Wall-crossing and operator ordering for ’t Hooft operators in N=2 gauge theories

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Based on the collaboration with

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Strings and Fields at YITP August 2019

1. Introduction

- Non-local objects play an important role in quantum field theories.

- In this talk, we focus on ’t Hooft line operators in a four-dimensional $\mathcal{N}$=2 supersymmetric gauge theory.

’t Hooft 78
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- An important physical quantity is the expectation values of the ’t Hooft line operators.

- The expectation values of ’t Hooft line operators in a four-dimensional theory on $S^1 \times \boldsymbol{R}^3$ may be evaluated by the localization method, turning on an Ω-deformation background.

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- In particular, we consider the U(N) gauge theory with 2N hypermultiplets in the fundamental representation.

- The minimal ’t Hooft line operators in the theory are in the fundamental or the anti-fundamental representation of U(N), which we denote by $T_{\square}$ and $T_{\overline{\square}}$ respectively.

- The expectation values of the minimal ’t Hooft line operators themselves are given by some universal formula and we are rather interested in their products.

- An example: $\langle T_{\square} \cdot T_{\bar{\square}} \rangle$

- It has been argued that the expectation value of the product may be evaluated by so-called the Moyal product.

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$$\langle T_{\square} \cdot T_{\bar{\square}} \rangle = \langle T_{\square} \rangle * \langle T_{\bar{\square}} \rangle$$

- However the Moyal product is generically non-commutative and there is an ambiguity with respect to the ordering.

$$\langle T_{\square} \cdot T_{\substack{\square\\\square}} \rangle = \langle T_{\square} \rangle * \langle T_{\substack{\square\\\square}} \rangle \ \text{ or } \ \langle T_{\substack{\square\\\square}} \rangle * \langle T_{\square} \rangle$$

How does this ordering ambiguity appear in the localization computation?

- Recently it has been argued that a non-perturbative contribution (monopole screening) to the expectation value can be computed from the Witten index of a supersymmetric quantum mechanics (SQM) which lives on the line defect.

Brennan, Dey, Moore 18, 19

- We point out that the discrete change associated with the different orderings precisely corresponds to the wall-crossing across different chambers in the Fayet-Iliopoulos parameter space of the SQM.

- In fact, the correspondence can be seen explicitly from a brane configuration in string theory.

- We will check the correspondence explicitly for the products of the two and three minimal ’t Hooft line operators.

Plan of Talk

1. Introduction

2. Branes and SQMs for ’t Hooft line operators

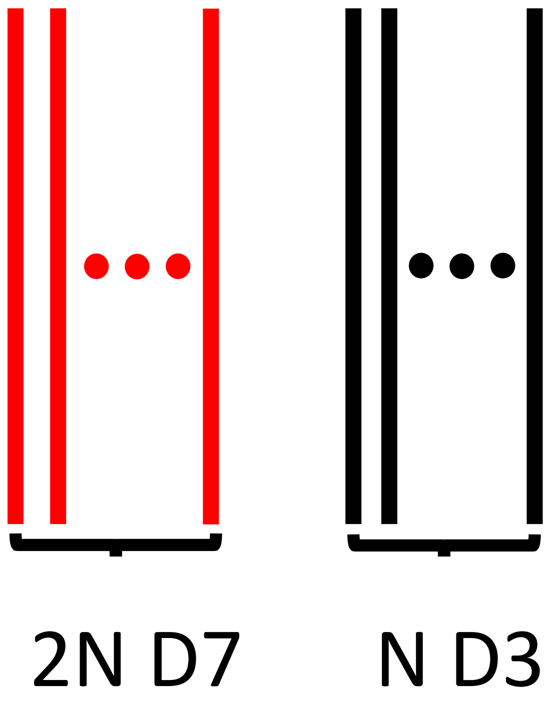
3. Wall-crossing and operator ordering

4. Conclusion

2. Branes and SQMs for ’t Hoof line operators

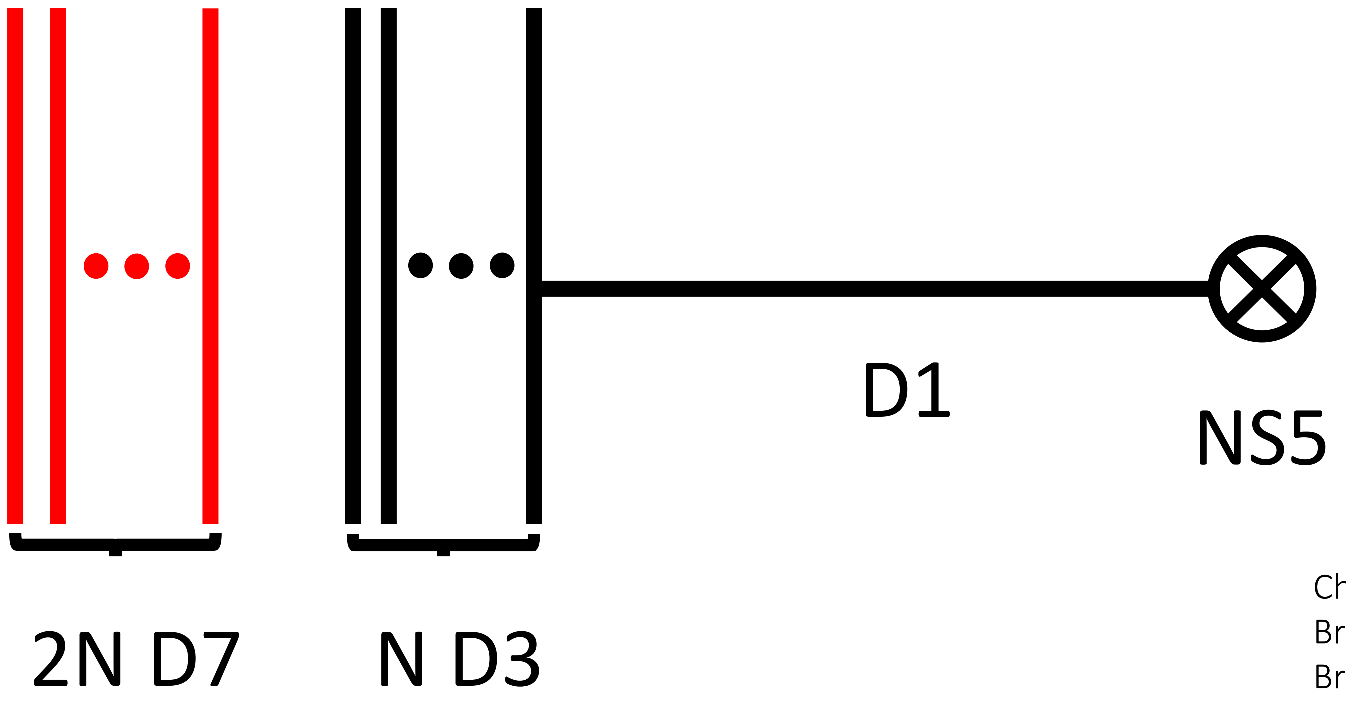
- A four-dimensional U(N) gauge theory can be realized on N D3-branes in type IIB string theory.

