Deformations of the Almheiri–Polchinski model

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[**Hideki Kyono, S.O. , Kentaroh Yoshida**]

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Motivation for Low Dimension Models

One of the most important issues of the String theory

is to understand **the Holographic Principle.** **(AdS / CFT)**

There is no proof of this conjecture.

It is difficult to conform it at Quantum Level .

We need a simple toy model of Quantum Holography.

AdS2 / CFT1

0. Introduction

Models of AdS2 / CFT1 correspondence

AdS2 : **2D dilaton gravity**

Ex. **Almheiri - Polchinski model**

[Almheili, Polchinski, “14]

[Jackiw, “85] [Teitelboin, “83]

It has a black hole as a vacuum solution.

Analytical solutions with matters

CFT1 : **Quantum Mechanics**

Ex. **Sachdev-Ye-Kitaev model**

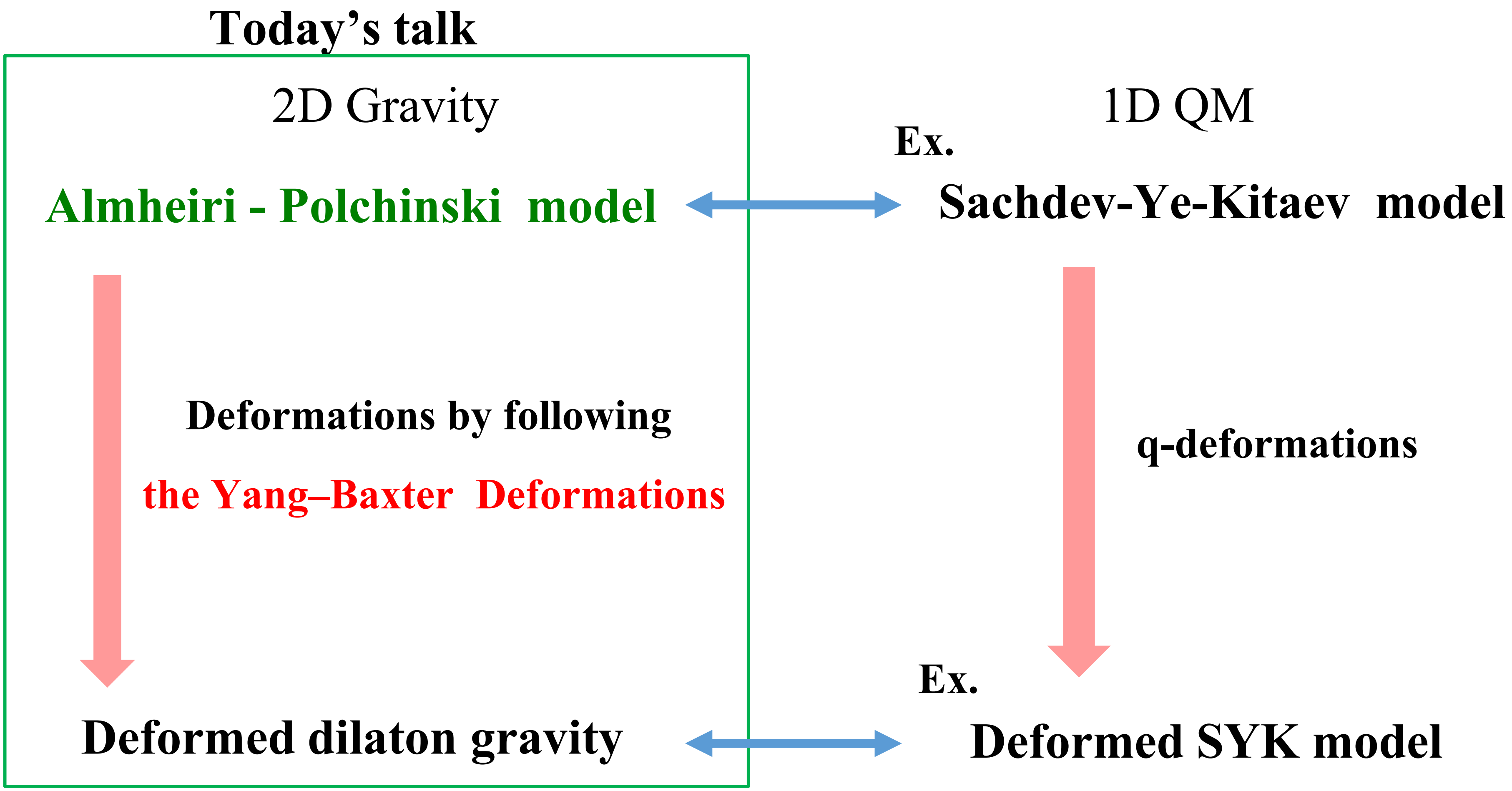
[Sachdev, Ye, “93] [Kitaev, “15]

