Branes in extended spacetime

Yuho Sakatani

Kyoto Prefectural University of Medicine (KPUM),
Institute for Basic Sciences (IBS), KOREA

based on arXiv:1607.04265,
in collaboration with Shozo Uehara (KPUM)

"Strings and Fields 2016"   9 Aug. 2016.

Introduction (1/2)

There exist various branes in String / M-theory :

String theory:

- Wave
- F-string
- D*p*-branes
- NS5-brane
- KK monopole
- ……

M-theory:

- Wave
- M2-brane
- M5-brane
- KK monopole
- M9-brane
- ……

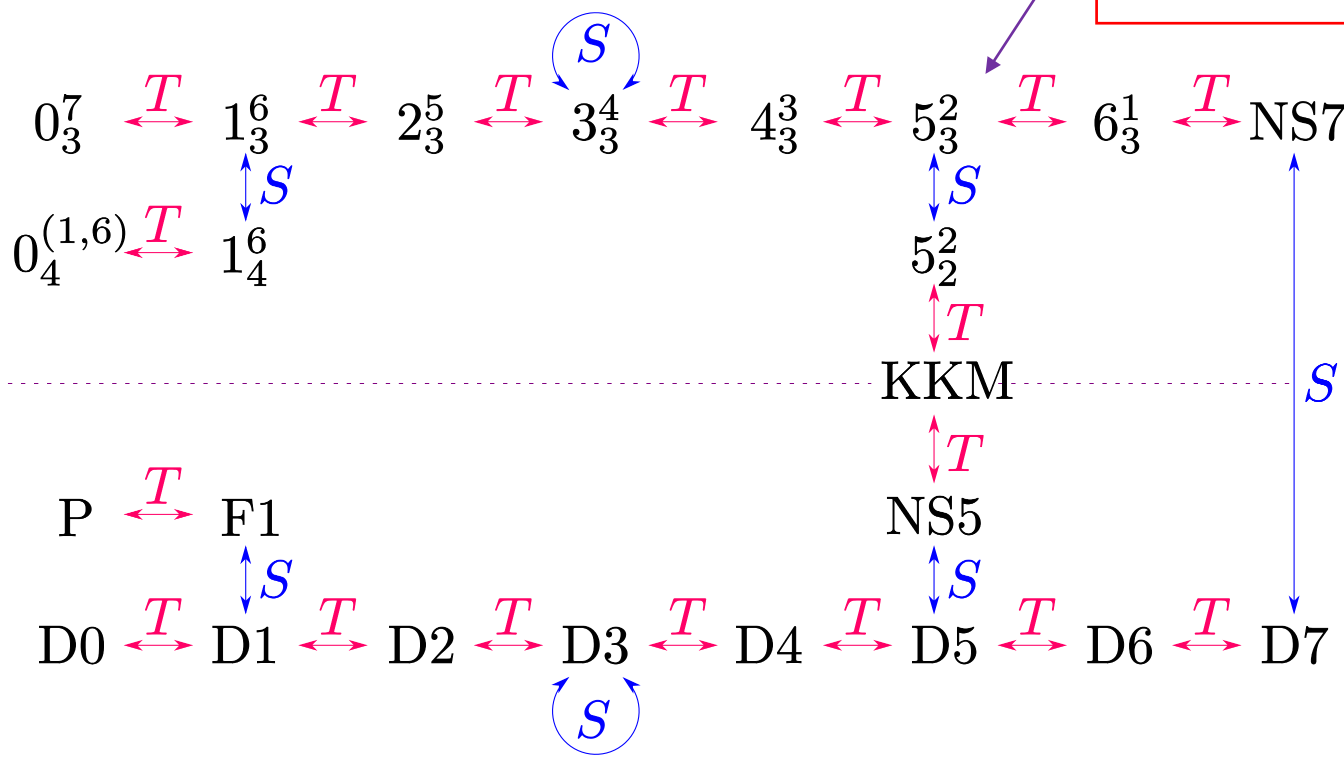
Introduction (2/2)

When String / M-theory is compactified on a torus, there is the U-duality symmetry.

