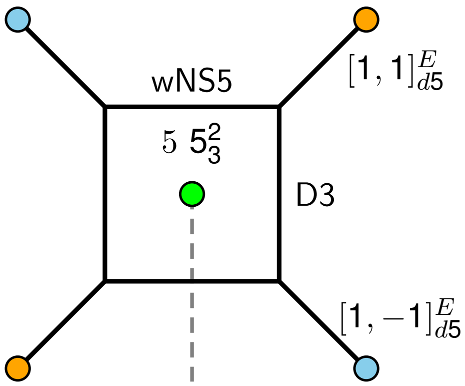


star-shaped quiver

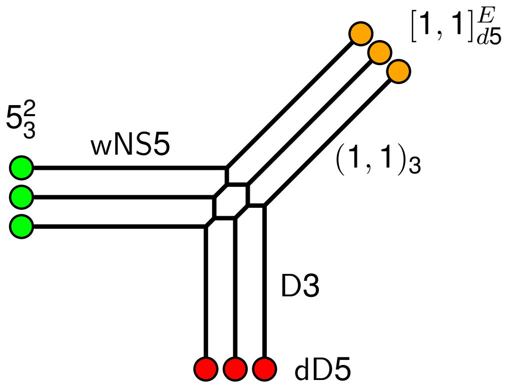
5^2_2-brane yields mirror of 3D T_3 theory.

Benini, Tachikawa and Xie: arXiv:1007.0992

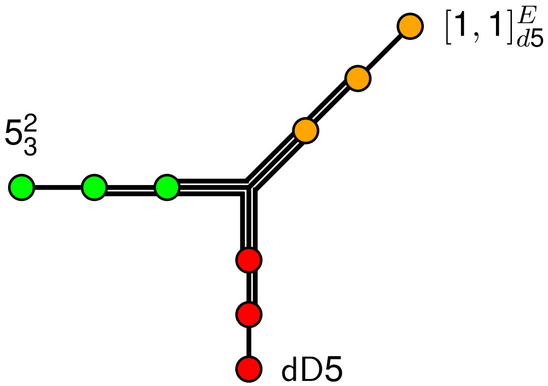
3D theory on “3-branes” with 5 exotic 5^2_3-branes :



HW →

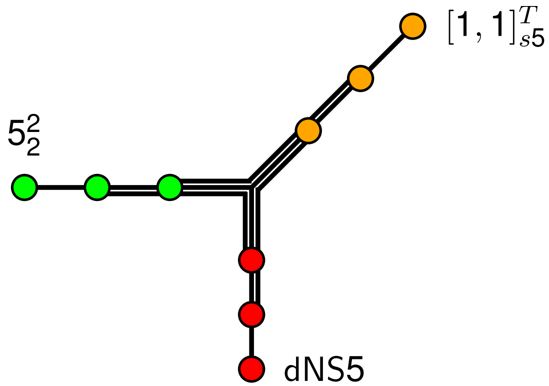


IIB	0	1	2	③	④	5	6	7	8	9
$5\ 5^2_3$	–	–	–	$\bullet^2$	$\bullet^2$	–	–	–		
wNS5	–	–	–	–	–				–	
D3	–	–	–							–
$(1,1)_3$	–	–	–						angle	



3D T_3 theory

mirror
⟷
S-duality



mirror of 3D T_3 theory

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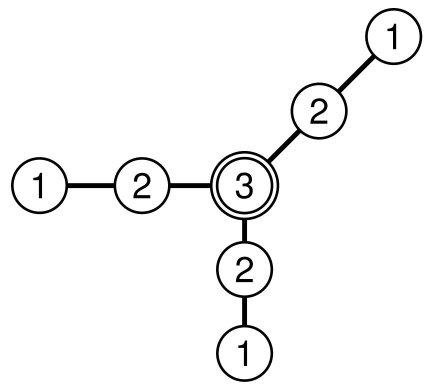


star-shaped quiver

5^2_3-brane gives 3D T_3 theory.

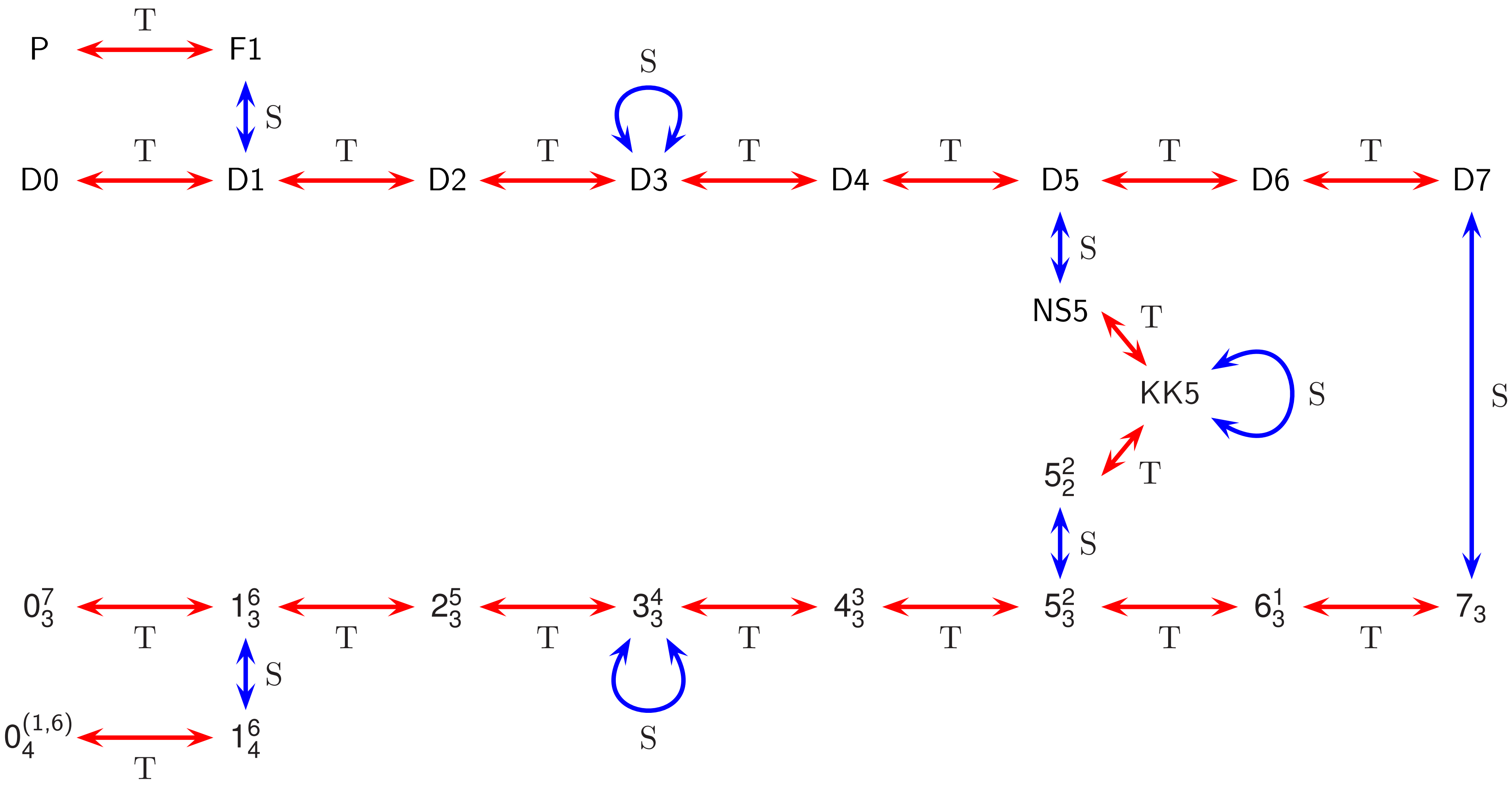
Exotic branes also play a role in generating (non-)Lagrangian theories.

Summary and Discussions

- Exotic brane has a monodromy (charge) with branch cut.
- When D3-brane cross the branch cut of 5^2_2-brane,
  it jumps to D3 + "D5 wrapped on two-torus".

- Exotic 5-branes are the building blocks of 3D T_3-theory and its mirror theory.
- They correctly provide 3D theory even when spacetime is compactified.

