Unified models of inflation, dark energy, and high-scale SUSY breaking in supergravity

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arXiv:1907.10373

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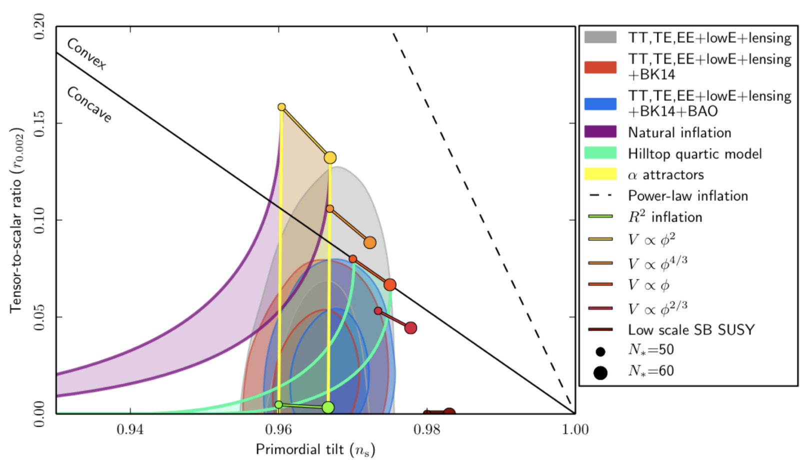
YITP Strings and Fields 2019

Introduction

- Supergravity – extension of (M)SSM, low-energy effective action of superstrings.
- Inflation – succesful idea solving the initial condition problems of pre-inflationary cosmology.
- Dark energy – describes present day expansion of the universe. Simplest case – small positive cosmological constant.
- SUSY breaking – in inflationary supergravity models SUSY is typically restored at the minimum. No evidence at LHC – perhaps SUSY is broken at a (very) high scale?

Inflationary models



[PLANCK '18]

N=1 D=4 supergravity

Superspace action:

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ \frac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})e^{-K/3} + W + \frac{1}{4} f_{AB} \mathcal{W}^A \mathcal{W}^B \right\} + \text{h.c.} \quad (1)$$

$K(\Phi_i, \overline{\Phi}_i, V)$ – Kähler potential; $W(\Phi_i)$ – superpotential;
$f_{AB}(\Phi_i)$ – gauge kinetic function;

Bosonic components:

$$e^{-1}\mathcal{L} = \frac{1}{2}R - K_{i\bar{j}} D_m \Phi^i \overline{D^m \Phi}^j - \frac{1}{4}\mathrm{Re}(f_{AB}) F^A_{mn} F^{B,mn} - \frac{i}{4}\mathrm{Im}(f_{AB}) \tilde{F}^A_{mn} F^{B,mn} - V_F - V_D \quad (2)$$

F- and D-term scalar potentials:

$$V_F = e^K \left[K^{i\bar{j}} (W_i + K_i W)(\overline{W}_{\bar{j}} + K_{\bar{j}} \overline{W}) - 3|W|^2 \right]$$
$$V_D = \frac{g^2}{2}\mathrm{Re}(f^{AB}) \mathscr{D}_A \mathscr{D}_B \ , \quad f^{AB} = f^{-1}_{AB} \quad (3)$$

$\mathscr{D}_A$ – Killing potentials (moment maps);
$D_m \Phi^i = \partial_m \Phi^i - g A^A_m X^i_A$; X^i_A – Killing vectors;

Starobinsky model in SUGRA

F-term:

- $K = -3\log(T + \overline{T} - |\Phi|^2)$ and $W = a\Phi(T - 1/2)$, Inflaton $= \mathrm{Re}\, T$ [Cecotti '87]; $\longrightarrow$ dual to old-minimal $R + R^2$ SUGRA
- $K = -3\log(T + \overline{T} - |\Phi|^2)$ and $W = a\Phi^2 + b\Phi^3$, Inflaton $\subset \Phi$ [Ellis et al. '13];
- $K = -2\log(T + \overline{T}) + |\Phi|^2$ and $W = a\Phi(T - b)$, Inflaton $= \mathrm{Re}\, T$ [Pallis, Toumbas '16];

$$\Longrightarrow \boxed{V_F \sim (e^{-\sqrt{\frac{2}{3}}\varphi} - 1)^2 + \ldots} \qquad (4)$$