Many-Body system of Fermions

↔

correspondence?

Motivation to study the Deformed Model



Today's talk

2D Gravity

1D QM

Almheiri - Polchinski model

Ex.

Sachdev-Ye-Kitaev model

Deformations by following

the Yang–Baxter Deformations

q-deformations

Deformed dilaton gravity

Ex.

Deformed SYK model

New Description of the Duality

Contents of This Talk

The Almheiri-Polchinski model

2D dilaton gravity model with a certain dilaton potential

$$S_\Phi = \frac{1}{16\pi G}\int d^2x\,\sqrt{-g}\,\left(\Phi^2 R - (2 - 2\Phi^2)\right)$$

(vacuum case)

EOM

$$\left\{\begin{array}{l} \boxed{4\partial_+\partial_-\omega + \mathrm{e}^{2\omega} = 0} \longleftarrow \text{Liouville equation} \\ 2\partial_+\partial_-\Phi^2 + \mathrm{e}^{2\omega}(\Phi^2 - 1) = 0 \\ -\mathrm{e}^{2\omega}\partial_\pm(\mathrm{e}^{-2\omega}\partial_\pm\Phi^2) = 0 \end{array}\right.$$

$\vdots$

We can obtain the general solutions.

AdS2

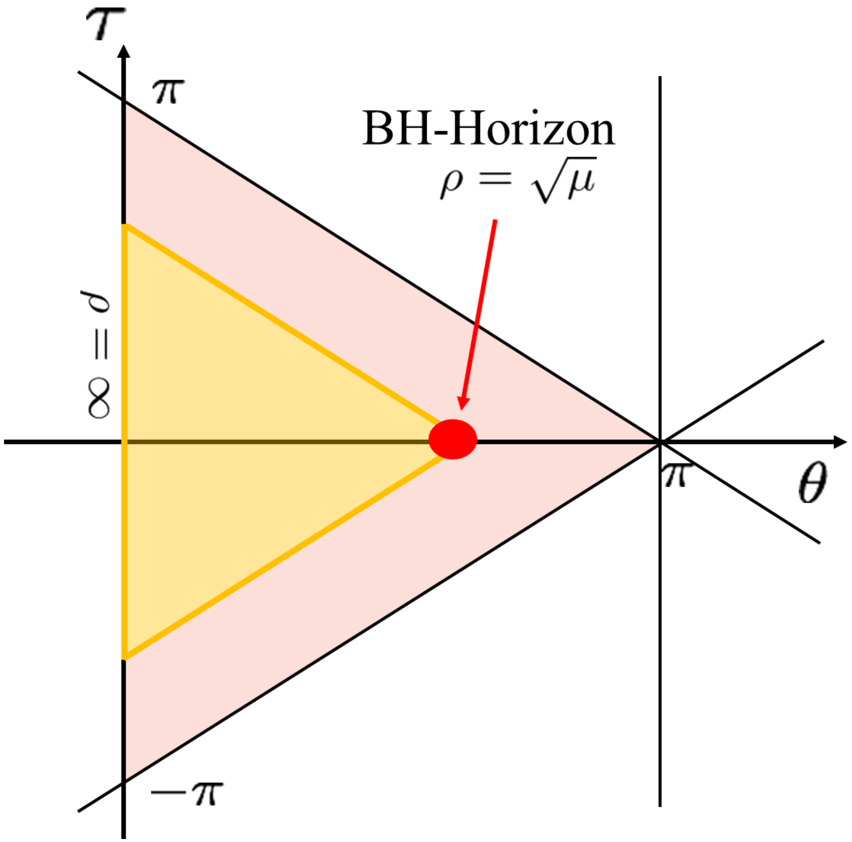
Conformal Gauge

$$ds^2 = -e^{2\omega}dx^+dx^- \qquad x^\pm \equiv t \pm z$$

Black Hole Solution

$$ds^2 = -4(\rho^2-\mu)dt^2 + \frac{d\rho^2}{\rho^2-\mu}$$

$$\Phi^2 = 1+\rho$$



Hawking Temperature

$$T_{\mathrm{H}} = \frac{1}{4\pi}\partial_\rho \sqrt{\frac{-g_{tt}}{g_{\rho\rho}}}\Bigg|_{\rho=\sqrt{\mu}} = \frac{\sqrt{\mu}}{\pi}$$

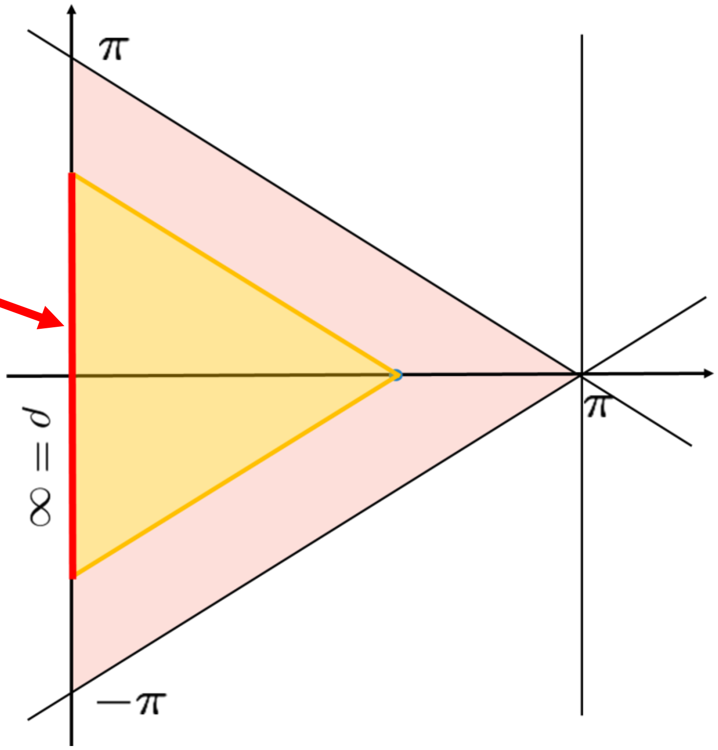
Bekenstein-Hawking Entropy

: Horizon Entropy

$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{eff}}} = \boxed{\frac{1+\pi T_{\mathrm{H}}}{4G}}$$

$$\begin{cases} A = 1 & \text{Area of the Horizon} \\ \dfrac{1}{G_{\mathrm{eff}}} = \dfrac{\Phi^2}{G} & \end{cases}$$

Entropy Evaluated on the Boundary



On–shell Action

$$S_{total} \equiv \underbrace{S_{\Phi}}_{\text{Bulk action}} + \underbrace{S_{\rm GH} + S_{\rm ct}}_{\text{Boundary terms}}$$

To cancel the divergence,

a counterterm is needed.