- $Non\text{-}Lagrangian\ theory\ from\ non\text{-}geometric\ background$


P
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NS5
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KK5
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5^2_2
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1^6_3
2^5_3
3^4_3
4^3_3
5^2_3
6^1_3
7_3
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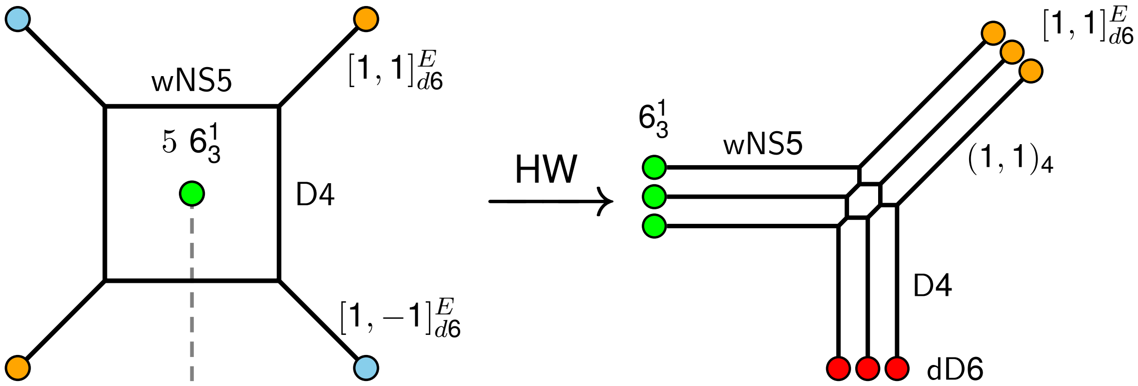
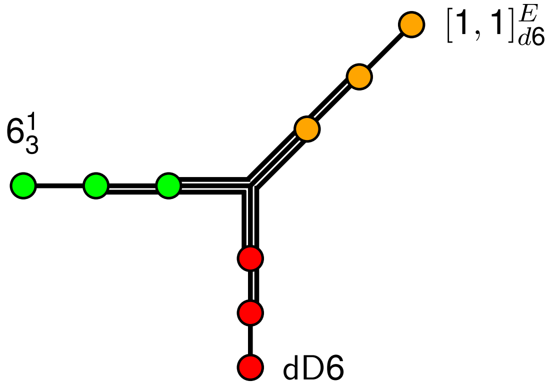
$0^{(1,6)}_4$
1^6_4
T

4D theory on "4-branes" with 5 exotic 6^1_3-branes :



IIA	0	1	2	3	④	5	6	7	8	9
$5\ 6^1_3$	–	–	–	–	$\bullet^2$	–	–	–		
wNS5	–	–	–	–	–				–	
D4	–	–	–	–						–
$(1,1)_4$	–	–	–	–					angle	



4D T_3 theory can be realized.

Uplift to M-theory : dD6 $\to$ KK6, 6^1_3 $\to$ KK6; D4 $\to$ M5, wNS5 $\to$ M5

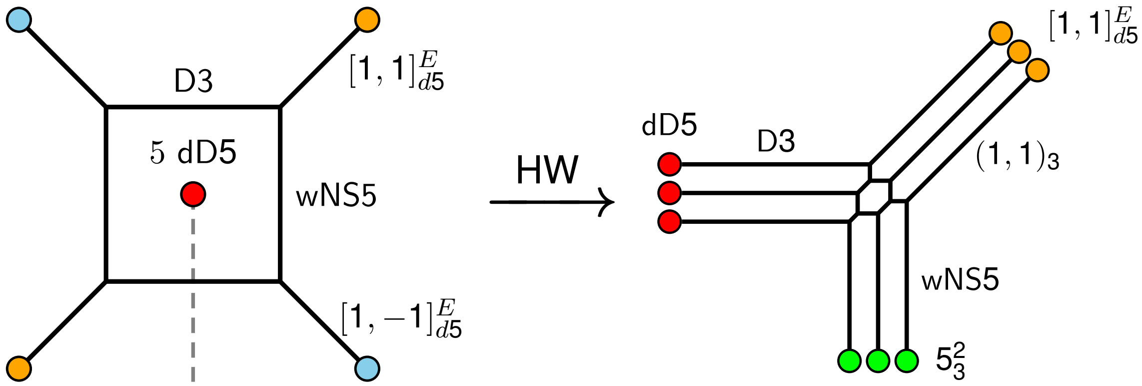
$$g_s \ell_s = R_\natural, \quad g_s \ell_s^3 = \ell_{\mathrm{P}}^3$$

Gaiotto: arXiv:0904.2715

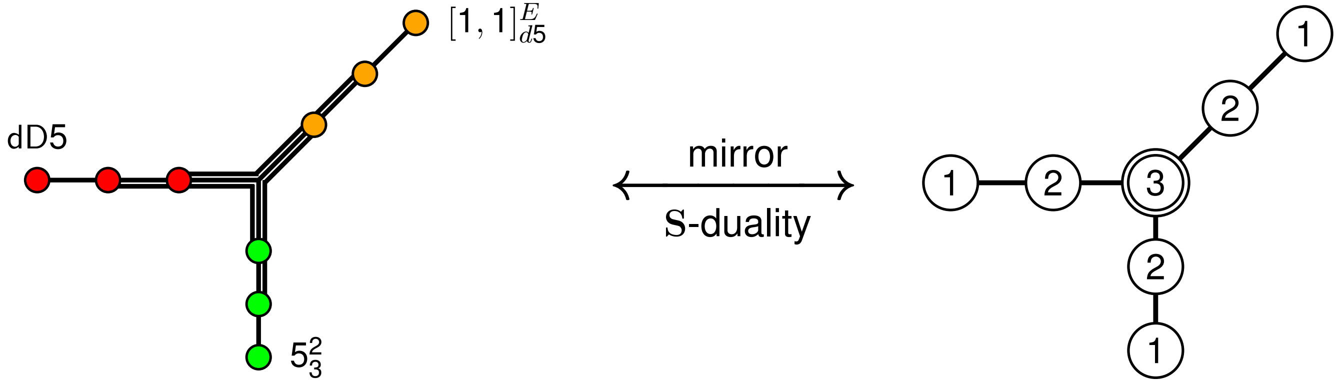
Thanks

Appendix

3D theory on “3-branes” with 5 defect D5-branes :



IIB	0	1	2	③	④	5	6	7	8	9
5 dD5	–	–	–			–	–	–		
D3	–	–	–						–	
wNS5	–	–	–	–	–					–
$(1,1)_3$	–	–	–						angle	

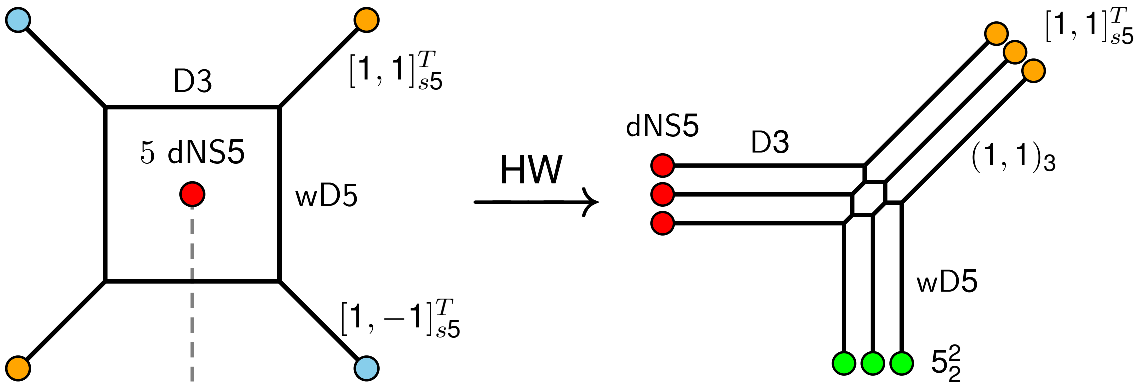


3D T_3 theory   star-shaped quiver

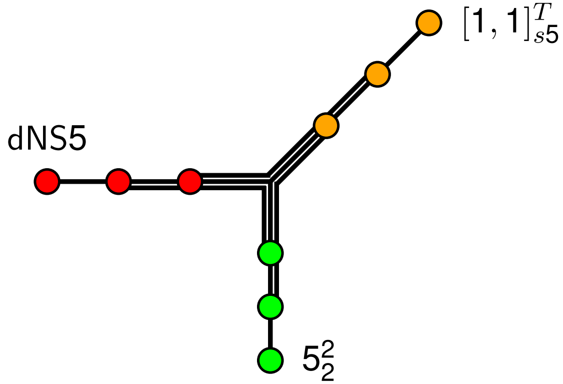
3D T_3 theory is realized.