Various branes are related by U-duality!

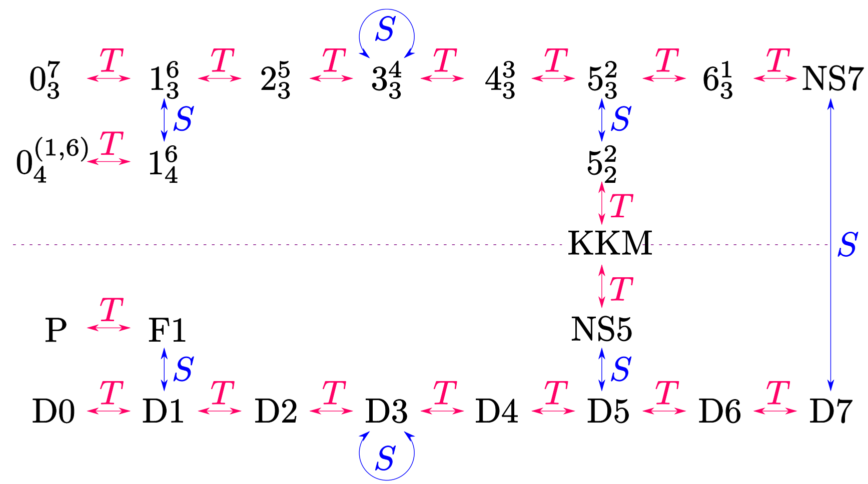
Exotic branes [Yata-san, Kimura-san's talk]

E.g. type II string on a 7-torus :



Motivation

We expect
all these branes
are a **single object**.



However, known actions for branes have **different forms**:

$$S_{\mathrm{F1}} = \frac{1}{2}\int_{\Sigma_2} G_{ij}\,\mathrm{d}X^i \wedge *\mathrm{d}X^j + \int_{\Sigma_2} B_2\,.$$

$$S_{\mathrm{D}p} = -\int_{\Sigma_{p+1}} \mathrm{d}^{p+1}\sigma\,\sqrt{-\det(G+B_2-F_2)} + \int_{\Sigma_{p+1}} \mathrm{e}^{B_2-F_2}\wedge C\,.$$

$$S_{\mathrm{KKM}} = -\int_{\Sigma} \mathrm{d}^6\sigma\;\mathrm{e}^{-2\phi}\;k^2\sqrt{-\det(G_{\mu\nu}D_\alpha X^\mu D_\beta X^\nu)} + \cdots\,.$$

We want to find **a single action** that reproduces **these**.

[∗ We consider only **bosonic** action for a **single brane**]

Main Result

* We have **not succeeded yet** in reproducing all actions from **a single action**.

Our proposed action for a *p*-brane has the form,

$$S = \frac{1}{p+1} \int_{\Sigma_{p+1}} \left(\boxed{\frac{1}{2} \mathcal{M}_{IJ} \mathcal{P}^I \wedge *\mathcal{P}^J} - \Omega_{p+1} \right) .$$

quadratic part is **common to all branes**
(U-duality inv.)

this part depends on the brane
~~(U-duality inv.)~~

Our approach is based on
the geometry in **Extended Field Theories**;
Double / **Exceptional Field Theory**.

Extended Field Theories
(brief sketch)

Extended Field Theories

We extend the spacetime dimensions in order to make the duality covariance manifest.

☆ **Double Field Theory** [Siegel '93; Hull, Zwiebach '09; ...]

- manifestly **T-duality covariant** formulation of supergravity

$$x^i \xrightarrow{\text{doubled}} (x^i,\, \tilde{x}_i)$$

$\Downarrow$ **generalization**

☆ **Exceptional Field Theory** [West '03; Hillmann '09; Berman, Perry '11; Hohm, Samtleben '13;...]

