Space-time structure and the Dirac zero modes from classical solutions in the Lorentzian type IIB matrix model

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1. Introduction

- The type IIB matrix model [Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)] is known as one of non-perturbative formulations of superstring theory. It is shown by numerical simulations that the (3+1)-dimensional expanding early universe appears in this model [Kim-Nishimura-Tsuchiya ('12), Nishimura-Tsuchiya ('18), Aoki-Hirasawa-Ito-Nishimura-Tsuchiya ('19)].
- We investigate the type IIB matrix model by numerically solving classical equations of motion, which is expected to be valid at late time since the action becomes large due to the expansion of space. Many solutions are obtained by the gradient decent method by assuming a quasi-direct-product structure for the (3+1) dimensions and the six extra dimensions. We find that these solutions generally represent expanding space-time with smooth structure. Assuming further some block diagonal structure in the extra dimensions, we observe emergence of zero modes of the Dirac operator in (3+1) dimensions as the matrix size is increased.

2. Classical solutions of the type IIB matrix model and algorithm of searching for classical solutions

- The type IIB matrix model

$$S = -\frac{1}{g^2}\,\mathrm{Tr}\left(\frac{1}{4}[A^M, A^N][A_M, A_N] + \frac{1}{2}\bar{\Psi}\Gamma^M[A_M, \Psi]\right) \quad (M = 0, 1, \ldots, 9)$$

$N \times N$ traceless Hermitian matrices

- A^M : transformed as the Lorentz vector
- Ψ : transformed as the Majorana-Weyl spinor

} under SO(9,1) transformation

Constraints corresponding to IR cutoffs: $\frac{1}{N}\,\mathrm{Tr}\,A_0^2 = \kappa, \quad \frac{1}{N}\,\mathrm{Tr}\,A_I^2 = 1 \quad (I = 1, \ldots, 9)$

- Classical equations of motion

$$[A^M, [A_M, A_0]] - \xi A_0 = 0$$

$$[A^M, [A_M, A_I]] - \zeta A_I = 0$$

ξ, ζ : Lagrange multipliers

- Quasi-direct-product structure [Nishimura-Tsuchiya ('13)]

We assume configurations with a quasi-direct-product structure in (3+1) dimensions and extra dimensions.

$$A_\mu = X_\mu \otimes M, \qquad A_a = 1_{N_X} \otimes Y_a$$

(X_μ, 1_{N_X}: N_X; M, Y_a: N_Y)

It is a general form preserving SO(3,1) symmetry and represents a direct-product space-time if $M = 1_{N_Y}$.

$(\mu = 0, \ldots, 3, \ a = 4, \ldots, 9, \ N = N_X N_Y)$

- Gradient descent algorithm
  - Begin with random configurations of X_μ, Y_a, M
  - Update them with following relations:

$$\delta X_\mu = -\epsilon\frac{\partial V}{\partial X_\mu^\dagger}, \quad \delta Y_a = -\epsilon\frac{\partial V}{\partial Y_a^\dagger}, \quad \delta M = -\epsilon\frac{\partial V}{\partial M^\dagger},$$

$$V = \mathrm{Tr}\left([A^M, [A_M, A_0]] - \xi A_0\right)^2 + \mathrm{Tr}\left([A^M, [A_M, A_I]] - \zeta A_I\right)^2$$

  - Repeat this procedure until $V \simeq 0$

- Typical solutions

In almost all of our solutions, the following equations are satisfied:

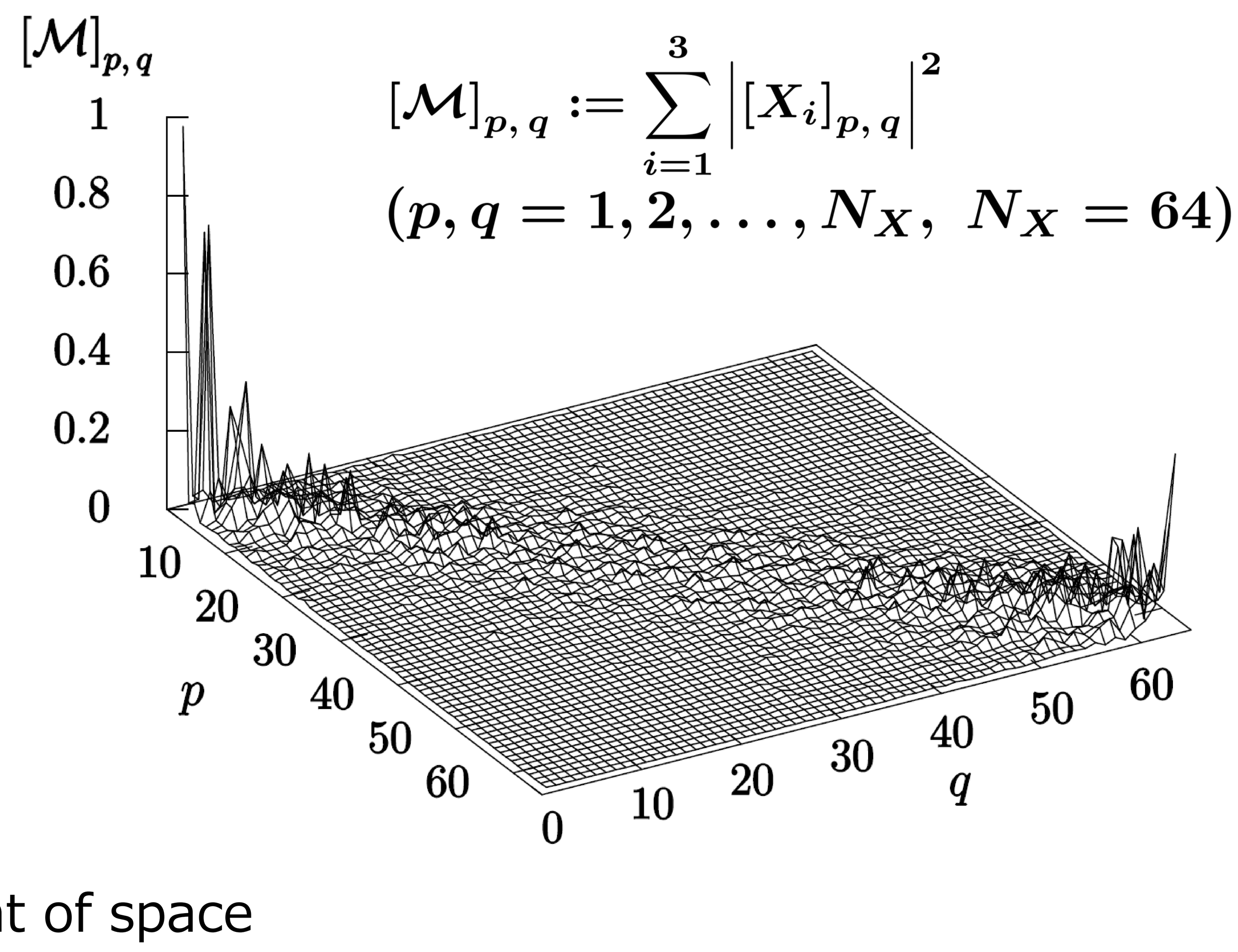
$$M^3 = M, \quad [M, Y_a] = 0, \quad (*) \quad \begin{aligned} &[X^\nu, [X_\nu, X_0]] - \zeta X_0 = 0, \\ &[X^\nu, [X_\nu, X_i]] - \xi X_i = 0 \quad (i = 1, 2, 3), \\ &[Y^b, [Y_b, Y_a]] - \xi Y_a = 0, \end{aligned}$$

One can see from the equation $(*)$ that Y_a have the following block-diagonal structure in a basis which diagonalize M:

$$M = \sqrt{\frac{N_Y}{N_Y - n^{[0]}}}\begin{pmatrix} -1_{n^{[-1]}} & & \\ & 0_{n^{[0]}} & \\ & & 1_{n^{[1]}} \end{pmatrix} \longleftrightarrow Y_a = \begin{pmatrix} Y_a^{[-1]} & & \\ & Y_a^{[0]} & \\ & & Y_a^{[1]} \end{pmatrix}.$$