Benini, Tachikawa and Xie: arXiv:1007.0992

3D theory on “3-branes” with 5 defect NS5-branes :

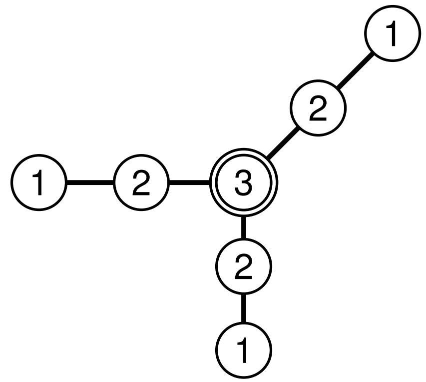


IIB	0	1	2	③	④	5	6	7	8	9
5 dNS5	–	–	–			–	–	–		
D3	–	–	–						–	
wD5	–	–	–	–	–					–
$(1,1)_3$	–	–	–						angle	



mirror of 3D T_3 theory

$=$

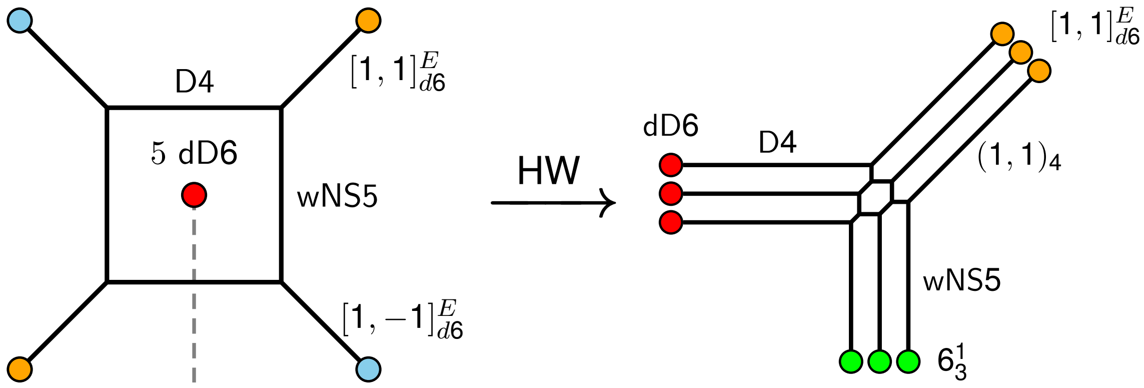


star-shaped quiver

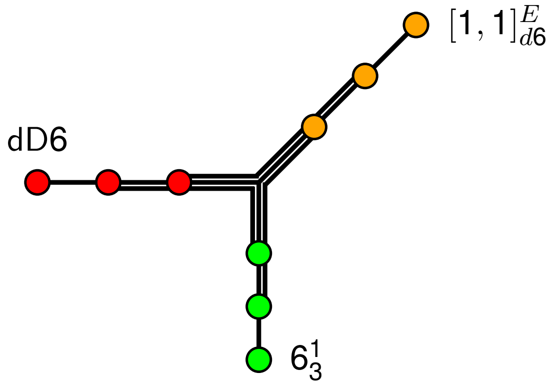
Mirror of 3D T_3 theory is realized.

Benini, Tachikawa and Xie: arXiv:1007.0992

4D theory on “4-branes” with 5 defect D6-branes :



IIA	0	1	2	3	④	5	6	7	8	9
5 dD6	–	–	–	–		–	–	–		
D4	–	–	–	–					–	
wNS5	–	–	–	–	–					–
$(1,1)_4$	–	–	–	–					angle	



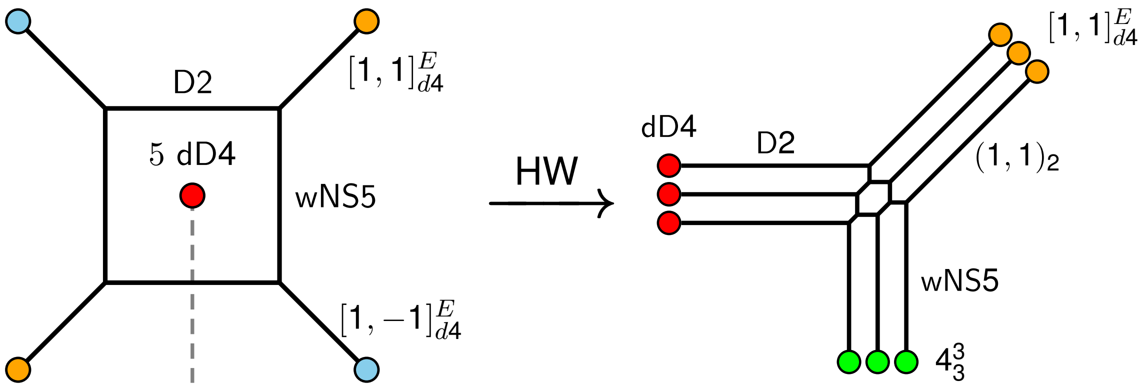
4D T_3 theory can be realized.

Uplift to M-theory : dD6 $\to$ KK6, $6^1_3 \to$ KK6; D4 $\to$ M5, wNS5 $\to$ M5

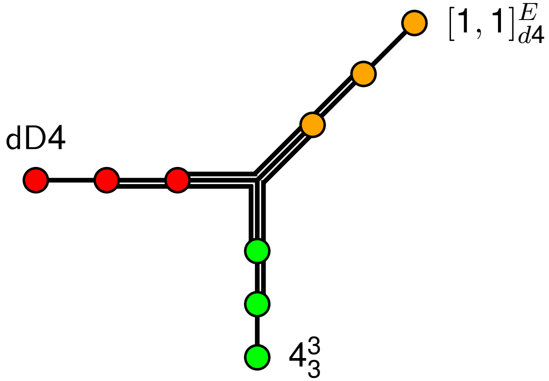
$$g_s \ell_s = R_\natural, \quad g_s \ell_s^3 = \ell_{\mathsf{P}}^3$$

Gaiotto: arXiv:0904.2715

2D theory on “2-branes” with 5 defect D4-branes :



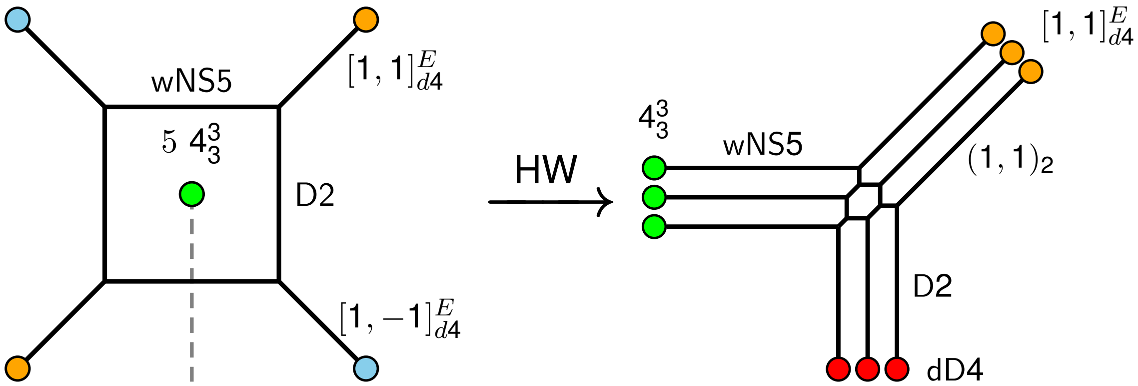
IIA	0	1	②	③	④	5	6	7	8	9
5 dD4	–	–				–	–	–		
D2	–	–							–	
wNS5	–	–	–	–	–					–
$(1,1)_2$	–	–							angle	



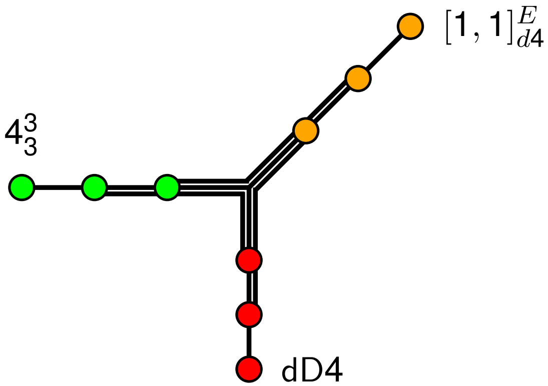
2D T_3 theory can be realized.