- manifestly **U-duality covariant** formulation of supergravity

$$x^i \xrightarrow{\text{much more dimensions}} (x^i,\, y_{ij},\, \cdots)$$

basic idea (1/4)

Analogy to the Kaluza-Klein theory

4 dim. Einstein-Maxwell theory : $\int d^4x \left[R(g) - \frac{1}{2} |dA_1|^2 \right].$

☆ **We extend the spacetime into 5dim. :** $(x^I) = (x^\mu, x^5).$

5 dim. generalized metric

4 dim

$$G_{IJ} = E^A{}_I \, E^B{}_K \, \eta_{AB} \,, \qquad (E^A{}_I) = \begin{pmatrix} \boxed{e^a_\mu} & 0 \\ e^5_5 \, A_\mu & e^5_5 \end{pmatrix}.$$

Metric ← metric + gauge field

$$\int d^5x \, R(G) \,. \quad \xrightarrow{\frac{\partial}{\partial x^5} = 0} \quad \int d^4x \left[R(g) - \frac{1}{2} |dA_1|^2 + \cdots \right].$$

5D gravitational action

☆ **5 dim. diffeo.** $\ni$ $\left\{ \begin{array}{l} \textbf{4 dim. diffeo.} \\ \textbf{Gauge sym. of } A_1 \end{array} \right.$

basic idea (2/4)

Double Field Theory

$$\int \mathrm{d}^d x\ \mathrm{e}^{2\phi}\left[R(G)+4|\mathrm{d}\phi|^2-\frac{1}{2}\,|\mathrm{d}B_2|^2\right].$$

---

doubled spacetime : $(x^I) = (x^i,\ \tilde{x}_i)$ **[Duff '89; Tseytlin '90; Siegel '93]**

(*2d* dim)

winding/dual coords.

generalized metric :

$$\mathcal{M}_{IJ} = E^A{}_I\, E^B{}_J\, \eta_{AB}\,, \qquad (E^A{}_I) = \begin{pmatrix} \boxed{e^a_i} & 0 \\ (e^{-1})^i_a\, B_{ij} & (e^{-1})^i_a \end{pmatrix}.$$

***d* dim**

$$\int \mathrm{d}^{2d} x\ \mathrm{e}^{-2d}\ \mathcal{R}(\mathcal{M})\,. \quad \xRightarrow{\frac{\partial}{\partial\tilde{x}_i}=0} \quad \int \mathrm{d}^d x\ \mathrm{e}^{2\phi}\left[R(G)+4|\mathrm{d}\phi|^2-\frac{1}{2}\,|\mathrm{d}B_2|^2\right].$$

[Siegel '93; Hohm, Hull, Zwiebach '10; Jeon, Lee, Park '10]

☆ ***2d* dim. generalized diffeo.** $\ni$

$$(V^I) = (v^i,\ \tilde{v}_i)$$

***d* dim. diffeo :** $x^i \to x^i + v^i$

Gauge sym. of B_2 **:**

$$B_2 \to B_2 + \mathrm{d}\tilde{v}$$

basic idea (3/4)

Exceptional Field Theory

$S_{\text{11d-SUGRA}}$ **(bosonic)**

$$\mathbb{R}^{11-d} \times T^d$$

$$(x^M) = (x^\mu,\, x^i)$$

external

internal extended

exceptional spacetime : $(\underline{x^\mu},\, \underbrace{x^i,\, y_{i_1 i_2},\, y_{i_1\cdots i_5},\, \cdots}_{(x^I)})$ [Duff, Lu '90; West '03]

	T^4	T^5	T^6	T^7	T^8
dimension x^I	10	16	27	56	248
duality group	SL(5)	SO(5,5)	E_6	E_7	E_8

generalized metric : [Duff, Lu '90; Berman, Perry '11; Berman, Godazgar, Perry, West '12; ....]

$$\mathcal{M}_{IJ} = E^A{}_I\, E^B{}_J\, \eta_{AB}\,, \qquad (E^A{}_I) = \begin{pmatrix} e_i^a & 0 & \cdots \\ \frac{1}{\sqrt{2}}(e^{-1})_a^{i_1 i_2}\, C_{i_1 i_2 j} & (e^{-1})_{a_1 a_2}^{i_1 i_2} & \cdots \\ \cdots C_{i_1\cdots i_5 j} \cdots & \cdots & \cdots \end{pmatrix}.$$

basic idea (4/4)

Exceptional Field Theory

$$\mathcal{L}_{\text{EFT}} \xrightarrow{\;\frac{\partial}{\partial y_{i_1 i_2}} = 0\,,\quad \frac{\partial}{\partial y_{i_1\cdots i_5}} = 0\;} S_{\text{11d-SUGRA}}$$

(manifestly U-duality cov.)