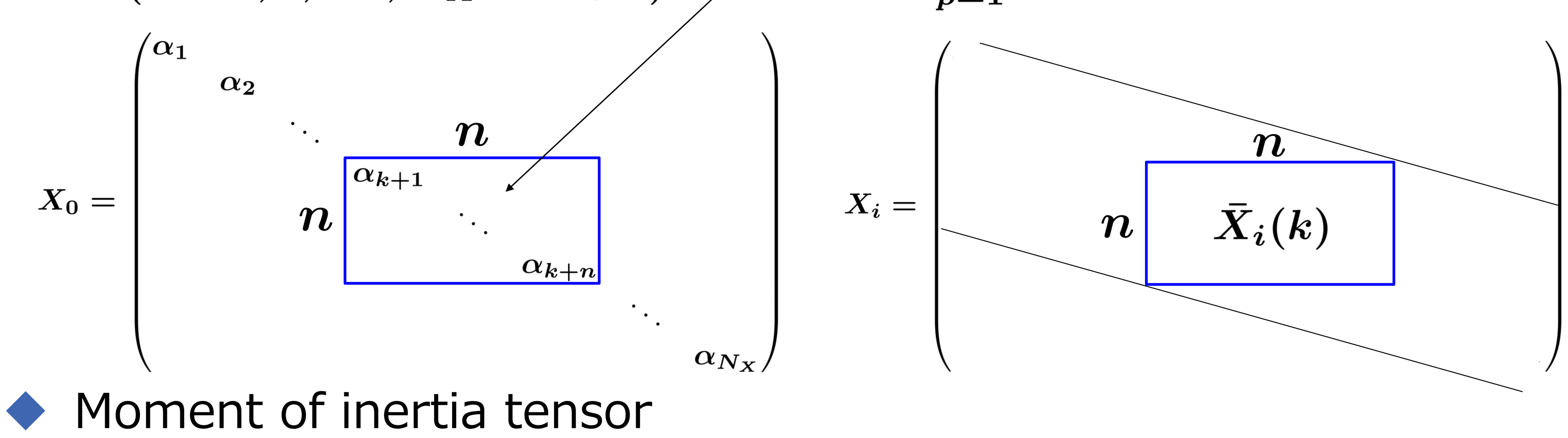
3. Space-time structure in the (3+1) dimensions

