S-matrix Unitarity in Higher Derivative Gravity with Matter

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Discuss "unitarity" of R^2 gravity and get some important suggestions about gravity theory

1 Motivation: Llewellyn Smith's conjecture

He suggests that there is an intimate connection between the high energy behavior of amplitudes and renormalizability.

The high energy behavior of amplitudes can be checked by the unitarity limit (= unitarity bound).

	Unitarity bound	UV renormalizability
QED	fulfilled	renormalizable
Yang-Mills theory	fulfilled	renormalizable
Weinberg-Salam model	fulfilled	renormalizable
4-Fermi theory	unfulfilled	non renormalizable
Massive vector theory	unfulfilled	non renormalizable

2 Question : Matter scattering and unitarity in R^2 gravity (PTEP 2018 no.3, 031E01)

A higher derivative gravity theory (R^2 gravity theory) is known to be a renormalizable theory.

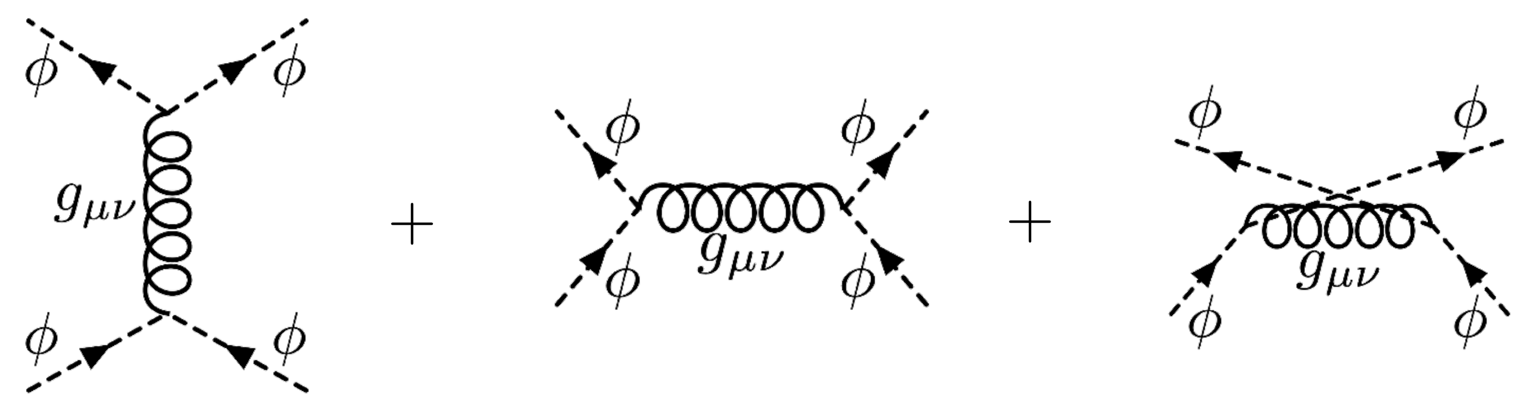
We are interested in the following two things:

What about unitarity in a higher derivative gravity theory?,

Is Llewellyn Smith's conjecture correct in a higher derivative gravity theory?

	Unitarity bound	UV renormalizability
R^2 gravity	?	renormalizable

We calculate *s*-, *t*-, *u*-channel graviton exchanges in ϕ-ϕ scattering.



Higher derivative gravity theory (R^2 gravity theory)

$$S=\int d^4x\sqrt{-g}\left(-\frac{1}{\kappa^2}R+\alpha R^2+\beta R_{\mu\nu}^2+\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-\frac{1}{2}m^2\phi^2-\frac{1}{4!}\lambda\phi^4\right)$$

・graviton propagator

$$G_{\mu\nu,\alpha\beta}=\frac{2}{\beta p^4+\kappa^{-2}p^2}P^{(2)}_{\mu\nu,\alpha\beta}+\frac{1}{2(3\alpha+\beta)p^4-\kappa^{-2}p^2}P^{(0)}_{\mu\nu,\alpha\beta},$$

$$P^{(2)}_{\mu\nu,\alpha\beta}\equiv\frac{1}{2}\left(\theta_{\mu\alpha}\theta_{\beta\nu}+\theta_{\mu\beta}\theta_{\alpha\nu}\right)-\frac{1}{3}\theta_{\mu\nu}\theta_{\alpha\beta},$$

$$P^{(0)}_{\mu\nu,\alpha\beta}\equiv\frac{1}{3}\theta_{\mu\nu}\theta_{\alpha\beta}$$

where $\theta_{\mu\nu}\equiv\eta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}$