Uplift to M-theory : dD4 $\to$ M5, $4^3_3 \to 5^3$; wNS5 $\to$ M5, D2 $\to$ M2

2D theory on "2-branes" with 5 exotic 4^3_3-branes :



IIA	0	1	②	③	④	5	6	7	8	9
$5\ 4^3_3$	–	–	$\bullet^2$	$\bullet^2$	$\bullet^2$	–	–	–		
wNS5	–	–	–	–	–				–	
D2	–	–								–
$(1,1)_2$	–	–							angle	



2D T_3 theory can be realized.

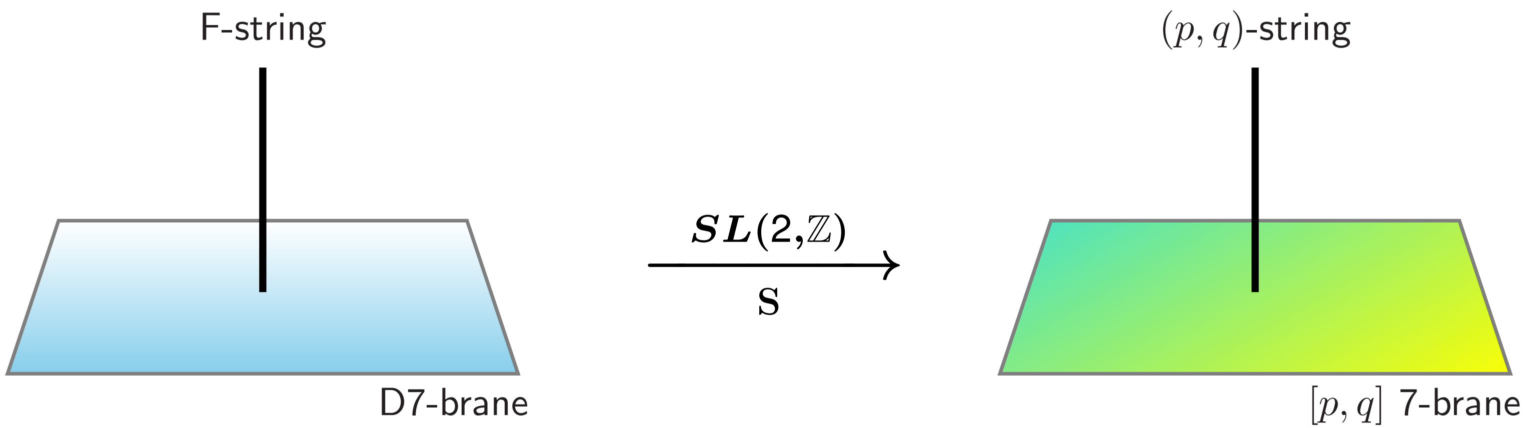
Uplift to M-theory : dD4 $\to$ M5, $4^3_3 \to 5^3$; wNS5 $\to$ M5, D2 $\to$ M2

F-string : couple to $B_{(2)}$

D-string : couple to $C_{(2)}$

D7(1234567) : couple to $\rho(z) = C + \mathrm{i}e^{-\phi}$ $\quad (z = x^8 + \mathrm{i}x^9 = r\,e^{\mathrm{i}\theta})$

$$\rho(z) = \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{z}\right) = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{r}\right)$$



$(1,0)$-string $=$ F1 $\qquad [1,0]$ 7-brane $=$ D7(1234567)

$(0,1)$-string $=$ D1 $\qquad [0,1]$ 7-brane $=$ 7_3(1234567)

Open D-string is ending on 7_3(1234567).

This is a setup in F-theory. We perform ST_{67}-duality and reduce 67-directions.

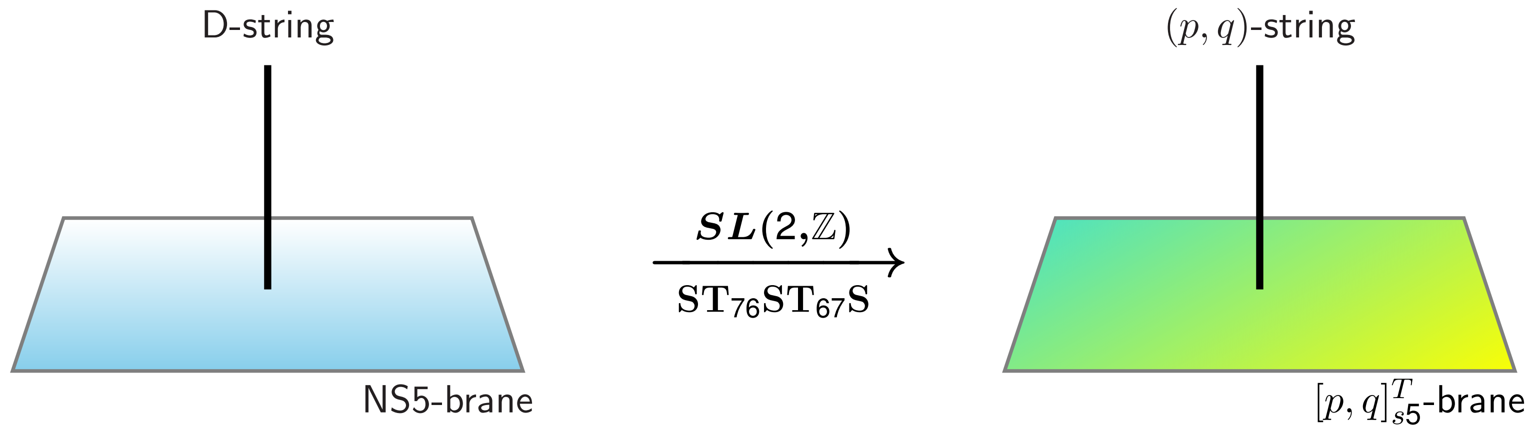
D-string : couple to $C_{(2)}$

D3-brane : couple to $C_{(4)}$

NS5(12345) : couple to $\rho(z) = B^{(2)}_{67} + \mathrm{i}e^{+2\phi}$

$$\rho(z) = \frac{\mathrm{i}}{2\pi}\log\Big(\frac{\Lambda}{z}\Big) = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\Big(\frac{\Lambda}{r}\Big)$$

$(z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta})$



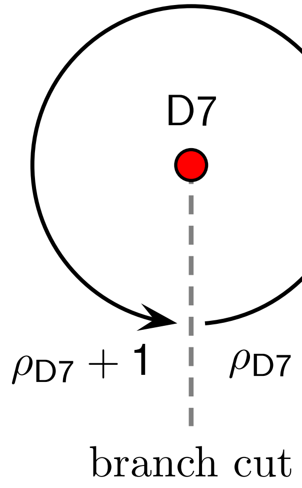
$(1,0)$-string $=$ D1

$(0,1)$-string $=$ D3 wrapped on T^2_{67}

$[1,0]^T_{s5}$-brane $=$ NS5(12345)

$[0,1]^T_{s5}$-brane $=$ 5^2_2(12345,67)

Open D3-brane wrapped on T^2_{67} is ending on exotic 5^2_2(12345,67).



$$\rho \;\to\; \rho + 1 \;=\; \frac{a\rho + b}{c\rho + d} \;\equiv\; M_{[1,0]} \cdot \rho \,, \qquad M_{[1,0]} \;=\; \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \;\in\; SL(2,\mathbb{Z})$$

or

$$K_{[1,0]} \cdot (\rho + 1) \;=\; \rho \,, \qquad K_{[1,0]} \;=\; (M_{[1,0]})^{-1} \;=\; \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