- 2N flavors are introduced by 2N D7-branes.



- In this setup, an ’t Hooft line defect is realized by a semi-infinite D1-brane ending on the D3-brane.

- It will be useful to make the D1-brane end on an NS5-brane.



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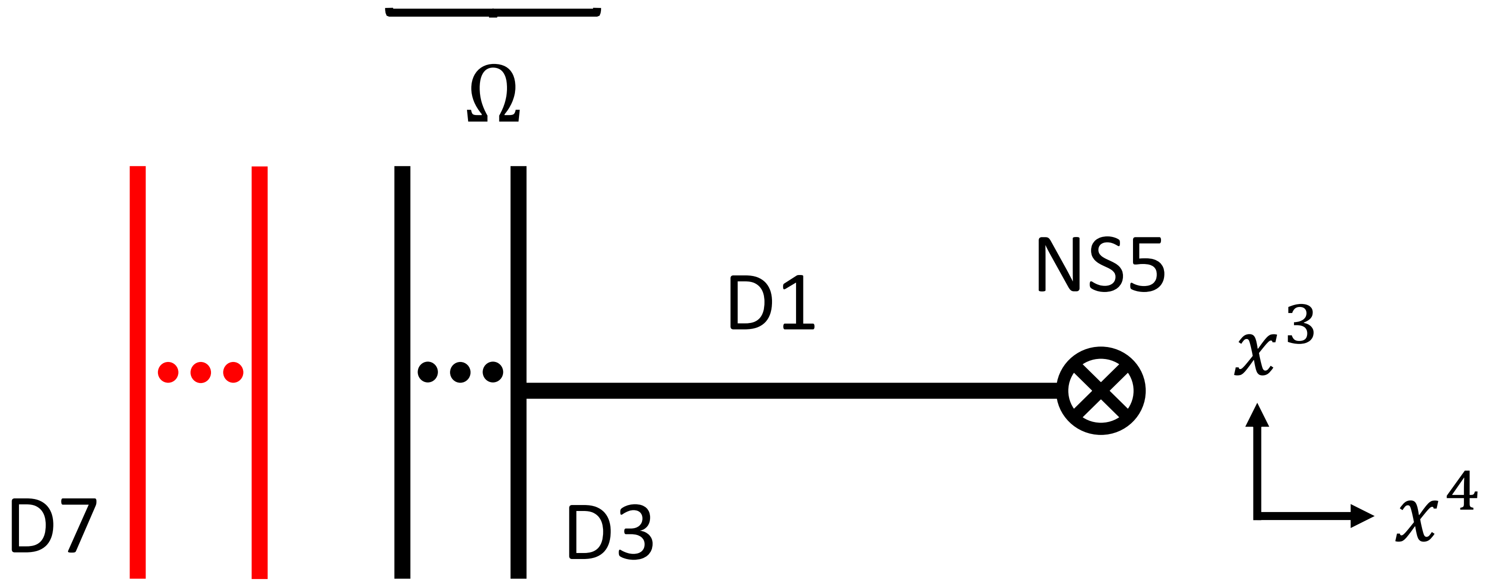
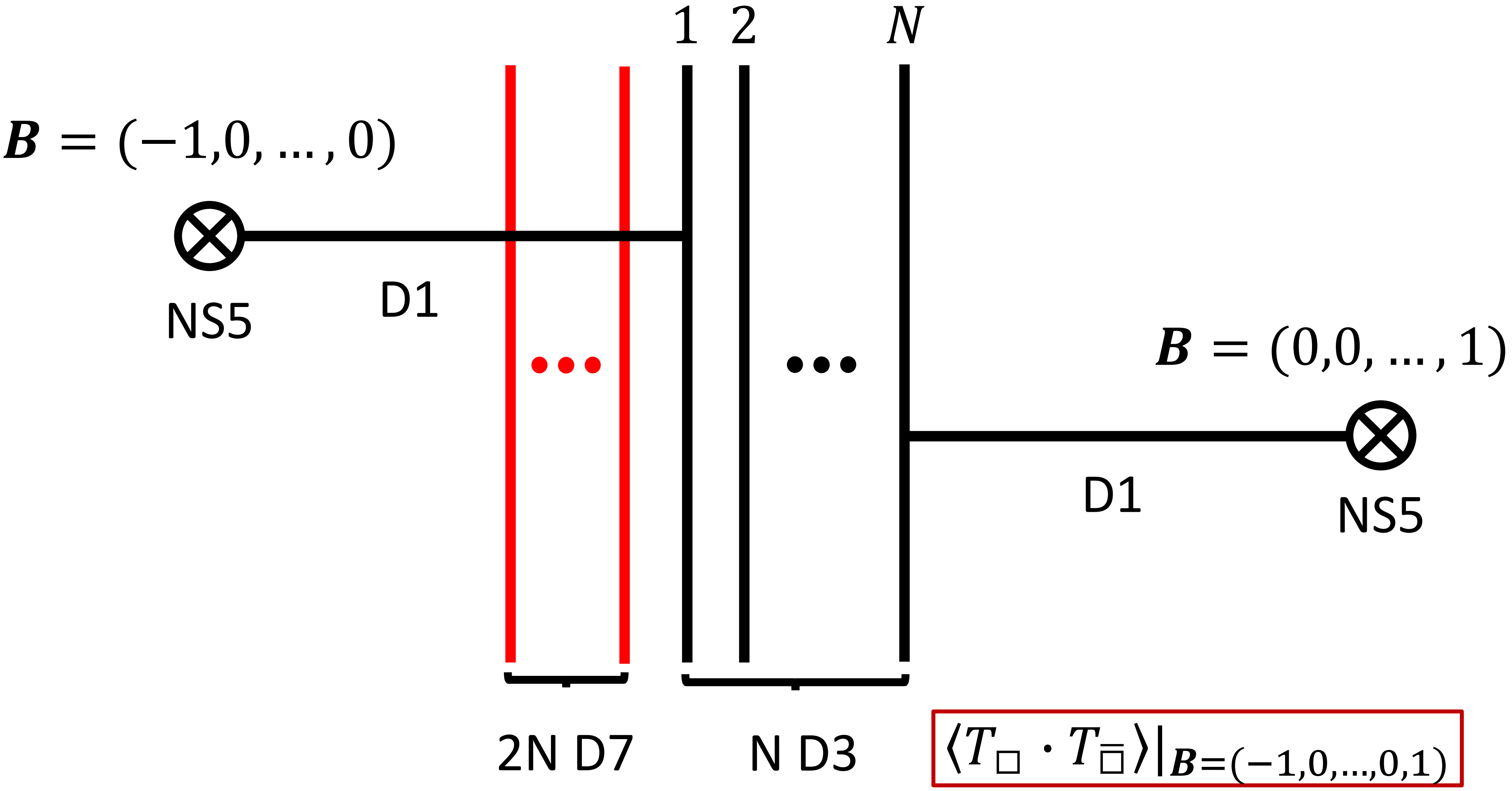
Brennan, Dey, Moore 18
Brennan 18