- Band-diagonal structure

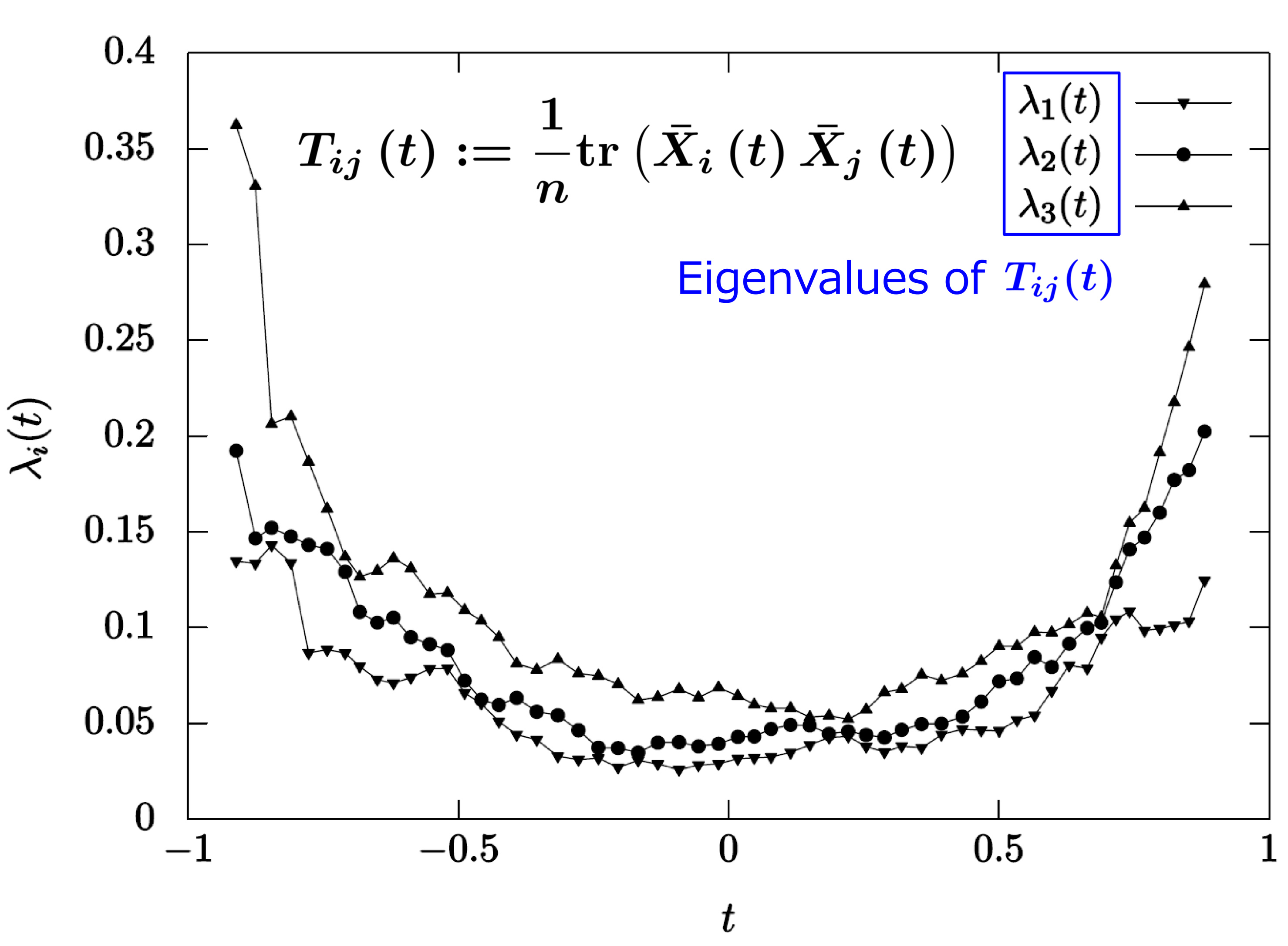


- Time evolution

t is defined as the average of the n eigenvalues of X_0 within the k-th $n \times n$ block: $t = t(k) := \frac{1}{n}\sum_{\tilde{p}=1}^{n}\alpha_{k+\tilde{p}} \ (\tilde{p} = 1, 2, \ldots, n)$.
$(k = 1, 2, \ldots, N_X - n + 1)$