[Hohm, Samtleben '13]

2-derivative action
for $\mathcal{M}_{IJ}$ (and some additional fields)

★ **generalized diffeo.** $\ni$

$(V^I) = (v^i,\ \tilde{v}_{i_1 i_2}\ \tilde{v}_{i_1\cdots i_5},\ \cdots)$

$$\left\{\begin{array}{l} \textbf{\textit{d} dim. diffeo :}\quad x^i \to x^i + v^i \\ \textbf{Gauge sym. of } C_3 :\quad C_3 \to C_3 + \mathrm{d}\tilde{v}_2 \\ \qquad\qquad\qquad\quad C_6 :\quad C_6 \to C_6 + \mathrm{d}\tilde{v}_5 \end{array}\right.$$

Branes in extended spacetime

[arXiv:1607.04265, YS, Shozo Uehara]

General construction

Let us consider a certain Extended spacetime

☆ with gen. coords. $(x^I) = (x^i, y_M)$, gen. metric $\mathcal{M}_{IJ}(x)$

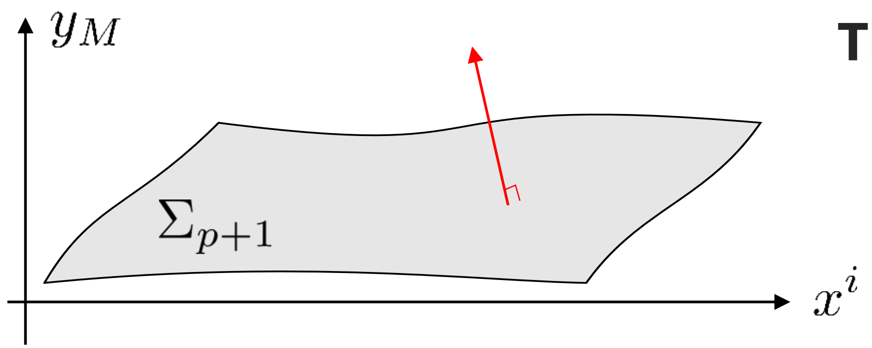
☆ We consider a (*p*+*1*)-dimensional worldvolume that has the **intrinsic metric** $\gamma_{\alpha\beta}(\sigma)$.

☆ $(x^I) = (x^i, y_M)$ ⟺ **1-form:** $\mathcal{P}^I(\sigma) = (\mathrm{d}X^i(\sigma), \mathcal{P}_M(\sigma))$

coords. on *d*-torus

***d* scalars** **auxiliary fields**

☆ worldvolume **gauge fields** : $\{A_q(\sigma)\}$



These describe the embedding of the *p*-brane into the Extended spacetime.

[Asakawa, Sasa, Watamura '12; Rey, YS '15]

Action (1/2)

Fields : $\boxed{\gamma_{\alpha\beta}(\sigma)\,,\ X^i(\sigma)\,,\ \mathcal{P}_M(\sigma)\,,\ \{A_q(\sigma)\}}$

$\{A_q(\sigma)\}$ only here

Action :

$$S = \frac{1}{p+1}\int_{\Sigma_{p+1}}\left(\boxed{\frac{1}{2}\mathcal{M}_{IJ}\,\mathcal{P}^I\wedge *\mathcal{P}^J} - \Omega_{p+1}\right).$$

only here $\gamma_{\alpha\beta}(\sigma)$

manifestly invariant under
- **duality transf.**
- **generalized diffeo.**

target diffeo.
+ gauge trsf. of *p*-form pot.