- The brane configuration is summarized as follows.

	0	1	2	3	4	5	6	7	8	9
D3	×	×	×	×						
D7	×	×	×	×			×	×	×	×
D1	×				×					
NS5	×					×	×	×	×	×





1
2
N
B = (−1,0, … , 0)
NS5
D1
B = (0,0, … , 1)
D1
NS5
2N D7
N D3
⟨T□ · T□⟩|B=(−1,0,…,0,1)

- We use the brane configuration to determine an SQM which lives on the line defect.

- The Witten index of the SQM gives a monopole screening contribution to the expectation value of the ’t Hooft line operator.

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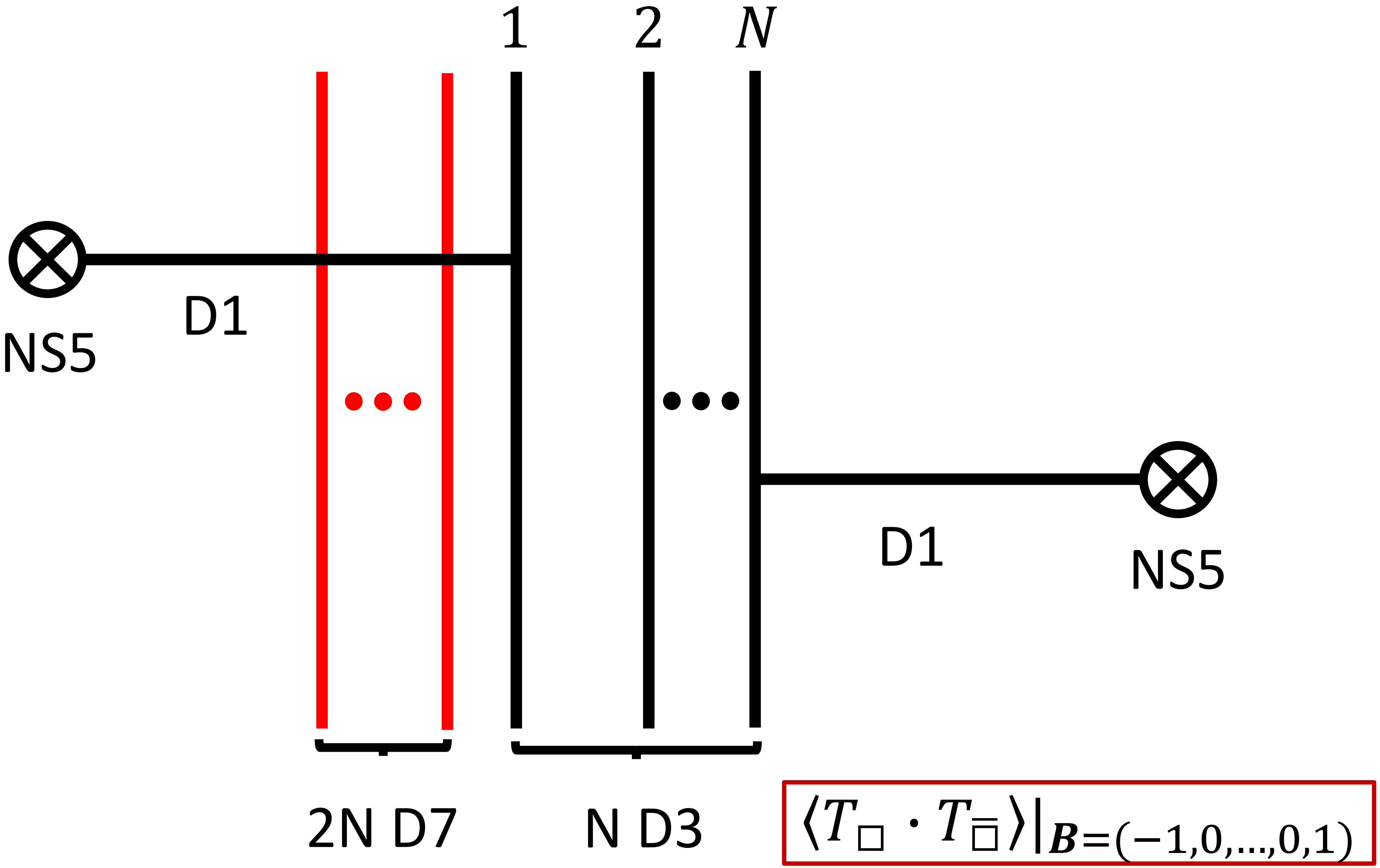