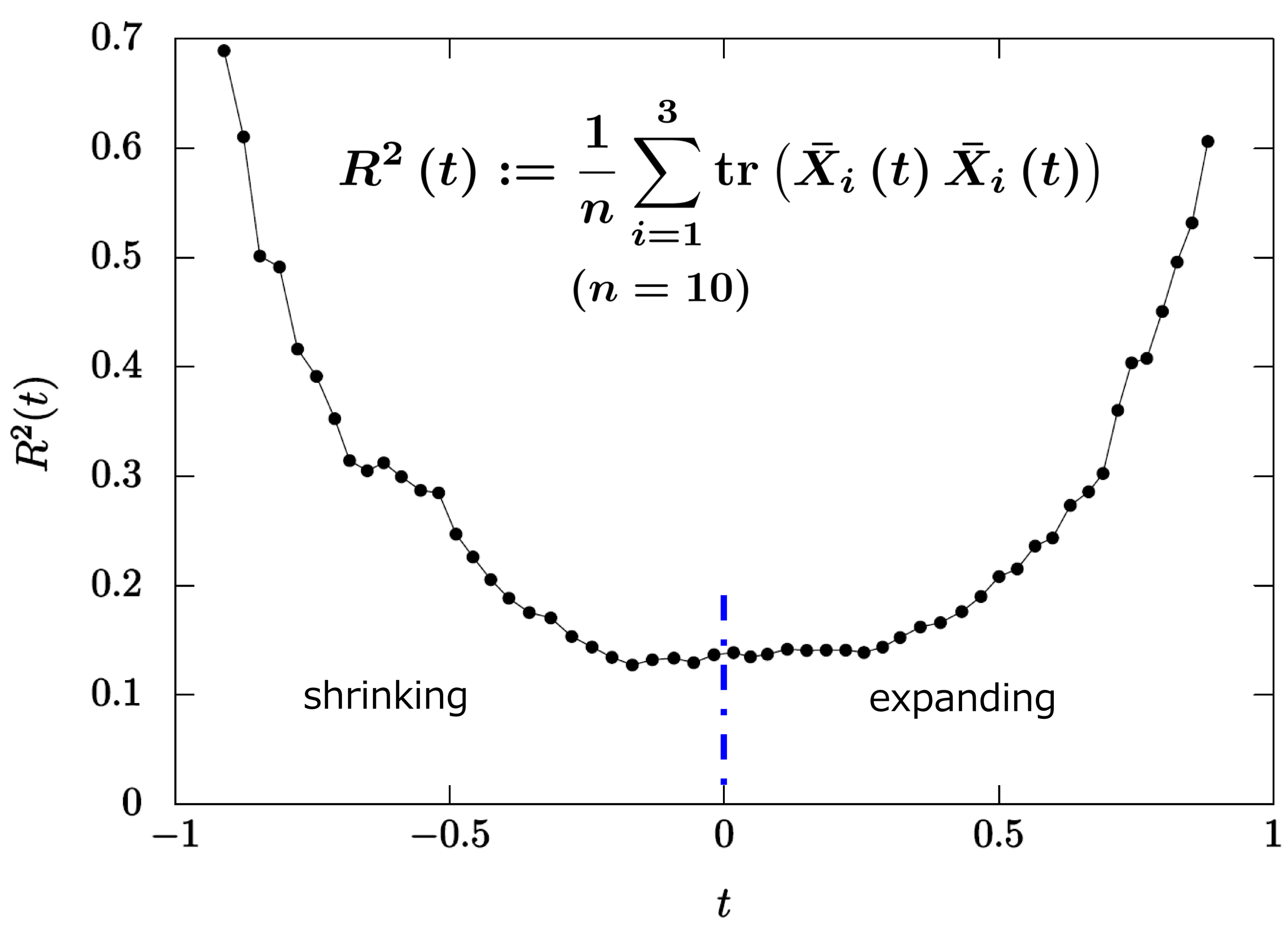


- Moment of inertia tensor



The behavior of $T_{ij}(t)$ is time-reversal symmetric as $R^2(t)$ is. The 3-dimensional space represented by the typical solution is SO(3) symmetric except at late time. The breaking of the SO(3) can be caused by a finite size effect.

- Extent of space



4. Emergence of zero modes of the Dirac operator

◆ 6-dimensional Dirac equation

- 10-dimensional Dirac equation: $\Gamma^M[A_M, \Psi] = 0$

  Ψ satisfies the chirality condition: $\Gamma_\chi \Psi = +\Psi$
- Decompose Γ :

$$\Gamma^\mu = \rho^\mu \otimes 1_8 \ , \ \Gamma^a = \rho_\chi \otimes \gamma^a \ , \ \Gamma_\chi = \rho_\chi \otimes \gamma_\chi$$

The chirality operators in 4 and 6 dimensions.

- Require Ψ to be chiral in the (3+1) dimensions:

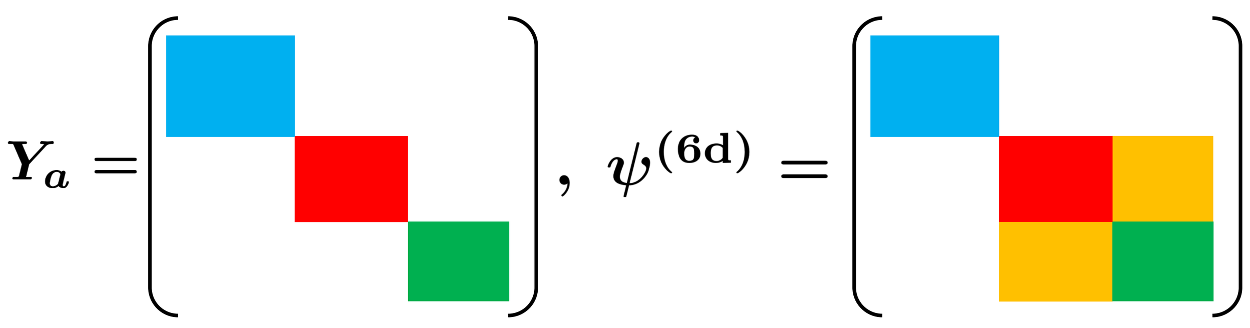
$$(\rho_\chi \otimes 1_8)\Psi = \pm\Psi$$

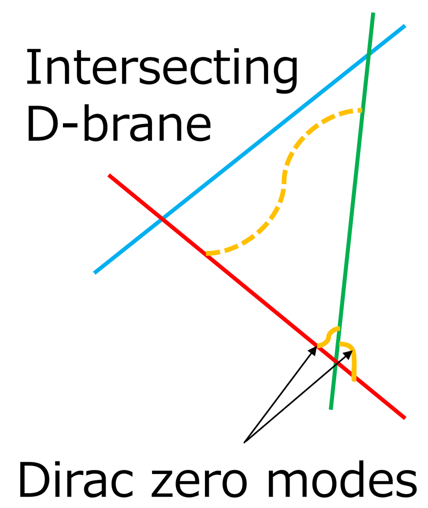
- Decompose Ψ : $\Psi = \psi^{(4d)} \otimes \psi^{(6d)}$
- 6-dimensional Dirac equation:

$$\gamma^a \left[Y_a, \psi^{(6d)}\right] = 0 \ , \quad \gamma_\chi \psi^{(6d)} = \pm\psi^{(6d)}$$

