Superconformal Index and Supersymmetry Enhancement of S-fold Theories

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Abstract

Recently concrete models of 4d $\mathcal{N}=3$ superconformal field theories called S-fold theories are constructed by Garcia-Etxebarria and Regalado. Although it is difficult to study these theories due to the lack of the Lagrangian description and the strong coupling, it is expected that there is a non-trivial supersymmetry enhancement for rank one and two theories by Aharony and Tachikawa. In this poster, we evaluate the first non-trivial finite rank corrections to the superconformal index of these theories by using AdS/CFT correspondence and check the supersymmetry enhancement. To evaluate the index in finite rank, we mainly focus on the D3-branes wrapping a non-trivial three cycle on AdS side interpreted as Pfaffian-like operators on CFT side. We see that our results agree with the results expected from the supersymmetry enhancement.

1. 4d $\mathcal{N}=3$ Theories and S-folds

SUSY Enhancement of S-fold Theory
[Aharony and Tachikawa '16]

Conjecture: $\mathcal{N}=4$ Rank 2 SYM = Rank 2 S-fold Theory

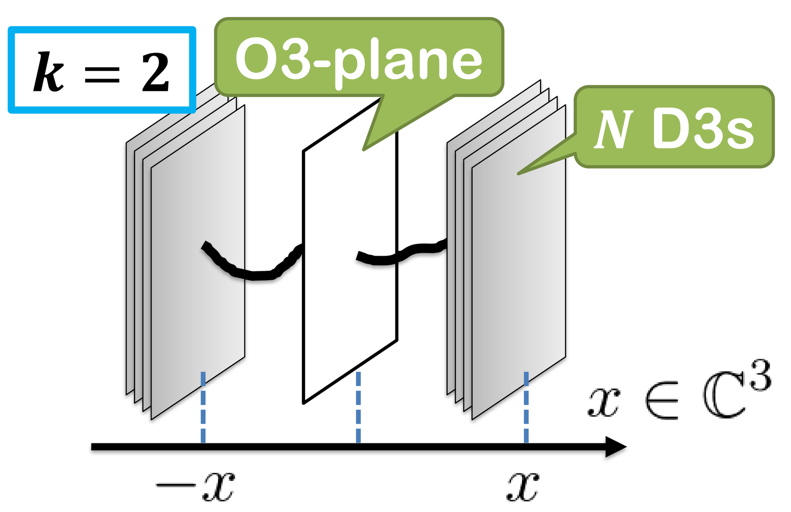
	$\mathcal{N}=4$ Rank 2 SYM	Rank 2 S-fold Theory	
SUSY	$\mathcal{N}=4$	$\mathcal{N}=3$	→ $\mathcal{N}=4$
R-sym.	$SU(4)_R$	$U(3)_R$	→ $SU(4)_R$

non-trivially enhanced

S-fold Theory
[Garcia-Etxebarria and Regalado '16]

One of the genuine $\mathcal{N}=3$ theory

$k=2$



[Witten '98]

Orientifold (trivial torsion)

$$\begin{cases} \text{Type IIB on } AdS_5 \times S^5/\mathbb{Z}_2 \\ \mathcal{N}=4\ SO(2N) \text{ on D3 WV} \end{cases}$$

with $\mathbb{Z}_2$ identification of string charge

$k=3,4,6$ Orientifold $\mathbb{Z}_2 \xrightarrow{\text{generalize}} \mathbb{Z}_k$ S-fold ($\mathcal{N}=3$) theory

$Q_I \xrightarrow{\text{S-fold}} (Q_1, e^{-\frac{4\pi i}{k}} Q_2, Q_3, Q_4)$ Q_2 is projected out.

- String charge identification is possible only for $k=3,4,6$.
- We denote an S-fold theory by $S(k,N)$.
- If an $\mathcal{N}=3$ theory has a dim. 2 CBO, SUSY is enhanced to $\mathcal{N}=4$.

Coulomb branch ops.