$\boxed{\Omega_{p+1}\ :\ \text{depends on the brane.}}$

***p*-brane associated with a winding coord.** $y_{i_1\cdots i_p}$

$$\mathrm{d}X^{i_1\cdots i_p} \equiv \frac{\mathrm{d}X^{i_1}\wedge\cdots\wedge \mathrm{d}X^{i_p}}{\sqrt{p!}}$$

$$\Omega_{p+1} \sim \mathcal{P}_{i_1\cdots i_p}\wedge \mathrm{d}X^{i_1\cdots i_p}$$ **(*p*+1)-form**

$$\Omega_{p+1} = \mathcal{P}_{i_1\cdots i_p}\wedge \mathrm{d}X^{i_1\cdots i_p} + \cdots$$

include gauge fields $\{A_q(\sigma)\}$ such that Ω_{p+1} is inv. under generalized diffeo.

Action (2/2)

Fields : $$\boxed{\gamma_{\alpha\beta}(\sigma)\,,\ X^i(\sigma)\,,\ \mathcal{P}_M(\sigma)\,,\ \{A_q(\sigma)\}}$$

Action : $$S = \frac{1}{p+1}\int_{\Sigma_{p+1}}\left(\boxed{\frac{1}{2}\mathcal{M}_{IJ}\,\mathcal{P}^I\wedge *\mathcal{P}^J} - \boxed{\Omega_{p+1}}\right).$$

manifestly invariant under
- duality transf.
- generalized diffeo.

invariant under
- ~~duality transf.~~
- generalized diffeo.

Duality non-invariance is reasonable :

E.g. Ω_{2+1} 2-brane **generic duality trsf.** $\Rightarrow$ Ω_{5+1} a certain brane, such as a 5-brane

Ω_{p+1} should transform covariantly under duality trsf.
(although, so far, the covariance is not clear to me).

Applications

$$S=\frac{1}{p+1}\int_{\Sigma_{p+1}}\left(\frac{1}{2}\,\mathcal{M}_{IJ}\,\mathcal{P}^I\wedge *\mathcal{P}^J-\Omega_{p+1}\right).$$

doubled spacetime:

$p=1$

standard string action (skip)

action for an exotic brane (skip)

$$S_{\text{WZ}}\sim\int\beta^{ij}\,\mathrm{d}\tilde{X}_i\wedge\mathrm{d}\tilde{X}_j\,.$$

$p=2$

E_6 exceptional spacetime (27 dim):

$p=5$

(non-)standard membrane action

non-standard M5-brane action

E_6 exceptional spacetime

generalized coords.

$$(x^I) = (x^i,\, y_{i_1 i_2},\, y_{i_1\cdots i_5})\,. \qquad \Longleftrightarrow \qquad (\mathcal{P}^I) = (\mathrm{d}X^i,\, \boxed{\mathcal{P}_{i_1 i_2}},\, \boxed{\mathcal{P}_{i_1\cdots i_5}})\,.$$

27 dims. = 6 + 15 + 6 **6**

21 auxiliary fields

generalized metric : $\mathcal{M}_{IJ} = \hat{\mathcal{M}}_{KL}\, L^K{}_I\, L^L{}_J$

$$(\hat{\mathcal{M}}_{IJ}) \equiv \begin{pmatrix} G_{ij} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & G^{i_1 i_2, j_1 j_2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & G^{i_1\cdots i_5, j_1\cdots j_5} \end{pmatrix},$$

$$(L^I{}_J) \equiv \begin{pmatrix} \delta^i_j & \mathbf{0} & \mathbf{0} \\ \frac{1}{\sqrt{2}}\, C_{i_1 i_2 j} & \delta^{j_1 j_2}_{i_1 i_2} & \mathbf{0} \\ -\frac{1}{\sqrt{5!}}\,(C_{i_1\cdots i_5 j} - 5\, C_{[i_1 i_2 i_3} C_{i_4 i_5]j}) & \frac{10\sqrt{2}}{\sqrt{5!}}\, \delta^{j_1 j_2}_{[i_1 i_2} C_{i_3 i_4 i_5]} & \delta^{j_1\cdots j_5}_{i_1\cdots i_5} \end{pmatrix}.$$