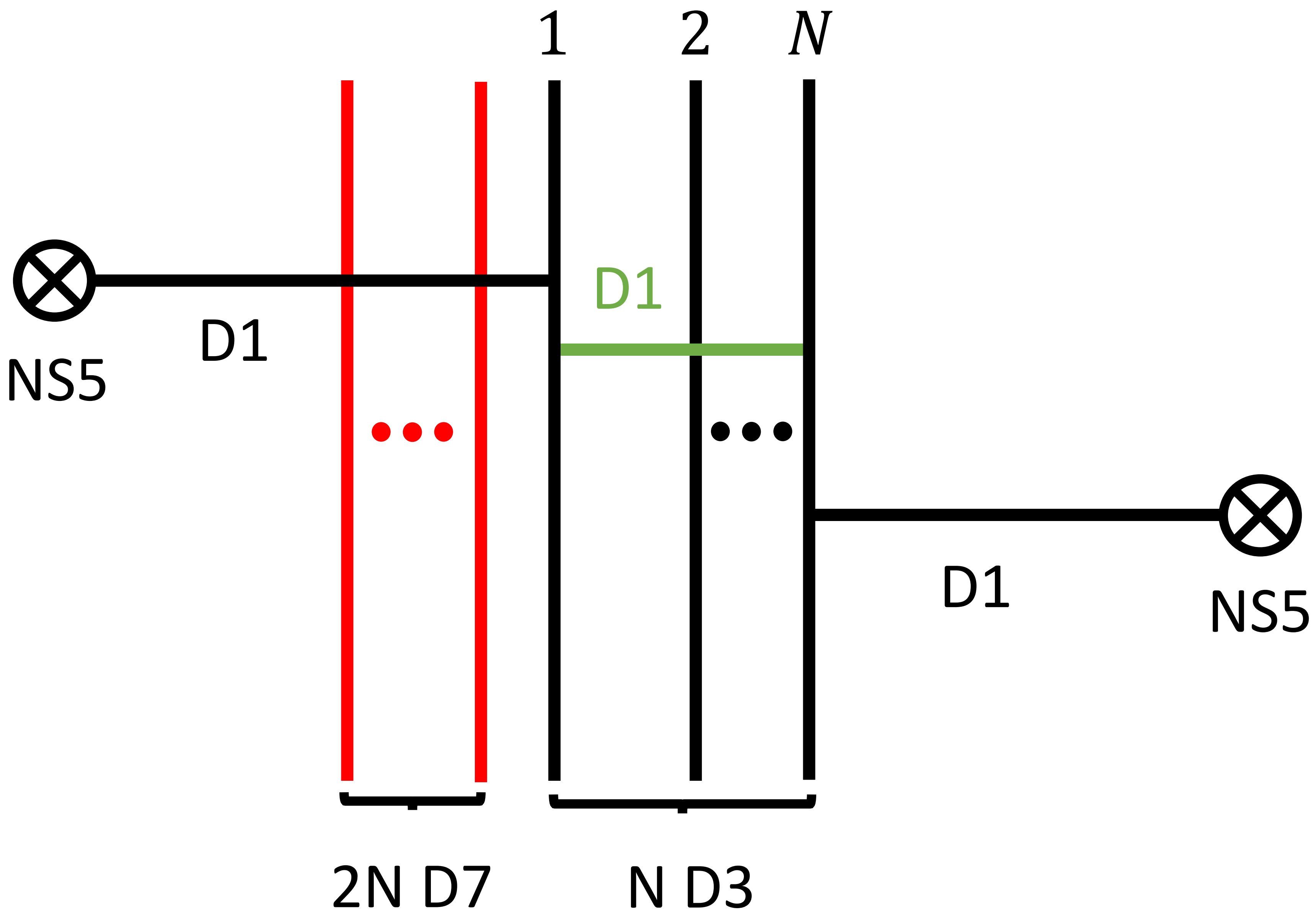
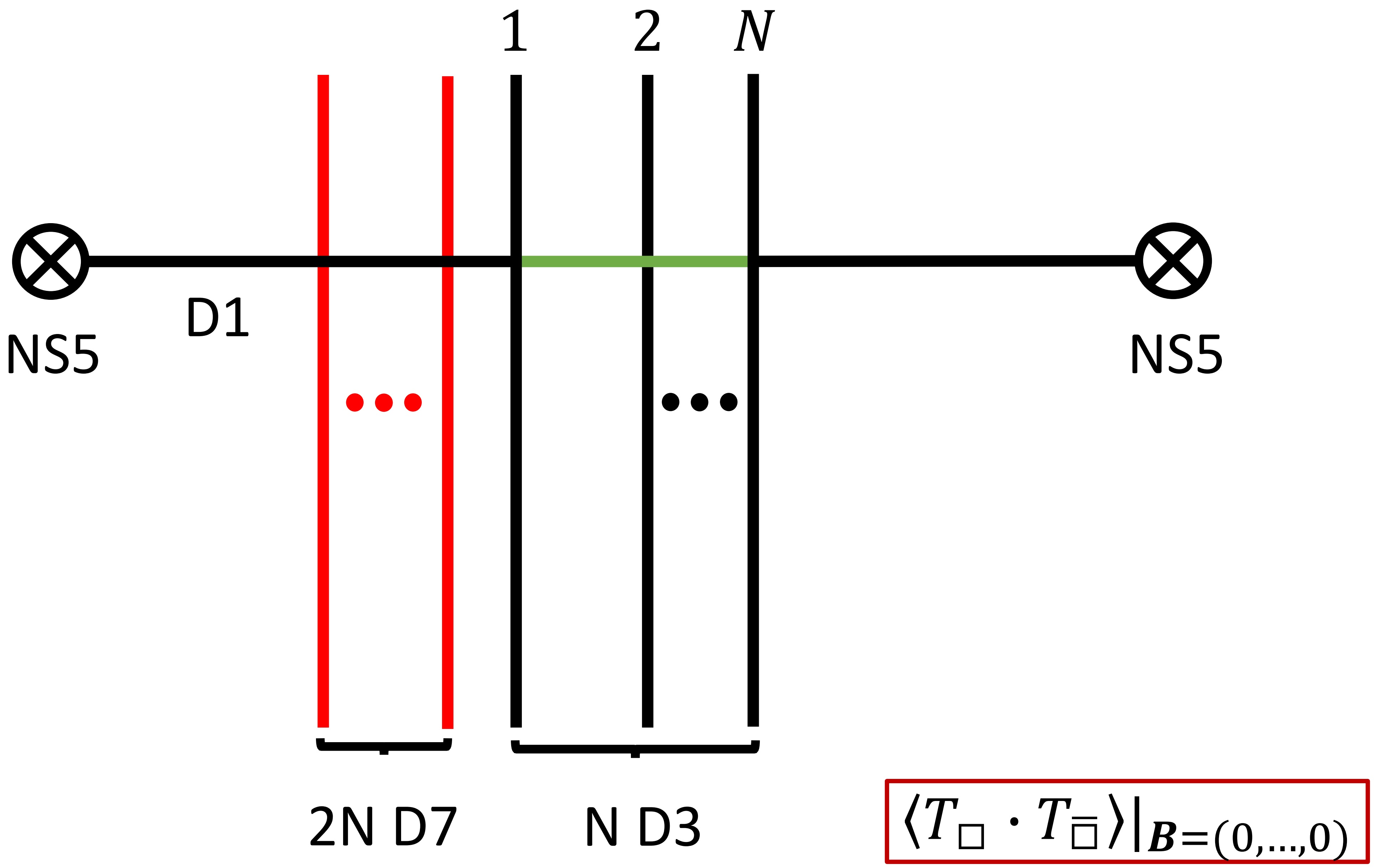
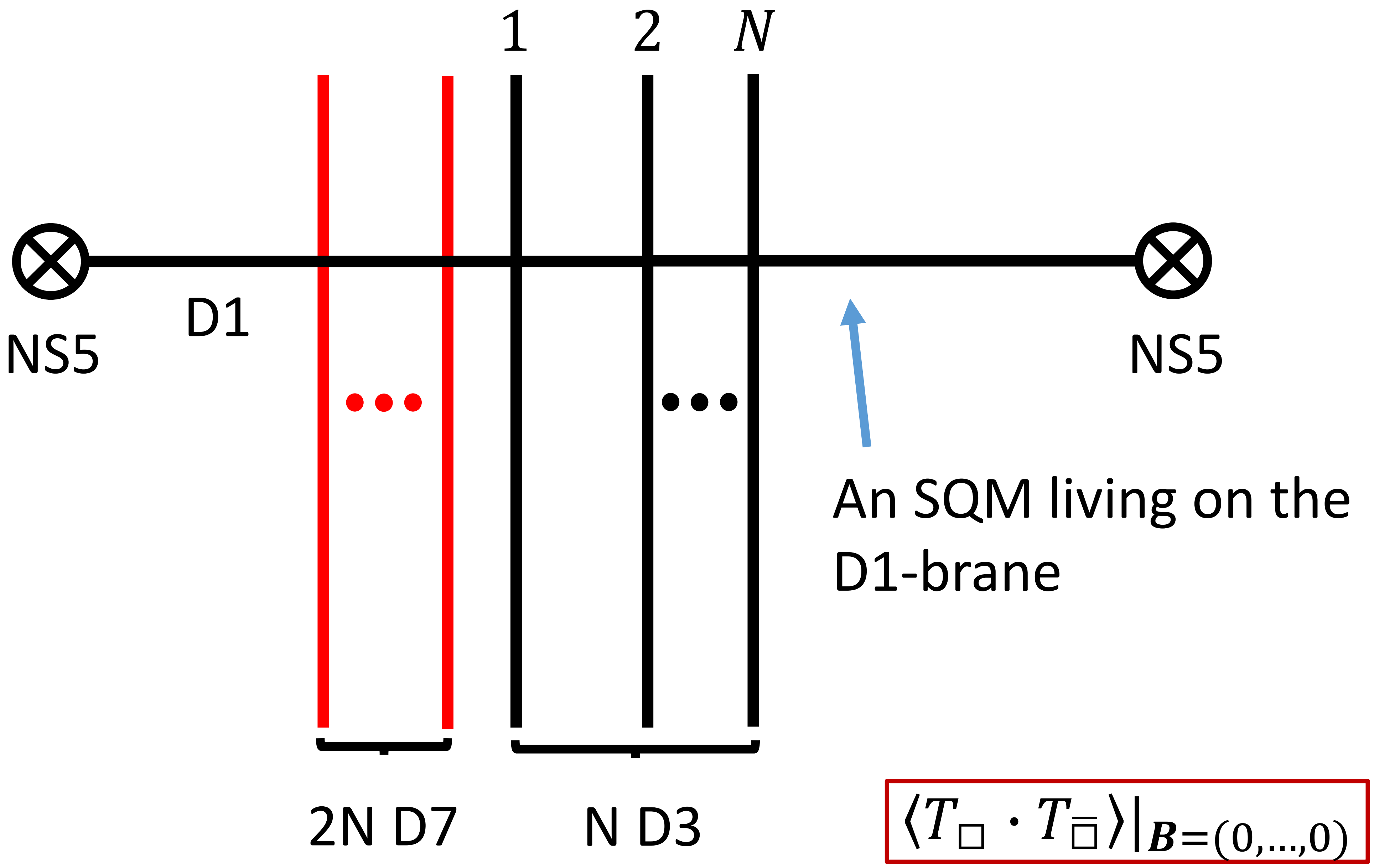
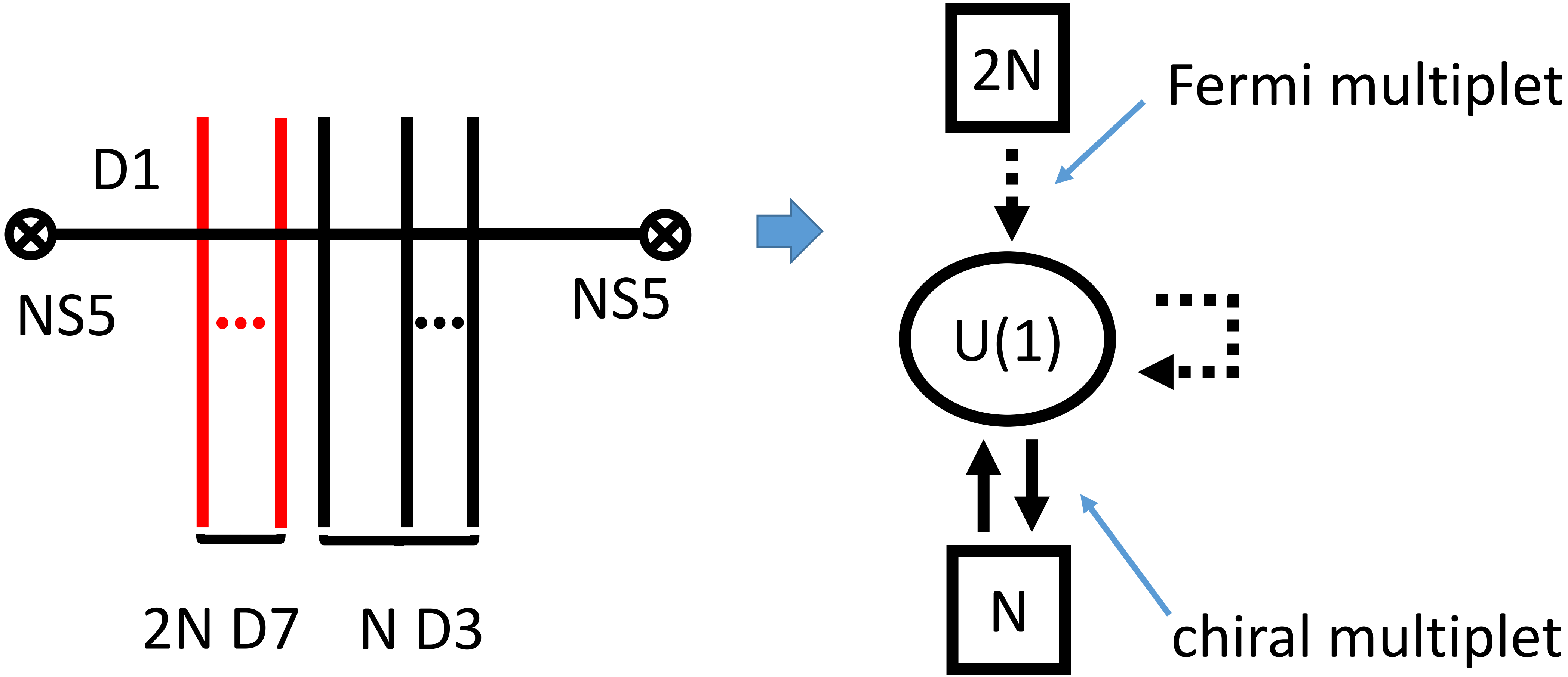