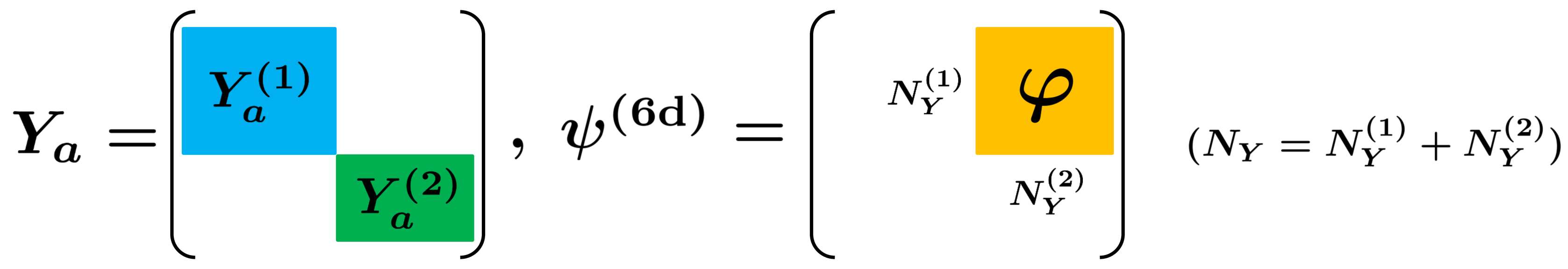
We examine spectra of the 6-dimensional Dirac operator.
zero eigenvectors → Dirac zero modes in the (3+1) dimensions

◆ Structure of Y_a and Dirac zero modes





◆ Ansatz for structure of Y_a and $\psi^{(6d)}$



We solve the following eigenvalue problem:

$$\gamma^a_{\alpha\beta}\left[Y_a^{(1)}\varphi_\beta - \varphi_\beta Y_a^{(2)}\right] = \mu\varphi_\alpha \ , \ \varphi_\alpha : \text{eigenvectors} \quad (\alpha = 1, \ldots, 8)$$

Mass in the (3+1) dimensions

If μ is eigenvalue of φ_α, $-\mu$ is also eigenvalue of φ_α.
We concentrate on the lowest and the second lowest eigenvalue μ_0, μ_1 among $4N_Y^{(1)}N_Y^{(2)}$ positive eigenvalues.

● 3d-3d ansatz

$$Y_1^{(1)} \neq 0, \ Y_2^{(1)} \neq 0, \ Y_3^{(1)} \neq 0, \ Y_4^{(1)} = Y_5^{(1)} = Y_6^{(1)} = 0$$

$$Y_1^{(2)} = Y_2^{(2)} = Y_3^{(1)} = 0, \ Y_4^{(2)} \neq 0, \ Y_5^{(2)} \neq 0, \ Y_6^{(2)} \neq 0$$

3-dimensional manifolds intersect at a point in the 6-dimensional space. We expect to obtain Dirac zero modes using this ansatz.

● 2d-4d ansatz

2-dimensinal and 4-dimensional manifolds intersect at a point in the 6-dimensional space. We also expect to obtain Dirac zero modes using this ansatz.

● 3d-4d, 4d-4d, 2d-3d ansatz

We cannot obtain Dirac zero modes and localized wave functions.

◆ |Each element of wave function|^2 $(N_Y^{(1)} = N_Y^{(2)} = 64)$

- Extract $(\varphi_L)_\alpha$ and $(\varphi_R)_\alpha$ from φ_α :

$$(\varphi_L)_\alpha = \frac{1-\gamma_\chi}{2}\varphi_\alpha \ , \ (\varphi_R)_\alpha = \frac{1+\gamma_\chi}{2}\varphi_\alpha$$

φ_α obtained in our numerical calculation contains the left-handed and the right-handed ones.