Dim. of CBO $k, 2k, \cdots, (N-1)k, N$

$k=2 \Rightarrow$ SUSY enhancement always occurs (orientifold)

$k=3,4,6 \Rightarrow$ SUSY enhancement occurs for $N=1,2$

These are equivalent to $\mathcal{N}=4$ gauge theories

k	3	4	6
$N=1$	$U(1)$	$U(1)$	$U(1)$
$N=2$	$SU(3)$	$SO(5)$	G_2

2. Superconformal Index

Purpose Confirm this conjecture by comparing BPS spectra

- This can be done by using superconformal index (SCI, index).
- For $S(k,2)$ we calculate it on AdS side because of no Lagrangian.
- For $\mathcal{N}=4$ gauge theory we calculate it by localization technique.
- Finally we check that $\mathcal{I}^{\text{AdS}}_{S(3,2)} \overset{?}{=} \mathcal{I}^{\text{Localization}}_{SU(3)}$ for $k=3$

Superconformal Index

$SU(4)_R \cong SO(6)_R$ Cartan: (R_x, R_y, R_z)

Superconformal index

$$\mathcal{I}(q,y,u,v) = \text{tr}[(-1)^F \overline{x}^{2\{\overline{S}^1_1, \overline{Q}^1_1\}} q^{E+J_2} y^{2J_1} \underbrace{u^{R_x - R_y} v^{R_y - R_z}}_{SU(3)_R \subset SU(4)_R}]$$

[Kinney, Maldacena, Minwalla, and Raju '05]

Counting only $[\overline{Q}^{\text{i}}_1, \mathcal{O}] = 0$, $[\overline{S}^1_{\text{i}}, \mathcal{O}] = 0$

- This is calculable for a Lagrangian theory.
- If there is a duality, SCI should be match for both theories.

Example: $\mathcal{N}=4$ $U(1)$ SYM

$$\mathcal{I}_{U(1)} = 1 + \chi_{(1,0)} q - \chi^J_1 q^{\frac{3}{2}} + (-\chi_{(0,1)} + \chi_{(2,0)}) q^2 + \cdots$$

(X, Y, Z) λ_{1a} $\bar{\lambda}^{i i}$ $(\text{X, Y, Z})^2$

R-sym. and SUSY from SCI

SUSY	Sym. in SCI	Character
$\mathcal{N}=4$	$SU(3)_R \subset SU(4)_R$	$\chi_{(n,m)} = \text{tr}[u^{R_x - R_y} v^{R_y - R_z}]$
$\mathcal{N}=3$	$U(2)_R \subset U(3)_R$	$u^{R_x}\chi_n = u^{R_x} \text{tr}\left[\left(\frac{v}{u}\right)^{R_y}\left(\frac{1}{v}\right)^{R_z}\right]$

We can read off SUSY of the theory from its SCI.

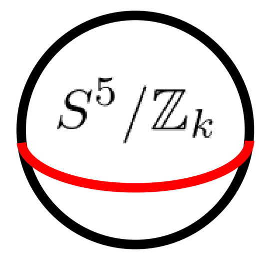
3. AdS Side Analysis of S-fold Theories

New objects in finite N

D3-branes wrapped around the non-trivial 3-cycle in $S^5/\mathbb{Z}_k$

Classified by wrapping number $m \in H_3(S^5/\mathbb{Z}_k, \mathbb{Z}) = \mathbb{Z}_k$