1
2
N
NS5
D1
D1
NS5
2N D7
N D3
$\langle T_{\square} \cdot T_{\bar{\square}} \rangle |_{\boldsymbol{B}=(-1,0,\ldots,0,1)}$


1
2
N
D1
NS5
D1
D1
NS5
2N D7
N D3


1
2
N
NS5
D1
NS5
2N D7
N D3
$\langle T_{\Box} \cdot T_{\bar{\Box}} \rangle |_{B=(0,\ldots,0)}$


1
2
N
D1
NS5
NS5
2N D7
N D3
An SQM living on the D1-brane
$\langle T_{\square} \cdot T_{\bar{\square}} \rangle|_{\boldsymbol{B}=(0,\ldots,0)}$


D1
NS5
NS5
2N D7
N D3
2N
Fermi multiplet
U(1)
N
chiral multiplet

3. Wall-crossing and operator ordering

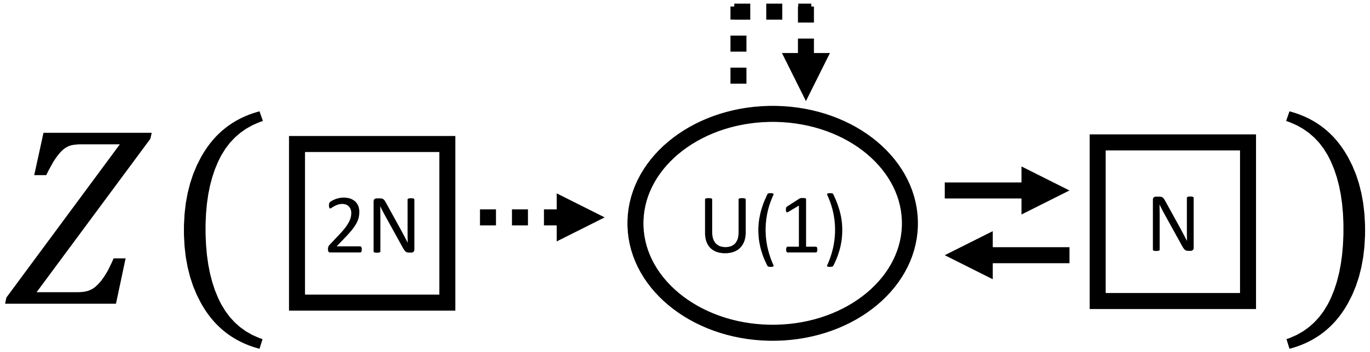
- We have two different ways to compute the $\boldsymbol{B} = (0,0,\ldots,0)$ screening sector .

$$\langle T_{\square} \cdot T_{\bar{\square}} \rangle|_{\boldsymbol{B}=\boldsymbol{0}}$$

$$\langle T_{\square} \rangle * \langle T_{\bar{\square}} \rangle = Z_{mono,1}(\boldsymbol{B} = \boldsymbol{0}) + \cdots$$

$$\langle T_{\bar{\square}} \rangle * \langle T_{\square} \rangle = Z_{mono,2}(\boldsymbol{B} = \boldsymbol{0}) + \cdots$$

vs



- In fact, the Witten index may be a piecewise constant function with respect to the Fayet-Iliopoulos parameter ζ for the U(1) gauge group.

Hwang, Kim, Kim, Park 14
Cordova, Shao 14
Hori, Kim, Yi 14

⇨ $$Z(\zeta > 0) \neq Z(\zeta < 0)$$ "wall-crossing"

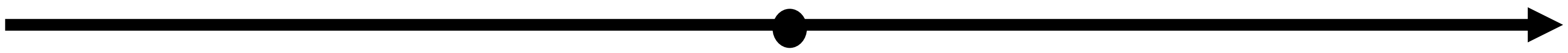
- It is then natural to expect that the two discrete changes are related to each other

- The explicit computation shows that they are indeed related like:

$$\langle T_{\square} \rangle * \langle T_{\overline{\square}} \rangle|_{\boldsymbol{B=0}} = Z(\zeta > 0)$$

$$\langle T_{\overline{\square}} \rangle * \langle T_{\square} \rangle|_{\boldsymbol{B=0}} = Z(\zeta < 0)$$

$$\langle T_{\overline{\square}} \rangle * \langle T_{\square} \rangle \qquad \langle T_{\square} \rangle * \langle T_{\overline{\square}} \rangle$$



ζ

$\zeta = 0$

Assel, Sciarappa 19
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- We can in fact see the relation explicitly from the brane realization.

- First the different orderings may mean different orderings of the insertion point of the ’t Hooft line operators on the x^3-axis.