$$S_{\rm ct} = \frac{1}{8\pi G}\int dt\,\sqrt{-\gamma_{tt}}\,(1-\Phi^2)$$

The Boundary Stress Tensor

$$\langle \hat{T}_{tt} \rangle \equiv \frac{-2}{\sqrt{-\hat{\gamma}_{tt}}}\frac{\delta S_{total}}{\delta \hat{\gamma}^{tt}}$$

$$\xrightarrow{\qquad}$$

$$\langle \hat{T}_{tt} \rangle \sim E = \int \frac{{\rm d}S}{T_{\rm H}}$$

Entropy

$$\boxed{S = \frac{\pi T_{\rm H}}{4G} + S_{T_{\rm H}=0}}$$

BH entropy is reproduced.

Contents of This Talk

Deformed Almheiri-Polchinski Model

We consider a new dilaton gravity model :

$$S_\Phi = \frac{1}{16\pi G}\int \mathrm{d}^2x\sqrt{-g}\left[\Phi^2 R + \frac{1}{\eta}\sinh\left(2\eta(\Phi^2-1)\right)\right]$$

$$\xrightarrow{\eta \to 0} \quad 2\Phi^2 - 2$$

The AP model

By introducing new variables:

$$\mathrm{d}s^2 = \mathrm{e}^{2\omega}\tilde{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$$

$$\omega_1 \equiv \omega + \eta\Phi^2 \qquad \omega_2 \equiv \omega - \eta\Phi^2$$

the action becomes the sum of two Liouville systems :

$$S_\Phi = \frac{1}{32\pi G\eta}\int \mathrm{d}^2x\sqrt{-\tilde{g}}\left[\left(\omega_1\tilde{R} + (\tilde{\nabla}\omega_1)^2 + \mathrm{e}^{2\omega_1}\right) - \left(\omega_2\tilde{R} + (\tilde{\nabla}\omega_2)^2 + \mathrm{e}^{2\omega_2}\right)\right]$$

General solutions of the Deformed Model

Deformed Metric

$$ds^2 = \frac{1-\eta^2(\beta^2+4\alpha\gamma)}{z^2-\eta^2\left(\alpha+\beta t+\gamma(-t^2+z^2)\right)^2}(-dt^2+dz^2)$$

This metric is the same as

a Yang-Baxter deformation of AdS2.

A class of integrable deformations

Deformed Dilaton

$$\Phi^2 = 1+\frac{1}{2\eta}\log\left|\frac{z+\eta\left(\alpha+\beta t+\gamma(-t^2+z^2)\right)}{z-\eta\left(\alpha+\beta t+\gamma(-t^2+z^2)\right)}\right|$$