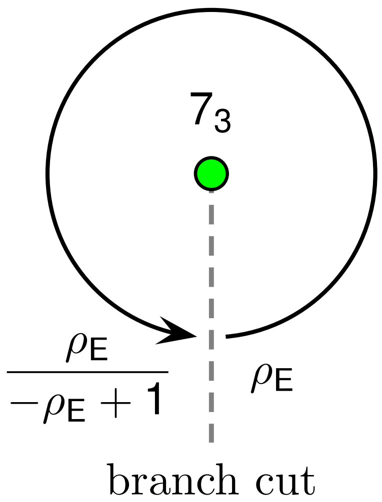
$M_{[p,q]}$: moving around the 7-brane

$K_{[p,q]}$: going across the branch cut

By $SL(2,\mathbb{Z})$, the monodromy matrix for general $[p,q]$ 7-brane is given as

$$K_{[p,q]} = g\, K_{[1,0]}\, g^{-1} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix} \qquad g \in SL(2,\mathbb{Z})$$

ex) monodromy $K_{[0,1]}$ for 7_3-brane : $\quad K_{[0,1]} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$



$$\rho_{\rm E} = -\frac{1}{\rho_{\rm D7}}$$

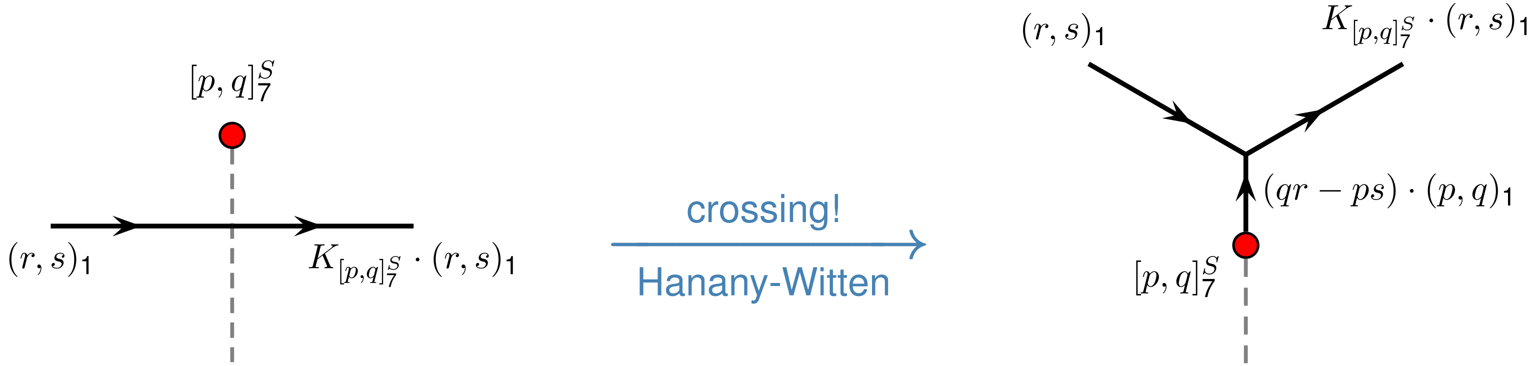
Consider an $(\boldsymbol{r}, \boldsymbol{s})$-string crossing the branch cut of $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane from the left.

The string charge is jumped by monodromy.

If the 7-brane goes across the string, the string is no longer crossing the branch cut.

Further, a new string and a junction appear (Hanany-Witten effect).

"$[\boldsymbol{p}, \boldsymbol{q}]_7^S$" means $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane

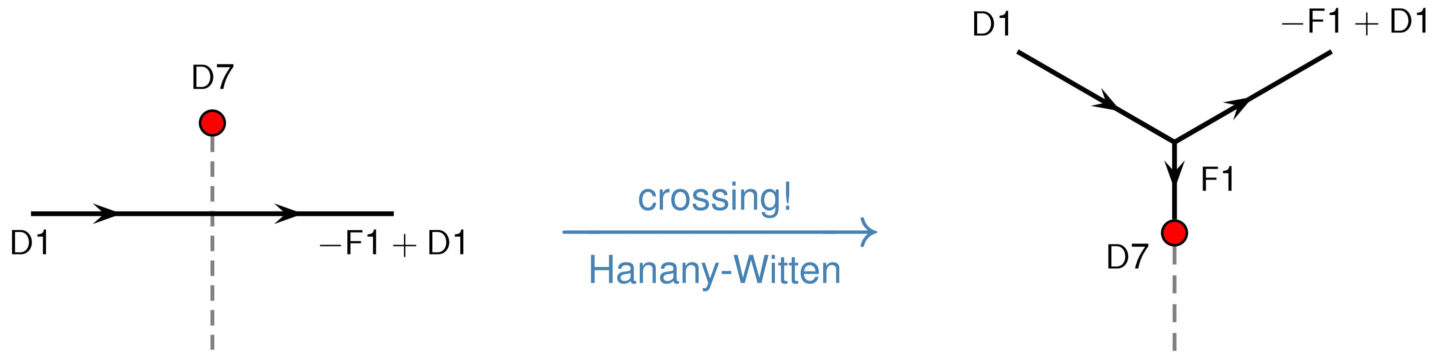


Note: 7-brane is stretched in 1234567-directions.

This is a string junction in F-theory.

Consider a D-string crossing the branch cut of D7-brane from the left.

A new string and a junction appear by Hanany-Witten effect.



Note: D7-brane is stretched in 1234567-directions.

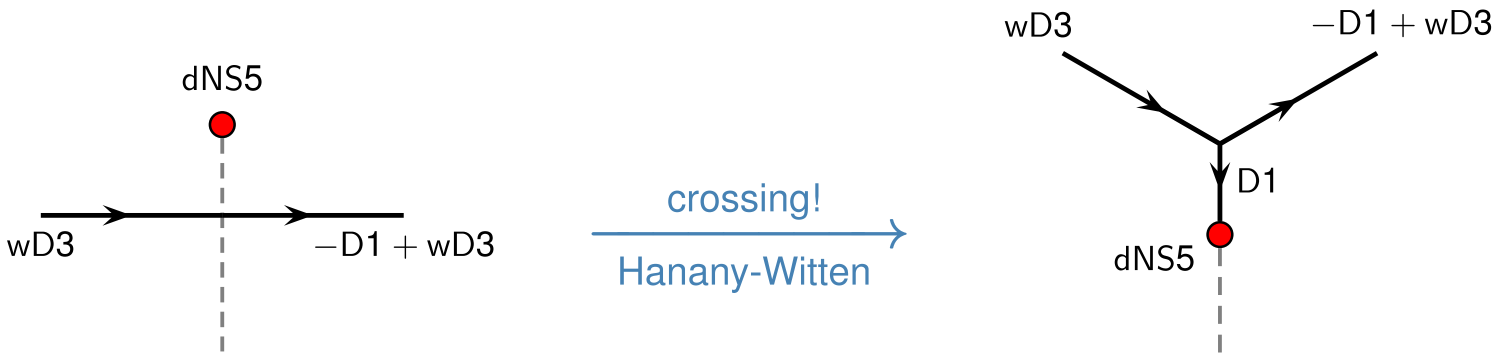
This is a string junction in F-theory.

Gaberdiel and Zwiebach: hep-th/9709013
DeWolfe and Zwiebach: hep-th/9804210
etc..

Consider a D3-brane wrapped on T^2_{ab} (wD3) and defect NS5-brane (dNS5).

If dNS5 goes across wD3,

a new D-string and a junction appear by Hanany-Witten effect.

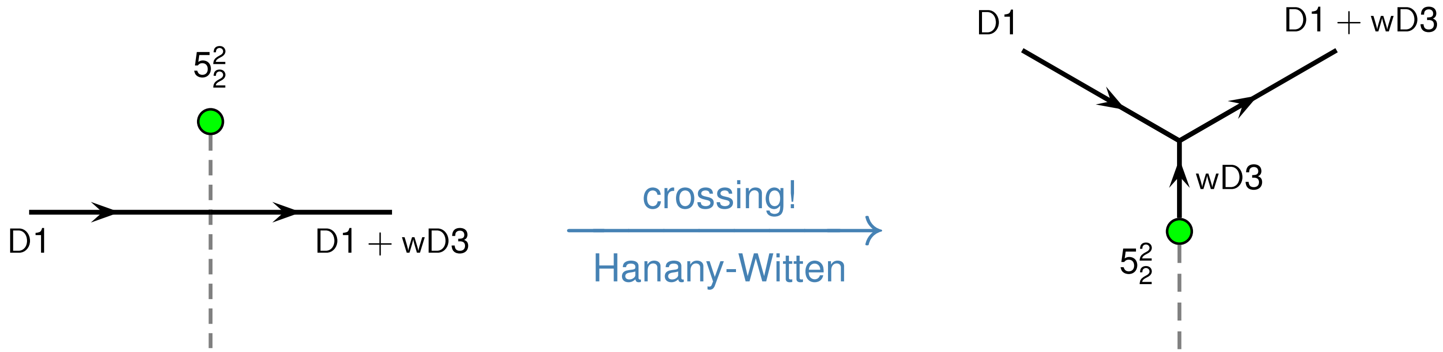


Consider a D-string and 5^2_2-brane.