D-term:

- $K = -3\log(T + \overline{T} - 2gV) + 3(T + \overline{T} - 2gV)$, Inflaton $= \mathrm{Re}\, T$ [Farakos et al. '13, Ferrara et al. '13]; $\longrightarrow$ dual to new-minimal $R + R^2$ SUGRA

$$\Longrightarrow \boxed{V_D \sim (e^{-\sqrt{\frac{2}{3}}\varphi} - 1)^2} \qquad (5)$$

...But after inflation SUSY is restored and minimum is Minkowski.

Alternative Fayet-Iliopoulos D-term

A class of alternative FI constructions is introduced in [Cribiori et al. '17, Kuzenko '18]. We consider the simplest model:

$$\mathcal{L}_{\rm FI} = g\xi \int d^2\Theta 2\mathcal{E}\overline{\mathcal{P}} \left(\frac{\mathcal{W}^2\overline{\mathcal{W}}^2}{\mathcal{P}\mathcal{W}^2\overline{\mathcal{P}}\overline{\mathcal{W}}^2} \mathcal{D}\mathcal{W} \right) + \text{h.c.}\ , \quad \mathcal{P} \equiv \mathcal{D}^2 - 8\overline{\mathcal{R}} \tag{6}$$

$$\Longrightarrow \boxed{V_D = \frac{1}{2} g^2 \xi^2} \tag{7}$$

- Does not require gauging of R-symmetry;
- Necessarily breaks SUSY, $\langle D \rangle \neq 0$;
- Generates positive cosmological constant;

  One cannot simply add this FI term to inflationary models (with SUSY Minkowski minimum), because then $\Lambda_{\rm C.C.} = \Lambda_{\rm SUSY}$. This problem is avoided if the minimum is AdS.

The setup

Consider the following $SU(1,1)/U(1)$ model with the FI term:

$$\boxed{\begin{aligned} K &= -\alpha \log(T + \overline{T}) \\ W &= \lambda + \mu T \ , \quad f = 1 \end{aligned}} \quad \& \quad \boxed{V_D = \frac{1}{2} g^2 \xi^2} \tag{8}$$

With the parametrization (for canonical ϕ)

$$T = e^{-\sqrt{\frac{2}{\alpha}}\phi} + it \ , \tag{9}$$

the resulting potential reads

$$V_F + V_D = \frac{\alpha - 3}{2^\alpha}(|\lambda|^2 + \omega_2 t + |\mu|^2 t^2) e^{\alpha\sqrt{\frac{2}{\alpha}}\phi} + \frac{(\alpha - 5)\omega_1}{2^\alpha} e^{(\alpha-1)\sqrt{\frac{2}{\alpha}}\phi} + \frac{(\alpha^2 - 7\alpha + 4)|\mu|^2}{2^\alpha \alpha} e^{(\alpha-2)\sqrt{\frac{2}{\alpha}}\phi} + \frac{g^2\xi^2}{2} \tag{10}$$

$\alpha \geq 3$ for stability

introducing
$$\begin{cases} \omega_1 \equiv \bar{\lambda}\mu + \lambda\bar{\mu} = 2\lambda_R\mu_R + 2\lambda_I\mu_I \ , \\ \omega_2 \equiv i(\bar{\lambda}\mu - \lambda\bar{\mu}) = 2\lambda_I\mu_R - 2\lambda_R\mu_I \ . \end{cases} \tag{11}$$