$$\delta^{j_1\cdots j_q}_{i_1\cdots i_q} \equiv \delta^{j_1}_{[i_1} \cdots \delta^{j_q}_{i_q]}\,, \qquad G^{i_1\cdots i_q, j_1\cdots j_q} \equiv \delta^{i_1\cdots i_q}_{k_1\cdots k_q}\, G^{k_1 j_1} \cdots G^{k_q j_q}\,.$$

$$\Rightarrow \begin{cases} \Omega_3 \equiv \mathcal{P}_{i_1 i_2} \wedge \mathrm{d}X^{i_1 i_2} + 3\, F_3\,, \\ \Omega_6 \equiv \mathcal{P}_{i_1\cdots i_5} \wedge \mathrm{d}X^{i_1\cdots i_5} + \mathcal{P}_{i_1 i_2} \wedge \mathrm{d}X^{i_1 i_2} \wedge F_3 + 6\, F_6\,. \end{cases}$$

(gauge inv. combinations)

membrane case (1/2)

Our action for a membrane :

$$S = \frac{1}{3}\int_\Sigma \left(\frac{1}{2}\mathcal{M}_{IJ}\,\mathcal{P}^I \wedge *\mathcal{P}^J - \Omega_3 \right) . \qquad \Omega_3 \equiv \mathcal{P}_{i_1 i_2} \wedge \mathrm{d}X^{i_1 i_2} + 3\,F_3 \, .$$

$$F_3 = \mathrm{d}A_2$$

eliminate the auxiliary fields $\mathcal{P}_{i_1 i_2}, \mathcal{P}_{i_1 \cdots i_5}$.

$$S = \frac{1}{6}\int_\Sigma G_{ij}\,\mathrm{d}X^i \wedge *\mathrm{d}X^j - \frac{1}{6}\int_\Sigma G_{i_1 i_2, j_1 j_2}\,\mathrm{d}X^{i_1 i_2} \wedge *\mathrm{d}X^{j_1 j_2} + \int_\Sigma \left(C_3 - F_3\right) .$$

membrane case (2/2)

$$S = \frac{1}{6}\int_\Sigma G_{ij}\,\mathrm{d}X^i \wedge *\mathrm{d}X^j - \frac{1}{6}\int_\Sigma G_{i_1 i_2, j_1 j_2}\,\mathrm{d}X^{i_1 i_2} \wedge *\mathrm{d}X^{j_1 j_2} + \int_\Sigma (C_3 - F_3)\,.$$

apparently different

membrane action [Bergshoeff, Sezgin, Townsend '87]

$$S = \frac{1}{2}\int_\Sigma \left(G_{ij}\,\mathrm{d}X^i \wedge *\mathrm{d}X^j - *1\right) + \int_\Sigma C_3\,.$$

e.o.m. for the intrinsic metric $\gamma_{\alpha\beta}$,

$$h_{\alpha\beta} \equiv G_{ij}\,\partial_\alpha X^i\,\partial_\beta X^j = \frac{\det h}{\det \gamma}\,(\gamma h^{-1}\gamma)_{\alpha\beta} \Rightarrow \gamma_{\alpha\beta} = h_{\alpha\beta}\,.$$

eliminate $\gamma_{\alpha\beta}$

$$\boxed{S = -\int_\Sigma \mathrm{d}^3\sigma\sqrt{-\det h} + \int_\Sigma C_3 - \int_{\partial\Sigma} A_2\,.}$$

5-brane case

Our action for a 5-brane :

$$S=\frac{1}{6}\int_\Sigma\Bigl(\frac{1}{2}\,\mathcal{M}_{IJ}\,\mathcal{P}^I\wedge *\mathcal{P}^J-\Omega_6\Bigr)\,.$$

$$\Omega_6=\mathcal{P}_{i_1\cdots i_5}\wedge \mathrm{d}X^{i_1\cdots i_5}+\mathcal{P}_{ij}\wedge\mathrm{d}X^{ij}\wedge F_3+6\,F_6\,.$$