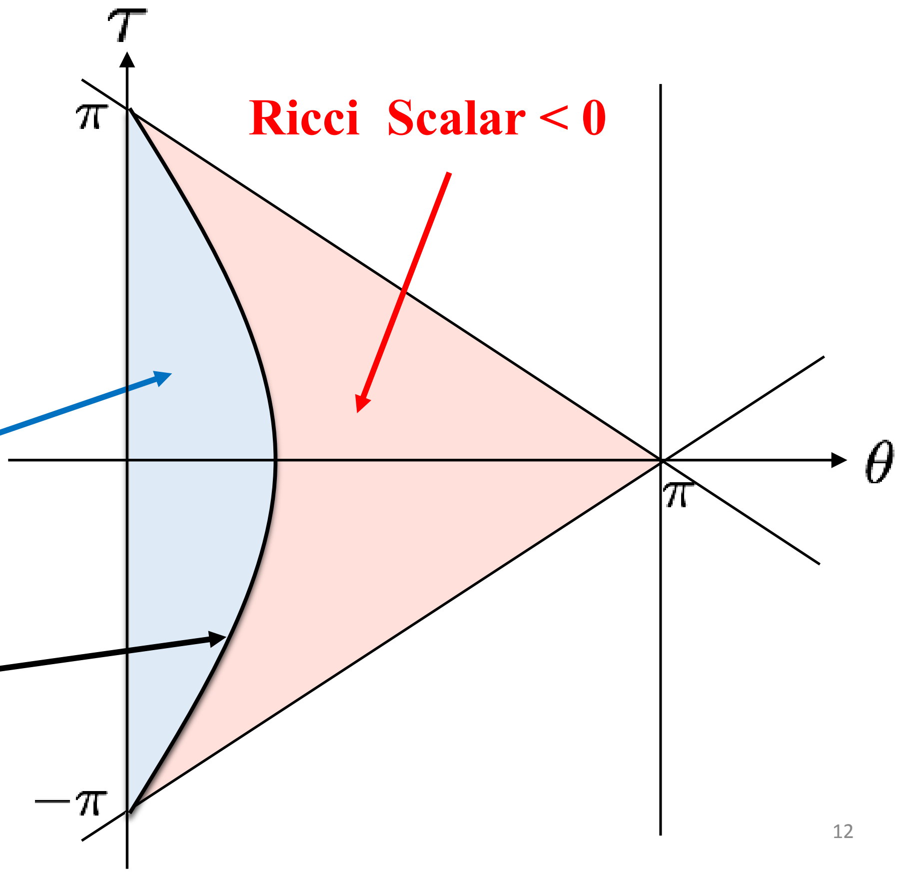
Geometry of the deformed AdS2

Ricci Scalar $$R = -2\,\frac{z^2 + \eta^2\left(\alpha + \beta t + \gamma(-t^2 + z^2)\right)^2}{z^2 - \eta^2\left(\alpha + \beta t + \gamma(-t^2 + z^2)\right)^2}$$

Ex.

$$\alpha = 1, \beta = 0, \gamma = 0$$

$$\boxed{\mathrm{d}s^2 = \frac{1}{z^2 - \eta^2}(-\mathrm{d}t^2 + \mathrm{d}z^2)}$$



Ricci Scalar < 0

Ricci Scalar > 0

Curvature Singularity

Black Hole Solution

By choosing the parameters as

$$\alpha = \frac{1}{2} \quad \beta = 0 \quad \gamma = \frac{\mu}{2}$$

we derive out a deformed Black Hole Solution.

$$\mathrm{d}s^2 = -4F(r)\,\mathrm{d}T^2 + \frac{\mathrm{d}r^2}{F(r)}$$

$$\Phi^2 = 1 + r$$

$$F(r) \equiv \frac{-1 - \eta^2\mu + (1 - \eta^2\mu)\cosh(2\,\eta\,r)}{2\eta^2}$$