Starobinsky case, $\alpha = 3$

Given $\alpha = 3$, the scalar potential takes the simple form

$$V = -\frac{\omega_1}{4} e^{\sqrt{\frac{8}{3}}\phi} - \frac{|\mu|^2}{3} e^{\sqrt{\frac{2}{3}}\phi} + \frac{g^2\xi^2}{2} \ , \tag{12}$$

and has a minimum at

$$\phi_0 = \sqrt{\frac{3}{2}} \log\left(-\frac{2|\mu|^2}{3\omega_1}\right) \quad \Rightarrow \quad V_0 = \frac{g^2\xi^2}{2} + \frac{|\mu|^4}{9\omega_1} \ , \tag{13}$$

provided that $\omega_1 < 0$. Defining $\varphi \equiv \phi - \phi_0$ the potential can be written as

$$\boxed{V = V_0 + \frac{|\mu|^4}{9|\omega_1|}\left(e^{\sqrt{\frac{2}{3}}\varphi} - 1\right)^2} \tag{14}$$

- SUSY is broken by $\langle D \rangle$ and possibly $\langle F \rangle$, and $m_{3/2} \geq m_\varphi/2$.
- The classical $\alpha = 3$ potential is t-flat $\longrightarrow$ the mass for the t has to be generated by e.g. loop corrections.

For $\alpha > 3$ the t is massive already at tree level!

The case $\alpha > 3$: vacuum solutions

Once the axion t acquires VEV, $t_0 = -\omega_2/(2|\mu|^2)$, we have effectively single-field scalar potential,

$$\boxed{\begin{aligned} V|_{t=t_0} = \frac{(\alpha-3)\omega_1^2}{2^{\alpha+2}|\mu|^2} e^{\sqrt{2\alpha}\phi} + \frac{(\alpha-5)\omega_1}{2^\alpha} e^{(\alpha-1)\sqrt{\frac{2}{\alpha}}\phi} \\ + \frac{(\alpha^2-7\alpha+4)|\mu|^2}{2^\alpha\alpha} e^{(\alpha-2)\sqrt{\frac{2}{\alpha}}\phi} + \frac{g^2\xi^2}{2} \ , \end{aligned}} \tag{15}$$

Critical points:

$$e^{\sqrt{\frac{2}{\alpha}}\phi_\pm} = \gamma_\pm \frac{|\mu|^2}{\omega_1} \quad \begin{cases} \gamma_+ \equiv \frac{2(-\alpha^2+7\alpha-4)}{\alpha(\alpha-3)} \\ \gamma_- \equiv \frac{2(2-\alpha)}{\alpha} \end{cases} \tag{16}$$

Since $\alpha > 3$, $\gamma_- < 0$ and (defining $\alpha_* \equiv \frac{1}{2}(7+\sqrt{33}) \approx 6.37$)

$$3 < \alpha < \alpha_* \longrightarrow \gamma_+ > 0 \tag{17}$$

$$\alpha = \alpha_* \longrightarrow \gamma_+ = 0 \tag{18}$$

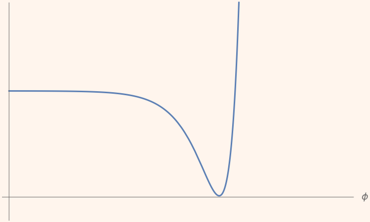
$$\alpha > \alpha_* \longrightarrow \gamma_+ < 0 \tag{19}$$

The case $\alpha > 3$: vacuum solutions

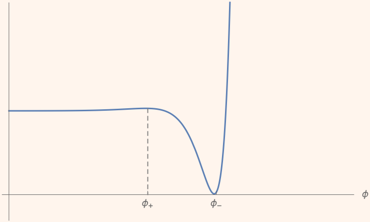
So,

$$(A)\ \omega_1 > 0 \ \text{ and } \ 3 < \alpha < \alpha_* : \ \text{Stable vacuum at } \phi_+$$
$$(B)\ \omega_1 < 0 \ \text{ and } \ 3 < \alpha \leq \alpha_* : \ \text{Stable vacuum at } \phi_-$$
$$(C)\ \omega_1 < 0 \ \text{ and } \ \alpha > \alpha_* : \ \text{Stable vacuum at } \phi_- \text{ and local max. at } \phi_+$$

all other cases unstable or runaway



(A)(B)