$F_3=\mathrm{d}A_2$ $\quad$ $F_6=\mathrm{d}A_5$

eliminate the auxiliary fields $\mathcal{P}_{i_1 i_2},\,\mathcal{P}_{i_1\cdots i_5}\,.$

$$S=-\frac{1}{12}\int_\Sigma \mathrm{d}^6\sigma\Bigl[\sqrt{-\gamma}\,\gamma^{\alpha\beta}\,h_{\alpha\beta}-\frac{\det h}{\sqrt{-\gamma}}\,\theta^\alpha{}_\beta\,(h^{-1}\gamma)^\beta{}_\alpha\Bigr]$$
$$+\int_\Sigma\Bigl(C_6-\frac{1}{2}\,H_3\wedge C_3-F_6\Bigr)\,,$$

3-form potential in 11d SUGRA

$$h_{\alpha\beta}\equiv G_{ij}\,\partial_\alpha X^i\,\partial_\beta X^j\,,\qquad H_3\equiv F_3-C_3\,,$$
$$\theta^\alpha{}_\beta\equiv\Bigl[1+\frac{\mathrm{tr}(H^2)}{6}\Bigr]\,\delta^\alpha_\beta-\frac{1}{2}\,(H^2)^\alpha{}_\beta\,.$$

5-brane case (linear)

eliminate $\gamma_{\alpha\beta}$ $\quad (\gamma_{\alpha\beta} \neq h_{\alpha\beta} \text{ in this case})$

$$S = -\int_\Sigma \mathrm{d}^6\sigma\sqrt{-h}\,\frac{\mathrm{tr}(\theta^{\frac{1}{2}})}{6} + \int_\Sigma \Big(C_6 - \frac{1}{2}H_3\wedge C_3 - F_6\Big)\,.$$

(not a known action)

In the weak-field approximation for H_3 **,**

$$S \sim -\int_\Sigma \mathrm{d}^6\sigma\sqrt{-h} + \frac{1}{4}\int_\Sigma H_3\wedge *_h H_3 + \int_\Sigma \Big(C_6 - \frac{1}{2}H_3\wedge C_3 - F_6\Big)\,.$$

[Bergshoeff, de Roo, Ortin '96]

e.o.m. for A_2 $\Rightarrow$ $\mathrm{d}(*_h H_3 - C_3) = \mathrm{d}(*_h H_3 + H_3) = 0\,.$

consistent with the linearized self-duality relation:

$$H_3 = -\,*_h\,H_3\,.$$

5-brane case (non-linear)

At the non-linear level,
e.o.m. for the gauge field A_2 becomes

$$\partial_\alpha \mathcal{E}^{\alpha\beta\gamma} = 0\,, \quad \mathcal{E}^{\alpha\beta\gamma} \equiv \frac{\partial \mathcal{L}}{\partial H_{\alpha\beta\gamma}}\,.$$

$$S = \int_\Sigma \mathrm{d}^6\sigma \mathcal{L} = \int_\Sigma \left[-\mathrm{d}^6\sigma \sqrt{-h}\, \frac{\mathrm{tr}(\theta^{\frac{1}{2}})}{6} + C_6 - \frac{1}{2} H_3 \wedge C_3 - F_6 \right].$$

$$\mathcal{E}^{\alpha\beta\gamma} \equiv \frac{\partial \mathcal{L}}{\partial H_{\alpha\beta\gamma}} = -\frac{1}{12} \left[\mathcal{C}^{[\alpha}{}_{\delta}\, H^{\beta\gamma]\delta} - (*_h C_3)^{\alpha\beta\gamma} \right].$$

$$\mathcal{C}_\alpha{}^\beta \equiv \frac{\mathrm{tr}(\theta^{-\frac{1}{2}})}{3}\, \delta_\alpha^\beta - (\theta^{-\frac{1}{2}})_\alpha{}^\beta\,.$$

consistent with the non-linear self-duality relation:

$$\mathcal{C}_{[\alpha_1}{}^{\alpha}\, H_{\alpha_2\alpha_3]\alpha} = -(*_h H_3)_{\alpha_1\alpha_2\alpha_3}\,.$$