・scattering amplitude (tree graphs : contact and *s*-, *t*-, *u*-channels)

$$\begin{aligned}M(\phi\phi\to\phi\phi)=-\lambda&+\frac{1}{(\beta s^2+\kappa^{-2}s)}\left(2tu-\frac{1}{3}(s-4m^2)^2\right)-\frac{1}{3\left(2(3\alpha+\beta)s^2-\kappa^{-2}s\right)}(s+2m^2)^2\\&+\frac{1}{(\beta t^2+\kappa^{-2}t)}\left(2us-\frac{1}{3}(t-4m^2)^2\right)-\frac{1}{3\left(2(3\alpha+\beta)t^2-\kappa^{-2}t\right)}(t+2m^2)^2\\&+\frac{1}{(\beta u^2+\kappa^{-2}u)}\left(2st-\frac{1}{3}(u-4m^2)^2\right)-\frac{1}{3\left(2(3\alpha+\beta)u^2-\kappa^{-2}u\right)}(u+2m^2)^2\end{aligned}$$

・scattering amplitude in high-energy limit

$$\begin{aligned}M(\phi\phi\to\phi\phi)&\simeq-\lambda+\frac{2}{\beta}\left(\frac{tu}{s^2}+\frac{us}{t^2}+\frac{st}{u^2}\right)-\frac{3(2\alpha+\beta)}{2\beta(3\alpha+\beta)}\\&=-\lambda+\frac{2}{\beta}g(\cos\theta)-\frac{3(2\alpha+\beta)}{2\beta(3\alpha+\beta)}=\mathcal{O}(s^0)\end{aligned}$$

where $g(x)=\frac{1}{4}(1-x)(1+x)-2(1+x)/(1-x)^2-2(1-x)/(1+x)^2$

Einstein gravity theory

$$S=\int d^4x\sqrt{-g}\left(-\frac{1}{\kappa^2}R+\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-\frac{1}{2}m^2\phi^2-\frac{1}{4!}\lambda\phi^4\right)$$

・graviton propagator

$$G^{\mu\nu\rho\sigma}=\frac{\kappa^2}{p^2}\left(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho}-\eta^{\mu\nu}\eta^{\rho\sigma}\right)$$

・scattering amplitude (tree graphs : contact and *s*-, *t*-, *u*-channels)

$$M(\phi\phi\to\phi\phi)=-\lambda+2\kappa^2\left[\left(\frac{tu}{s}+\frac{su}{t}+\frac{st}{u}\right)+6m^2-2m^4\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right)\right]$$

・scattering amplitude in high-energy limit

$$M\simeq-\lambda+\kappa^2[2sf(\cos\theta)+\mathcal{O}(s^0)]=\mathcal{O}(s^1)$$

where $f(x)=\frac{1}{4}(1-x^2)+2(1+x^2)/(1-x^2)$

We examine the unitarity bound in the UV limit :

$$s\to\infty,\ t\simeq-\frac{1}{2}s(1-\cos\theta),\ u\simeq-\frac{1}{2}s(1+\cos\theta)$$

where θ is the scattering angle in the center-of-mass frame

	Unitarity bound	UV renormalizability
Einstein gravity + matter	unfulfilled	non renormalizable
R^2 gravity + matter	fulfilled	renormalizable

3 Consideration : S-matrix Unitarity (arXiv:1805.00262 to appear in PTEP)

A higher derivative gravity theory does not have (Commonly used) unitarity because of the negative norm.
Therefore, there is no guarantee that the unitarity bound will hold in a higher derivative gravity theory.
What should we evaluate to the high energy behavior of amplitudes in a higher derivative gravity theory?

A positive norm theory

Unitarity (= $S^\dagger S=1$ & positive norm) ⟺ Renormalizability

There is a guarantee that Unitarity bound holds.

We analyzed a higher derivative scalar theory as a simple toy model.

From our investigation of a higher derivative scalar theory, we have studied that the high energy behavior of amplitudes can be evaluated by S-matrix unitarity ($S^\dagger S=1$), regardless of the norm positivity.

A theory, regardless of the norm positivity

S-matrix unitarity ⟺ Renormalizability

We expected that there is a connection between S-matrix unitarity and renormalizability.

We are going to be able to evaluate the high energy behavior of amplitudes by S-matrix unitarity ($S^\dagger S=1$) in comparing a higher derivative gravity theory and Einstein gravity theory.

In future work, we will calculate *s*-, *t*-, *u*-channel graviton exchanges in two-graviton elastic scattering.