(C)

The case $\alpha > 3$: scalar masses and SUSY breaking

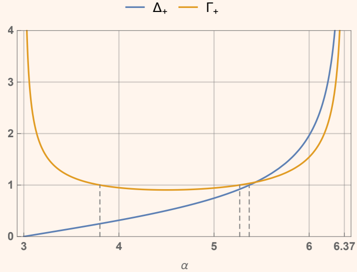
$$\phi_0 = \phi_+ \longrightarrow D, F \neq 0 \ , \quad \phi_0 = \phi_- \longrightarrow D \neq 0, F = 0 \tag{20}$$

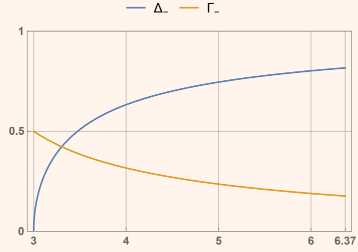
The scalar and gravitino masses have the similar form

$$M^2 = f(\alpha) \frac{|\mu|^{2(\alpha-1)}}{\omega_1^{\alpha-2}} \tag{21}$$

so that their mass ratios depend only on α ($\varphi = \phi - \phi_0$):

$$\Delta_\pm \equiv \left. \frac{m_t}{m_\varphi} \right|_{\phi_0 = \phi_\pm} \ , \quad \Gamma_\pm \equiv \left. \frac{m_{3/2}}{m_\varphi} \right|_{\phi_0 = \phi_\pm} \tag{22}$$





Inflationary observables

Trading $g\xi$ for the cosmological constant V_0, and restoring κ we have

$$V = V_0 + \kappa^2 \left(\frac{\gamma}{2}\right)^{\alpha} \frac{|\mu|^{2(\alpha-1)}}{\omega_1^{\alpha-2}} \left[\frac{\alpha-3}{4} e^{\sqrt{2\alpha}\kappa\varphi} + \frac{\alpha-5}{\gamma} e^{(\alpha-1)\sqrt{\frac{2}{\alpha}}\kappa\varphi} + \frac{\alpha^2-7\alpha+4}{\alpha\gamma^2} e^{(\alpha-2)\sqrt{\frac{2}{\alpha}}\kappa\varphi} - \frac{\alpha(\gamma+2)^2}{4\gamma^2} + \frac{(\gamma+2)(3\gamma+14)}{4\gamma^2} - \frac{4}{\alpha\gamma^2}\right] . \tag{23}$$

Next,

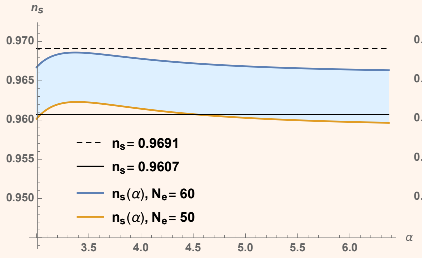
- calculate the predictions for $n_s \simeq 1 + 2\eta_i - 6\epsilon_i$ and $r \simeq 16\epsilon_i$,
- use the observed value of $A_s \simeq \frac{\kappa^4 V(\varphi_i)}{24\pi^2\epsilon_i}$ to fix the composite parameter $|\mu|^{2(\alpha-1)}/\omega_1^{\alpha-2}$ – this fixes m_φ, m_t, and $m_{3/2}$.