$\delta_{\alpha_1}^{\alpha}$ **weak field**

5-brane case (known result)

Our result: $\mathcal{C}_{[\alpha_1}{}^{\alpha} H_{\alpha_2\alpha_3]\alpha} = -(*_h H_3)_{\alpha_1\alpha_2\alpha_3}\,.$

$$\mathcal{C}_{\alpha}{}^{\beta} \equiv \frac{\operatorname{tr}(\theta^{-\frac{1}{2}})}{3}\,\delta_{\alpha}^{\beta} - (\theta^{-\frac{1}{2}})_{\alpha}{}^{\beta}\,.$$

Known result: [Howe, Sezgin '97; Howe, Sezgin, West '97; Sezgin, Sundell '98]

$$C_{[\alpha_1}{}^{\alpha} H_{\alpha_2\alpha_3]\alpha} = -(*_h H_3)_{\alpha_1\alpha_2\alpha_3}$$

$$C_{\alpha}{}^{\beta} = K^{-1}\left\{\left[1+\frac{1}{12}\operatorname{tr}(H^2)\right]\delta_{\alpha}^{\beta} - \frac{1}{4}(H^2)_{\alpha}{}^{\beta}\right\}, \quad K \equiv \sqrt{1+\frac{\operatorname{tr}(H^2)}{24}}\,.$$

From the non-linear self-duality relation, we can show $C_{\alpha}{}^{\beta} = \mathcal{C}_{\alpha}{}^{\beta}\,.$ ⇨ **Consistent!**

Summary

- **We proposed a simple action,**

$$S = \frac{1}{p+1}\int_\Sigma \left(\frac{1}{2}\mathcal{M}_{IJ}\,\mathcal{P}^I \wedge *\mathcal{P}^J - \Omega_{p+1}\right).$$

- **In the doubled spacetime, the action reproduces the conventional string sigma model action.**

- **In the exceptional spacetime (for E_6 EFT), we considered the following cases:**

  $\Omega_3 \Rightarrow$ **membrane action (not conventional but equivalent)**

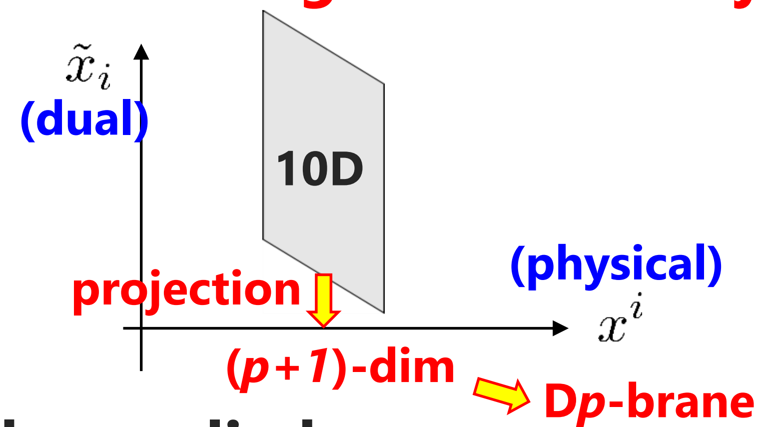
  $\Omega_6 \Rightarrow$ **M5-brane action (at least at the linearized level)**

  It will be equivalent even at the non-linear level.

- **We can also consider actions for exotic branes.**

A goal of this project

According to [Hull '05; Asakawa, Sasa, Watamura '12],
D*p*-brane can be interpreted as a single 10-dim. object
in the doubled spacetime:



I expect a similar idea can be applied
to all branes that are connected by the U-duality chain.
Namely, all branes will be a single 10?/11?-dim. object
in the exceptional spacetime, and the action is given by

$$S \sim \int_{\Sigma_{10?11?}} \left(\frac{1}{2}\,\mathcal{M}_{IJ}\,\mathcal{P}^I \wedge *\mathcal{P}^J - \Omega_{10?11?}\right).$$

projection

(p+1)-dim $\Longrightarrow$ p-brane

I want to find this Ω