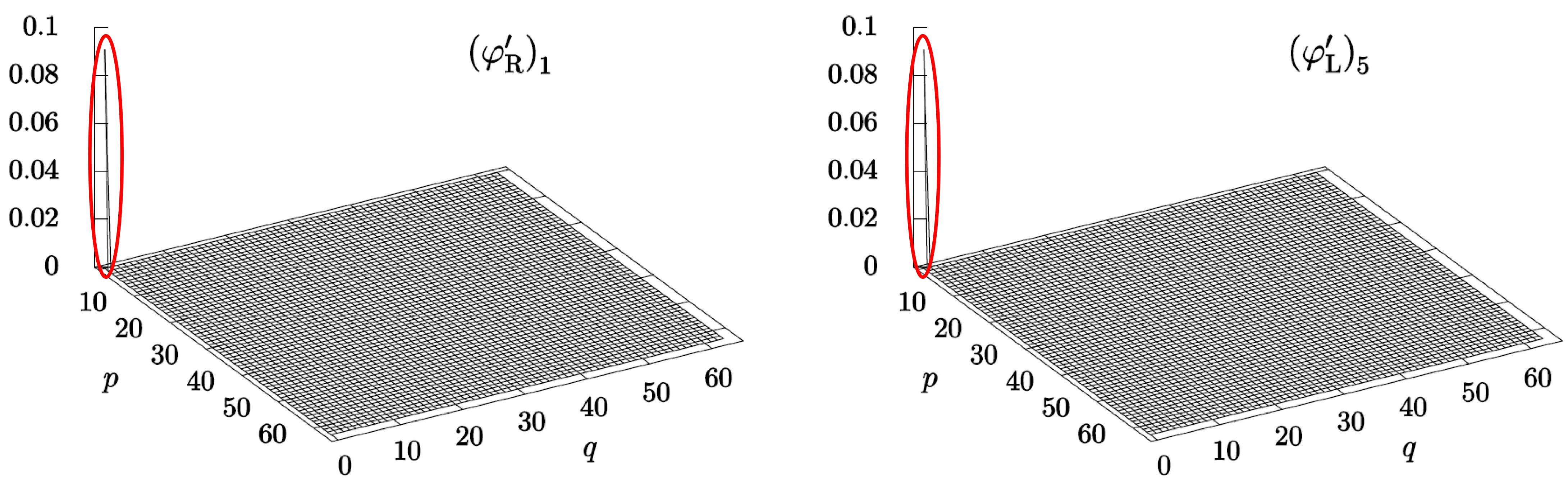
- Transform $(\varphi_L)_\alpha$ and $(\varphi_R)_\alpha$:

$$(\varphi'_L)_\alpha = U(\varphi_L)_\alpha V^\dagger \ , \ (\varphi'_R)_\alpha = U(\varphi_R)_\alpha V^\dagger \ ,$$