$$\Big\downarrow\ \eta \to 0$$

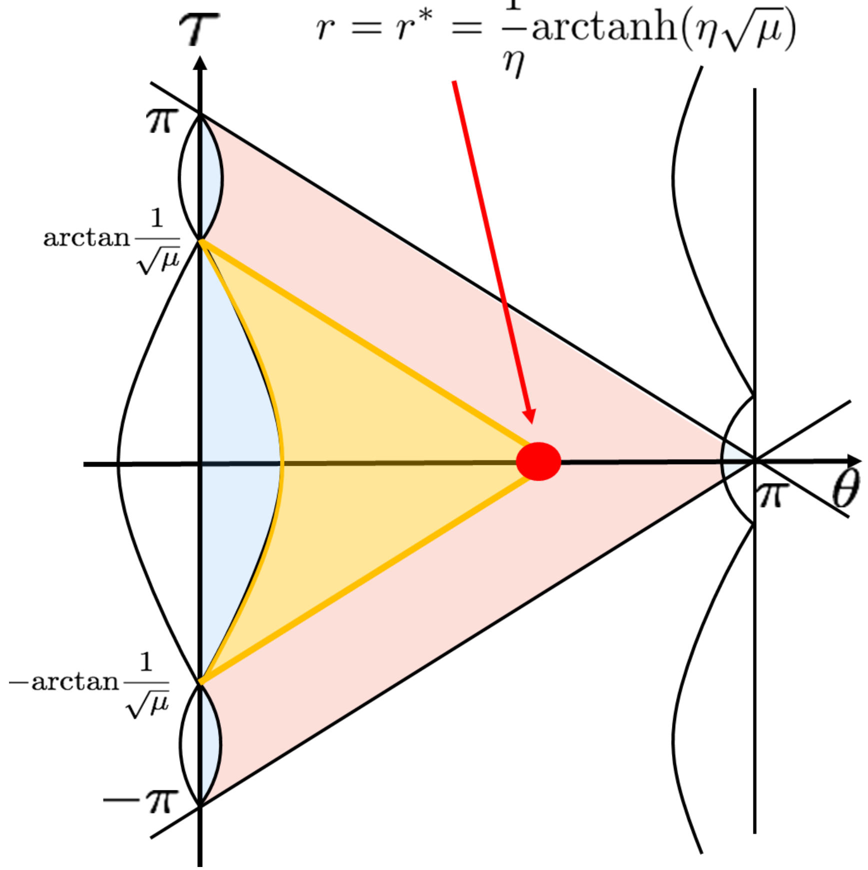
The AP model

$$\mathrm{d}s^2 = -4(\rho^2 - \mu)\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{\rho^2 - \mu}$$

$$\Phi^2 = 1 + \rho$$

BH-Horizon

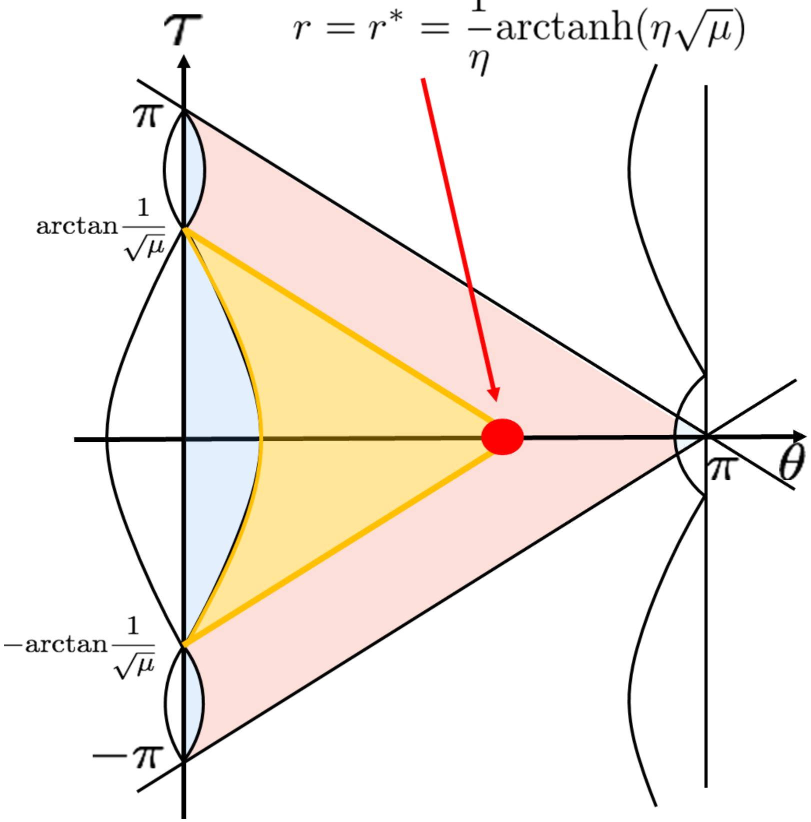
$$r = r^* = \frac{1}{\eta}\mathrm{arctanh}(\eta\sqrt{\mu})$$