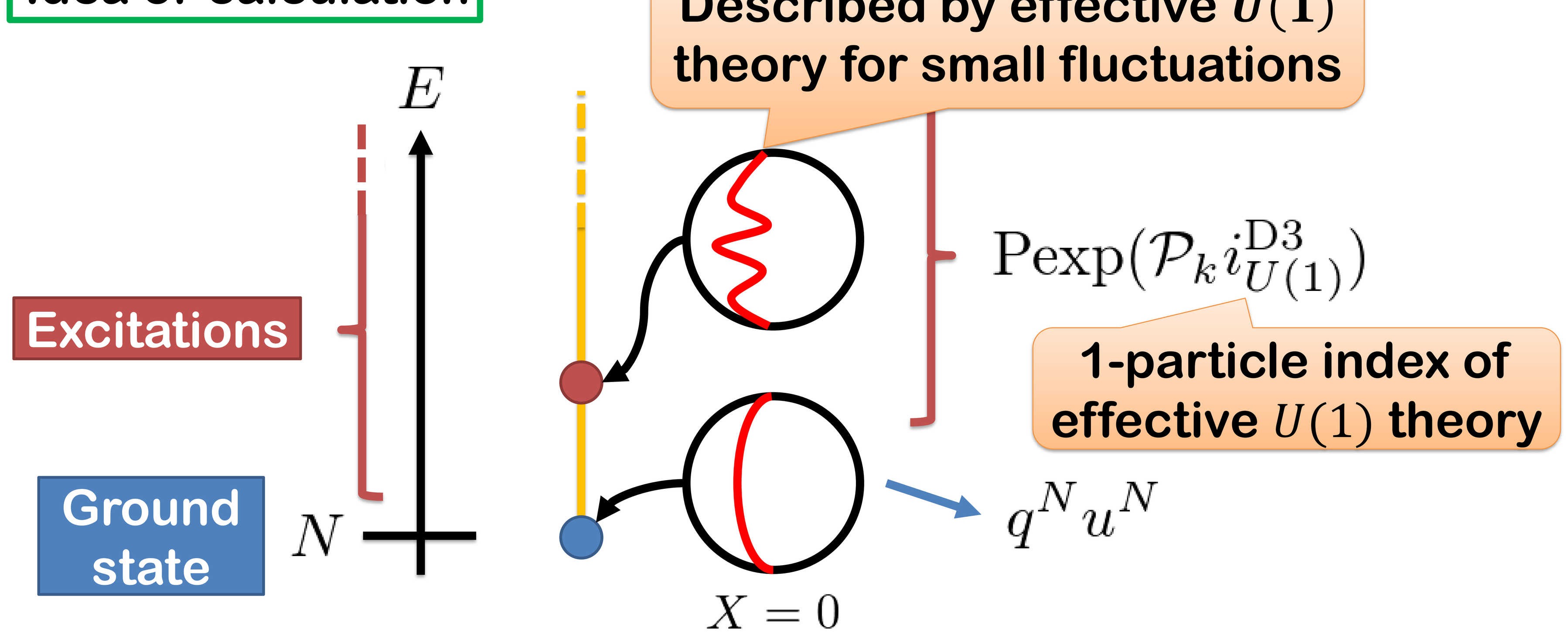
Interpretation on CFT side: $\text{Pf}(\Phi) = \epsilon_{i_1 \cdots i_{2N}} \Phi_{i_1 i_2} \cdots \Phi_{i_{2N-1} i_{2N}}$ =



Contribution	$1+\cdots$	$q^N+\cdots$	$q^{2N}+\cdots$	$\cdots$	$q^{kN+1}+\cdots$
KK modes	○	○	○	$\cdots$	○
D3 with $\lvert m\rvert=1$		○	○	$\cdots$	○
D3 with $\lvert m\rvert=2$			○	$\cdots$	○
$\vdots$					
D3 with $\lvert m\rvert=k$					○

Our focus: D3-branes with $m=\pm 1$

Idea of calculation



Index from D3 is contribution from effective $U(1)$ on wrapped D3

Results

For $S(3,2)$, $(k=3, N=2)$

[Imamura and Yokoyama '16]
[RA and Imamura '19]

$$\mathcal{I}_{\text{KK}} = 1 + u\chi_1 q^2 - u\chi^J_1 q^{\frac{5}{2}} + \cdots + \chi^J_1(-2 + u^3 + u^{-1}\chi_1 - 2u^2\chi_1 - u\chi_2 + \chi_3) q^{\frac{9}{2}} + \cdots$$

$$\mathcal{I}_{\text{KK}}\mathcal{I}_{\text{D3}} = (u^2 + \chi_2) q^2 - \chi^J_1 \chi_1 q^{\frac{5}{2}} + \cdots + \chi^J_1 (1 - u^3 - 2u^{-1}\chi_1 + u^2\chi_1 - \chi_3) q^{\frac{9}{2}} + \cdots$$

Each contribution has $\mathcal{N}=3$ SUSY, but combining them...

$$1 + \underbrace{(u^2 + u\chi_1 + \chi_2)}_{\chi_{(2,0)}} q^2 - \chi^J_1 \underbrace{(u + \chi_1)}_{\chi_{(1,0)}} q^{\frac{5}{2}} + \cdots - \chi^J_1 \underbrace{(1 + u^{-1}\chi_1 + u^2\chi_1 + u\chi_2)}_{\chi_{(1,1)}} q^{\frac{9}{2}} + \cdots$$

SUSY is enhanced and it is equivalent to $I_{SU(3)}$ up to $\mathcal{O}(q^5)$!

Similarly we have $\mathcal{I}^{\text{AdS}}_{S(4,2)} = \mathcal{I}_{SO(5)} + \mathcal{O}(q^4)$ $\mathcal{I}^{\text{AdS}}_{S(6,2)} = \mathcal{I}_{G_2} + \mathcal{O}(q^4)$

Summary

- We confirmed SUSY enhancement of rank 2 S-fold theories by using SCI up to appropriate order of fugacity q.
- In order to include higher terms we should consider multiple branes but it may be difficult because of strings between branes.