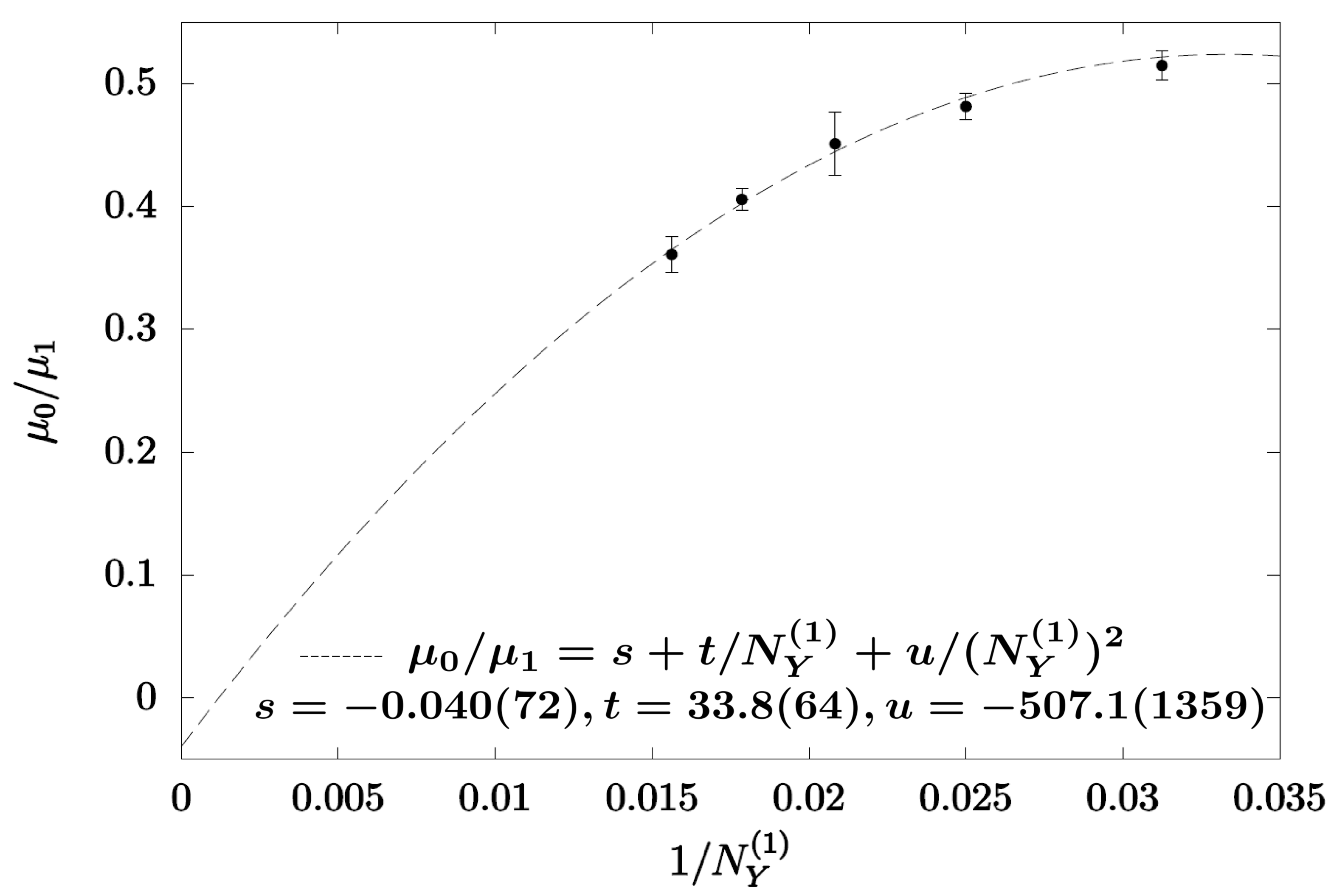
where $U \in \mathrm{SU}\left(N_Y^{(1)}\right)$, $V \in \mathrm{SU}\left(N_Y^{(2)}\right)$.

Here, we choose U and V such that $(\varphi_R)_1 = U^\dagger(\varphi'_R)_1 V$ becomes the singular value decomposition.

Wave function is localized at a point. → D-branes intersect at a point.



◆ Ratio of eigenvalues $(N_Y^{(1)} = N_Y^{(2)} = 32, 40, 48, 56, 64)$



In the $N_Y^{(1)} \to \infty$, s converges at 0 within error.

We can obtain Dirac zero modes $N_Y^{(1)} \to \infty$.

5. Conclusion and discussion

- ◆ **By assuming a quasi-direct-product structure, we searched for classical solutions of the type IIB matrix model. This structure favors a block-diagonal structure which can yield D-branes intersecting in extra dimensions.**
- ◆ **The band-diagonal structure of X_i ensures the locality of the time.**
- ◆ **We found that the 3-dimensional space obtained in our solutions is smooth and SO(3) symmetry is respected as almost all of late time.**
  Breaking of SO(3) symmetry at the last regime is considered to be a finite matrix size effect.
- ◆ **In 3d-3d ansatz, we found solutions that give Dirac zero modes in the $N_Y^{(1)} \to \infty$ limit.**
  Wave functions corresponding to the lowest eigenvalue μ_0 are localized at a point.
  This is consistent with the picture of intersecting D-branes.
- ◆ **What is important is that Dirac zero modes are obtained as solutions of equations of motion.**
  Cf.) Aoki ('11), Chatzistavrakidis-Steinacker-Zoupanos ('11), Nishimura-Tsuchiya ('13), Aoki-Nishimura-Tsuchiya ('14)
  →In these studies, matrix configurations in extra dimensions are given by hand.
- ➢ **We would like to identify Higgs modes in the spectra of fluctuation of Y_a, and determine the Yukawa coupling from the overlap of wave functions between Dirac zero modes and Higgs modes.**
- ➢ **We would like to see whether Dirac fermions are obtained at the low energy by using the renormalization group.**