PLANCK 2018:

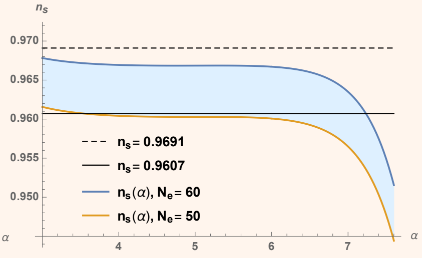
$$n_s = 0.9649 \pm 0.0042 \ (68\%\mathrm{CL}) \ , \quad r < 0.064 \ (95\%\mathrm{CL}) \ ,$$
$$\log(10^{10} A_s) = 2.975 \pm 0.056 \ (68\%\mathrm{CL}) \ \Rightarrow \ A_s \approx 1.96 \times 10^{-9}.$$

Constraining α

Numerical results for $n_s(\alpha)$:



(a) $\omega_1 > 0$



(b) $\omega_1 < 0$

$\alpha > \alpha_{\text{max}} \approx 7.235$ is incompatible with observations at 68%CL.

Predictions for n_s and r

- Starobinsky-like inflation ($3 \leq \alpha \leq \alpha_*$):

α	3	4		5	6		α_*
$\mathrm{sgn}(\omega_1)$	$-$	$+$	$-$	$+/-$	$+$	$-$	$-$
n_s	0.9650	0.9649	0.9640	0.9639	0.9634	0.9637	0.9632
r	0.0035	0.0010	0.0013	0.0007	0.0005	0.0004	0.0003
$-\kappa\varphi_i$	5.3529	3.5542	3.9899	3.2657	3.0215	2.7427	2.5674
$-\kappa\varphi_f$	0.9402	0.7426	0.8067	0.7163	0.6935	0.6488	0.6276

Predictions for n_s, r for integer α and $\alpha_* \equiv (7+\sqrt{33})/2$. $N_e = 55$ is assumed.

- Hilltop inflation ($\alpha_* < \alpha \leq \alpha_{\max}$): taking $\alpha = 7$ and $N_e = 60$ ($N_e = 55$ gives incompatible n_s when $\alpha = 7$), we find $n_s \approx 0.9635$, $r \approx 0.0002$.

Fixing SUSY breaking scale and scalar masses

α	3	4		5		6		7	
$\mathrm{sgn}(\omega_1)$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$-$	
m_φ	2.83	2.95	2.73	2.71	2.71	2.53	2.58	1.86	$\times 10^{13}$ GeV
m_t	0	0.93	1.73	2.02	2.02	4.97	2.01	1.56	$\times 10^{13}$ GeV
$m_{3/2}$	≥ 1.41	2.80	0.86	2.56	0.64	3.91	0.49	0.29	$\times 10^{13}$ GeV
$\langle F \rangle$	?	$\neq 0$	0	$\neq 0$	0	$\neq 0$	0	0	$\times 10^{31}$ GeV^2
$\langle D \rangle$	8.31	4.48	5.08	3.76	3.76	3.25	2.87	1.73	$\times 10^{31}$ GeV^2

The masses of inflaton (m_φ), axion (m_t) and gravitino ($m_{3/2}$), and the VEVs of F- and D-fields derived from our models by fixing the amplitude A_s according to PLANCK data.

Summary

- We studied the models with $K = -\alpha \log(T + \overline{T})$, $W = \lambda + \mu T$, and the alternative FI term. The Kähler potential with $\alpha = 1, 2, .., 7$ can be obtained from e.g. M-theory on G_2 [Ferrara, Kallosh '16].
- $3 \leq \alpha \leq \alpha_* \rightarrow$ Starobinsky-like inflation, $\alpha > \alpha_* \rightarrow$ hilltop inflation ($\alpha_* \approx 6.372$).
- For $\alpha > 3$ the axion t has non-tachyonic (tree-level) mass comparable to the inflaton mass.
- Spectral tilt n_s is compatible with PLANCK 2018 data for $3 \leq \alpha \leq \alpha_{\text{max}} \approx 7.235$.
- Observations of the scalar amplitude A_s fix SUSY breaking scale, $m_{3/2} \sim 10^{13}$ GeV (or larger if we allow fractional α).
- Some models have mixed F-/D-term SUSY breaking, while the others have pure D-term breaking ($\langle D \rangle$ is fixed by A_s, $\langle F \rangle$ is not).
- Small positive C.C. (dark energy) can be obtained by fine-tuning the parameters.

ご清聴ありがとうございました。