Bekenstein-Hawking Entropy

$$ds^2 = -4F(r)\,dT^2 + \frac{dr^2}{F(r)}$$

$$\Phi^2 = 1 + r$$



BH-Horizon

$$r = r^* = \frac{1}{\eta}\mathrm{arctanh}(\eta\sqrt{\mu})$$

Hawking Temperature

$$T_{\mathrm{H}} = \frac{1}{4\pi}\partial_r\sqrt{-\frac{g_{tt}}{g_{rr}}}\bigg|_{r=r^*} = \frac{\sqrt{\mu}}{\pi}$$

Bekenstein-Hawking Entropy

: Horizon Entropy

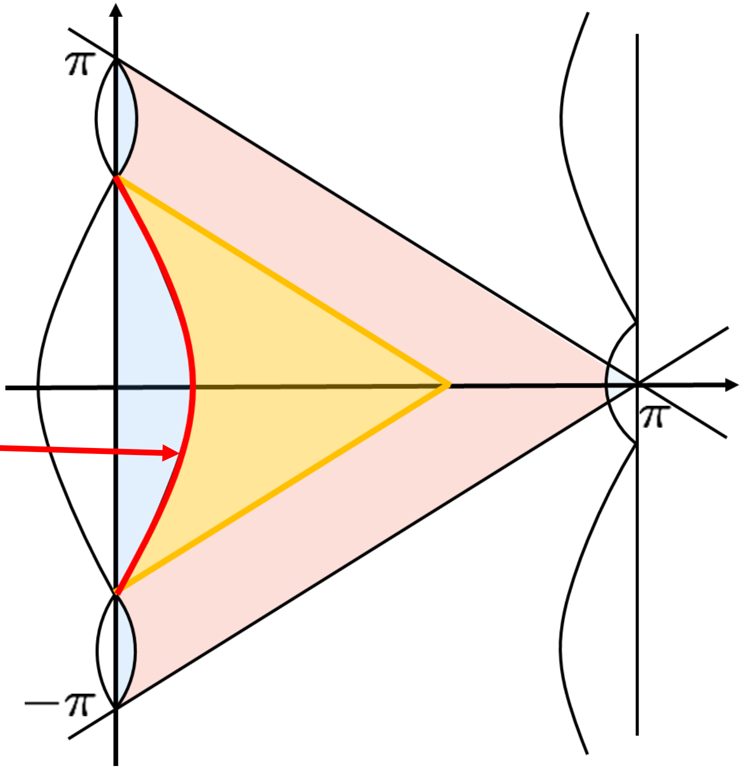
$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{eff}}}$$

$$\boxed{= \frac{\mathrm{arctanh}(\pi T_{\mathrm{H}}\,\eta)}{4G\eta} + \frac{1}{4G}} \xrightarrow{\eta \to 0}$$

The AP model

$$S_{\mathrm{BH}} = \frac{1 + \pi T_{\mathrm{H}}}{4G}$$

Entropy Evaluated on the Holographic Screen



Our proposals

1. The singularity surface is the Holographic screen.

2. An appropriate Counterterm

$$S_{\mathrm{ct}} = -\frac{1}{8\pi G}\int dt\,\sqrt{-\gamma_{tt}}\,\sqrt{F(\Phi^2-1)-\frac{1}{\eta^2}\log(1-\eta^2\mu)}$$

Entropy

$$\boxed{S = \frac{\mathrm{arctanh}(\pi T_{\mathrm{H}}\eta)}{4G\eta} + S_{T_{\mathrm{H}}=0}}$$

BH entropy is reproduced.

Conclusion

1. We discuss the deformed AP model motivated by the Yang–Baxter deformations.
2. We obtain the Deformed Black Hole solution.
3. Bekenstein-Hawking Entropy is reproduced by the boundary stress tensor with an appropriate counterterm.

Open problems

Interpretation of the counterterm?

What is the boundary dual theory of the deformed model?

(q-deformed SYK model?)