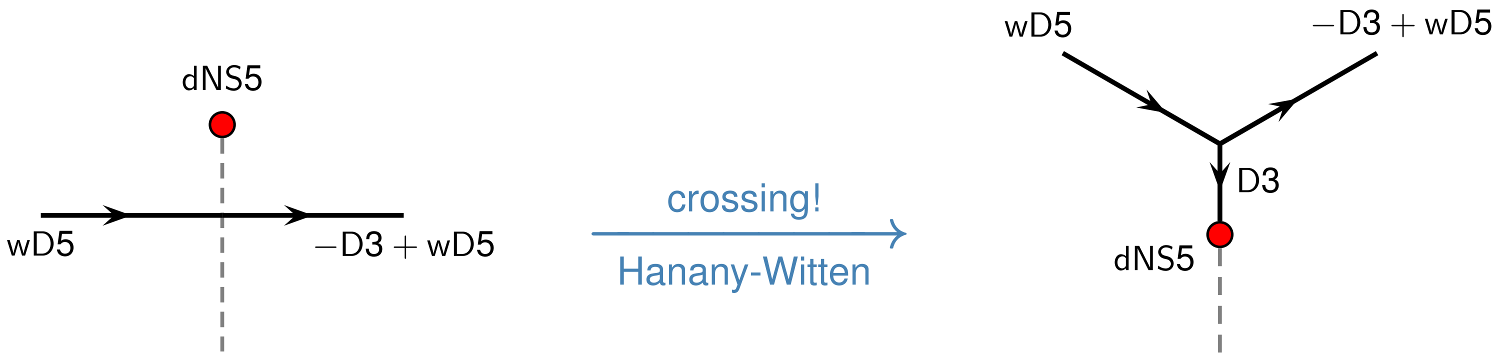
D-string charge is jumped by monodromy.

If 5^2_2-brane goes across D-string,

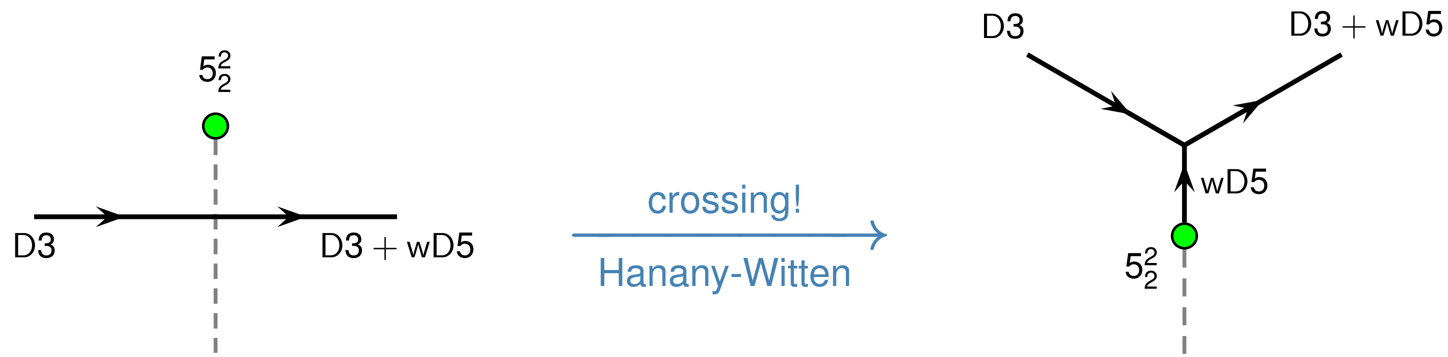
a new wD3 and a junction appear (Hanany-Witten effect).



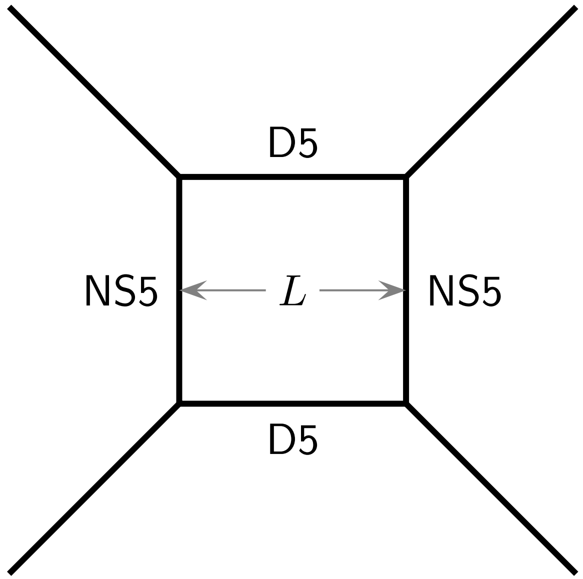
- defect NS5-brane and D5-brane wrapped on T^2_{ab} :



- 5^2_2-brane and D3-brane :



5D SUSY gauge theory can be realized by brane construction :



IIB	0	1	2	3	4	5	6	7	8	9
n D7	—	—	—	—	—	—	—	—		
D5	—	—	—	—	—				—	
NS5	—	—	—	—	—					—
$(1,1)_5$	—	—	—	—	—				angle	

color : N_c D5 between 2 NS5 $=$ $SU(N_c)$ gauge symmetry

flavor : N_f D5 outside 2 NS5 $=$ N_f flavors

coupling : $$\frac{1}{g_{\rm YM}^2} \simeq \frac{L}{g_s\, \ell_s^2}$$ (This can be derived from Dirac-Born-Infeld action of D5-brane.)

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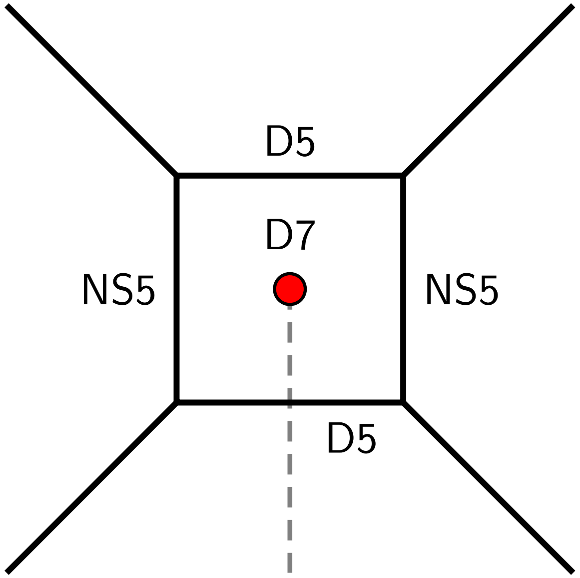

IIB	0	1	2	3	4	5	6	7	8	9
n D7	—	—	—	—	—	—	—	—		
D5	—	—	—	—	—				—	
NS5	—	—	—	—	—					—
$(1,1)_5$	—	—	—	—	—				angle	

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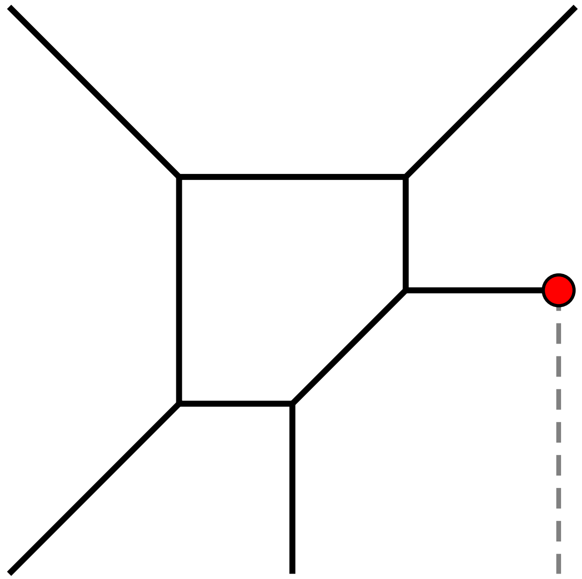

IIB	0	1	2	3	4	5	6	7	8	9
n D7	—	—	—	—	—	—	—	—		
D5	—	—	—	—	—				—	
NS5	—	—	—	—	—					—
$(1,1)_5$	—	—	—	—	—				angle	

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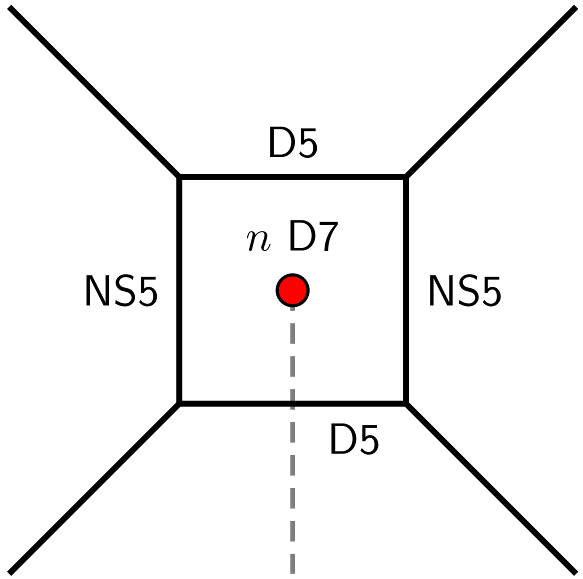


IIB	0	1	2	3	4	5	6	7	8	9
n D7	–	–	–	–	–	–	–	–		
D5	–	–	–	–	–				–	
NS5	–	–	–	–	–					–
$(1,1)_5$	–	–	–	–	–				angle	

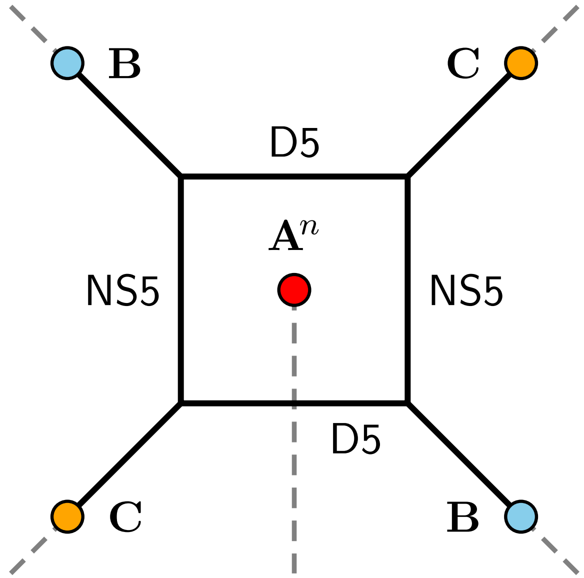
This config. indicates 5D $SU(2)$ gauge symmetry with n flavors on 01234-directions.

In the $L \propto 1/g_{\rm YM}^2 \to 0$ limit, this gauge theory flows to SCFT with E_{n+1} symmetry.

(UV fixed point)

Let us consider a config. with $n = 5, 6, 7$ D7-branes.

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with n flavors on D5-brane can be illustrated as



$\mathbf{A}$-brane : D7-brane

$\mathbf{B}$-brane : $[1, -1]$ 7-brane

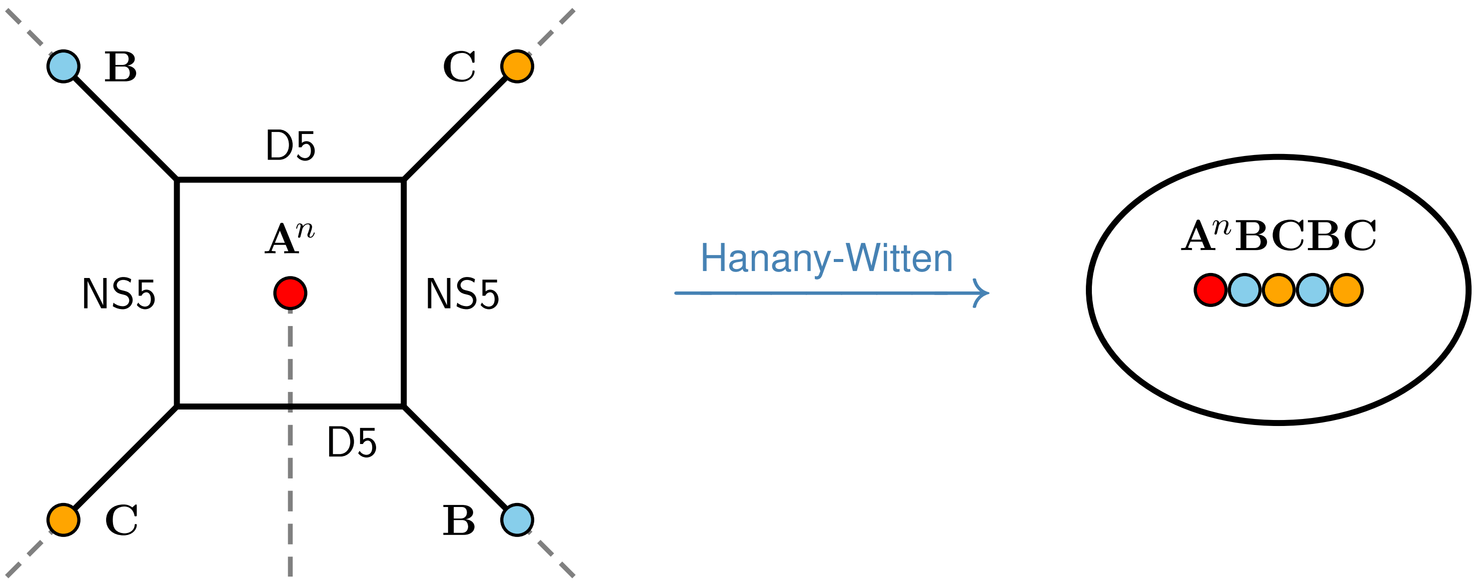
$\mathbf{C}$-brane : $[1, 1]$ 7-brane

Without changing the 5D gauge theory on D5-brane,

semi-infinite (p, q) 5-branes are terminated by $[p, q]$ 7-branes.

Let us consider a config. with $n = 5, 6, 7$ D7-branes.

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with n flavors on D5-brane can be illustrated as



Moving $\mathbf{B}$- and $\mathbf{C}$-branes along the (p, q) 5-branes and going inside the "box",

the (p, q) 5-branes are annihilated by the Hanany-Witten effect.

Further, the "box" becomes "loop" by back reaction of $\mathbf{A}^n$-, $\mathbf{B}$-, $\mathbf{C}$-, $\mathbf{B}$-, and $\mathbf{C}$-branes.

(skipped drawing the branch cuts.)

Let us consider a config. with $n = 5, 6, 7$ D7-branes.

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with n flavors on D5-brane can be illustrated as

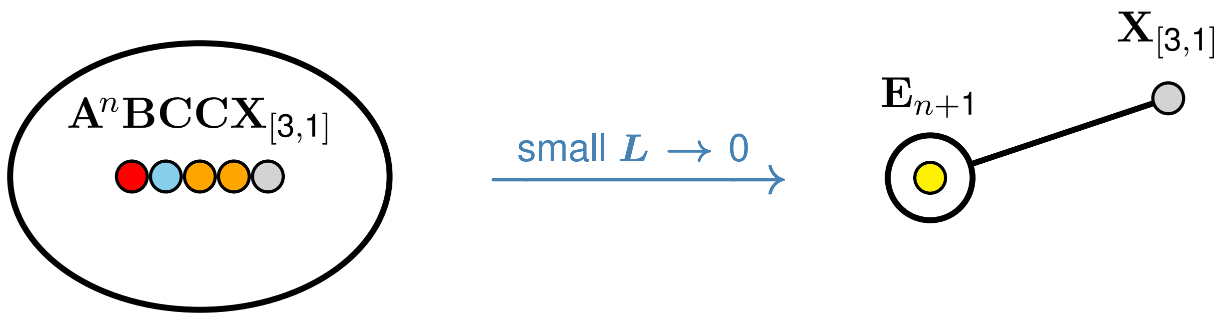
Perform the re-ordering of 7-branes with branch cuts.

When a 7-brane goes across another's branch cut, its monodromy is modified.

$$\mathbf{A}^n\mathbf{BCBC} \ \rightarrow \ \mathbf{A}^n\mathbf{BCCX}_{[3,1]} \qquad \text{with } \ \mathbf{X}_{[3,1]}\text{-brane} \equiv [3,1]\text{-brane}$$

Let us consider a config. with $n = 5, 6, 7$ D7-branes.

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with n flavors on D5-brane can be illustrated as



Perform the small loop limit $L \to 0$. This implies the strong coupling limit $g_{\text{YM}} \to \infty$.

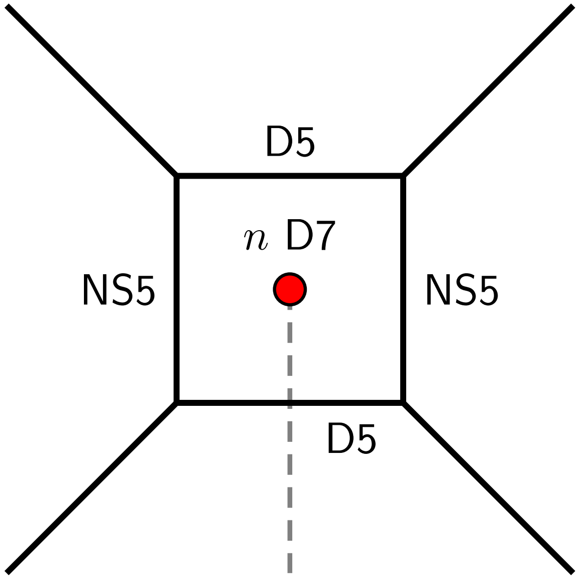
There exists a non-trivial UV fixed point of 5D gauge theory $\to$ CFT.

$\mathbf{A}^n$-, $\mathbf{B}$-, and $\mathbf{C}^2$-branes are collapsed to $\mathbf{E}_{n+1}$-brane.

$\mathbf{X}_{[3,1]}$-brane is gone far away from $\mathbf{E}_{n+1}$-brane.

Open string ending on 5-brane loop and $\mathbf{E}_{n+1}$-brane provides $\mathbf{E}_{n+1}$ symmetry.

We argued 5D SUSY gauge theory and its strong coupling limit by brane construction.



IIB	0	1	2	3	4	5	6	7	8	9
n D7	—	—	—	—	—	—	—	—		
D5	—	—	—	—	—				—	
NS5	—	—	—	—	—					—
$(1,1)_5$	—	—	—	—	—				angle	

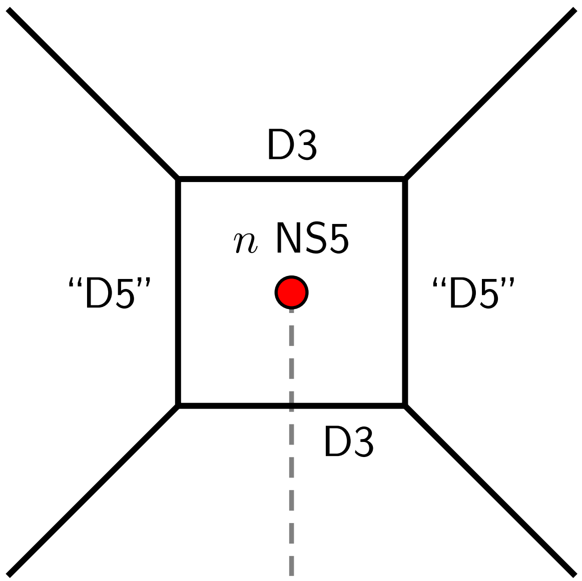
Perform string dualities :

ST_{67}-dual $\longrightarrow$ 5D

ST_{47}-dual $\longrightarrow$ 4D

ST_{34}-dual $\longrightarrow$ 3D

$\mathbf{ST}_{34}$-dualized system :

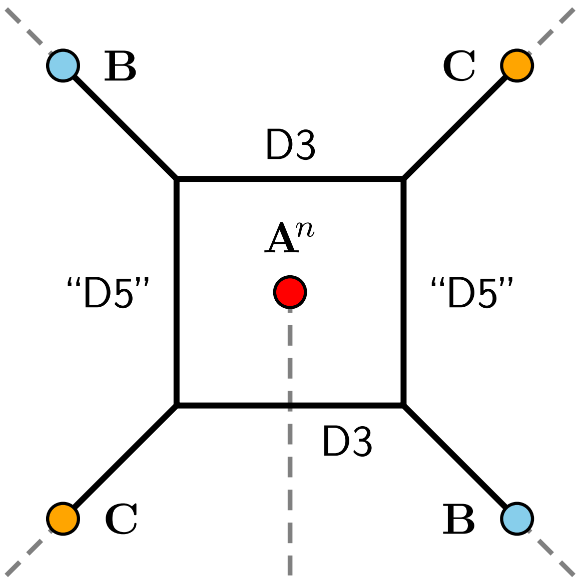


IIB	0	1	2	③	④	5	6	7	8	9
n NS5	–	–	–			–	–	–		
D3	–	–	–						–	
“D5”	–	–	–	–	–					–
$(1,1)_3$	–	–	–						angle	

We can see 3D $SU(2)$ gauge symmetry with n flavors on 012-directions.

There exits IR fixed point.

$\mathbf{ST}_{34}$-dualized system :



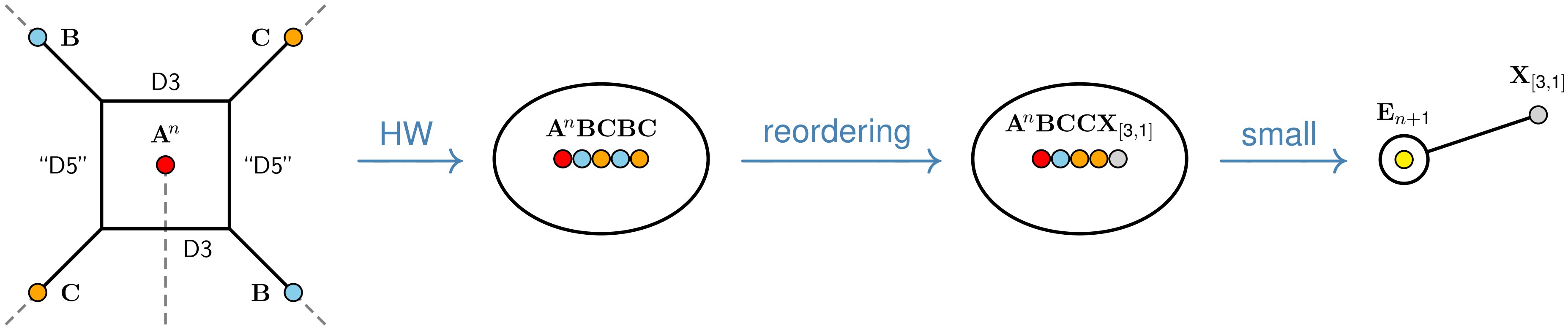
IIB	0	1	2	③	④	5	6	7	8	9
n NS5	—	—	—			—	—	—		
D3	—	—	—						—	
"D5"	—	—	—	—	—					—
$(1,1)_3$	—	—	—						angle	

$\mathbf{A}$-brane : $[1,0]^T_{s5}$-brane $=$ NS5

$\mathbf{B}$-brane : $[1,-1]^T_{s5}$-brane

$\mathbf{C}$-brane : $[1,1]^T_{s5}$-brane

3D $\mathcal{N} = 4$ $SU(2)$ gauge with n flavors $\rightarrow$ SCFT with E_{n+1} symmetry



3D gauge coupling is given by Dirac-Born-Infeld action of D3-brane :

$$\frac{1}{g_{\text{YM}}^2} \simeq \frac{L}{g_s}$$

Then the strong coupling limit is given by

$$L \rightarrow 0$$