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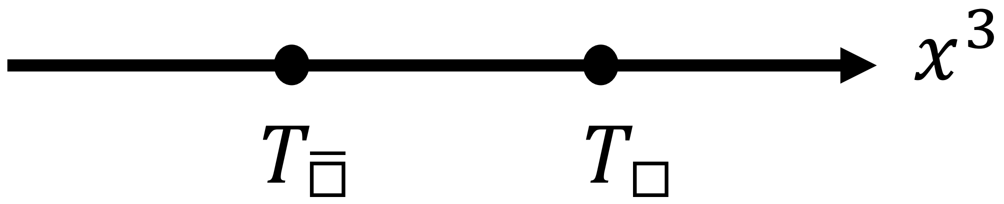
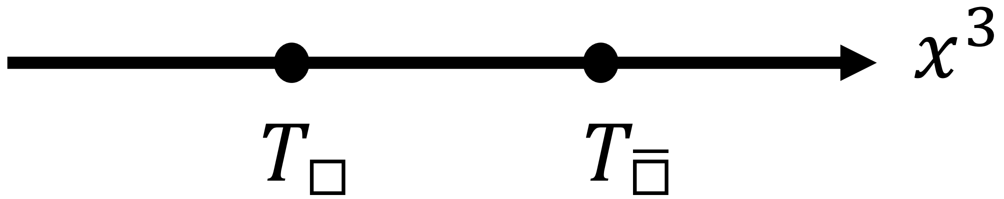
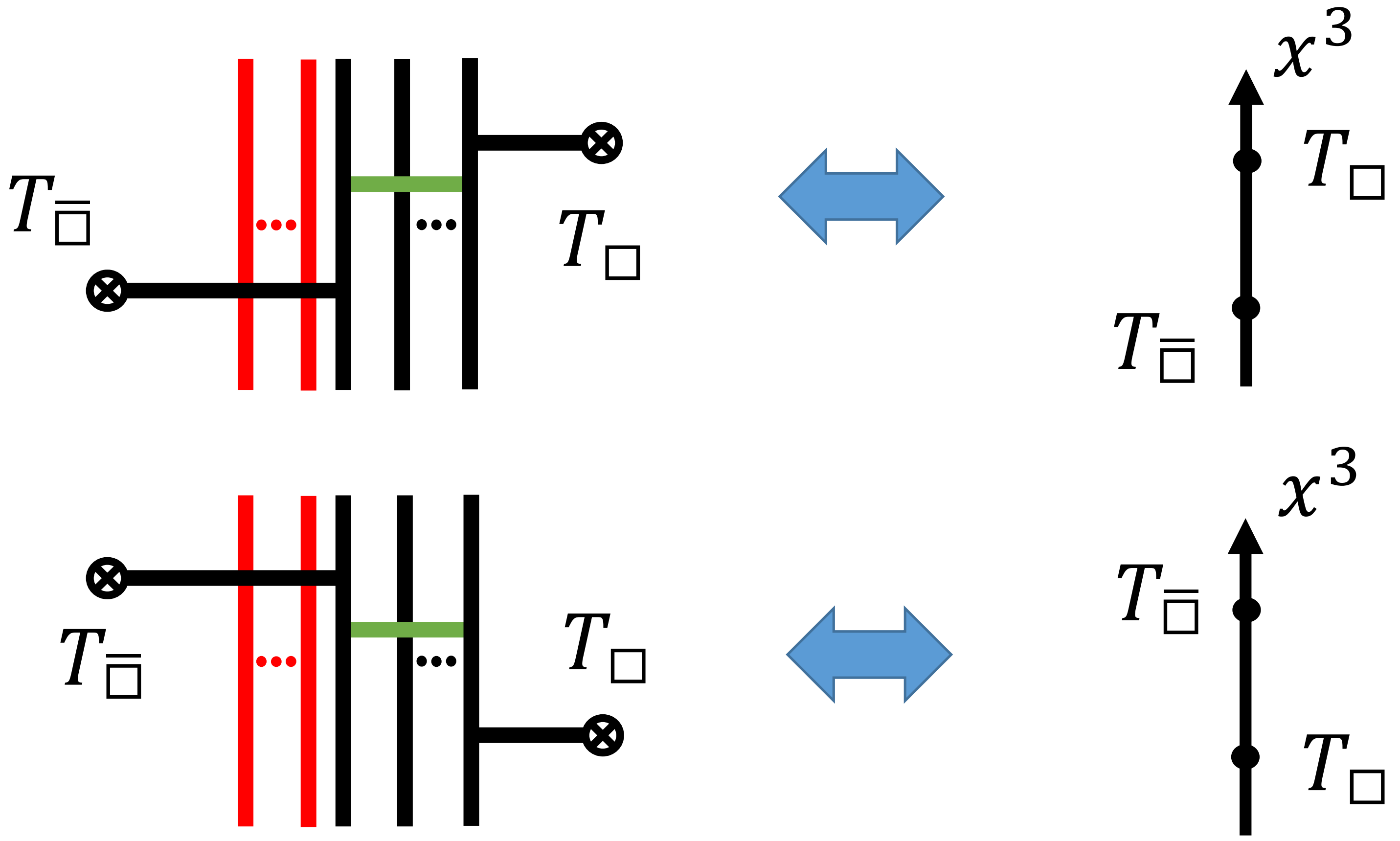
$\langle T_{\square}\rangle * \langle T_{\bar{\square}}\rangle$ ⇔



$\langle T_{\bar{\square}}\rangle * \langle T_{\square}\rangle$ ⇔



- In the brane construction, the position of the ’t Hooft line operators is the position of the corresponding D1-branes (or NS5-branes).



- On the other hand, in terms of the SQM realized on the D1-brane, the difference of the position of the two NS5-branes is the FI parameter.

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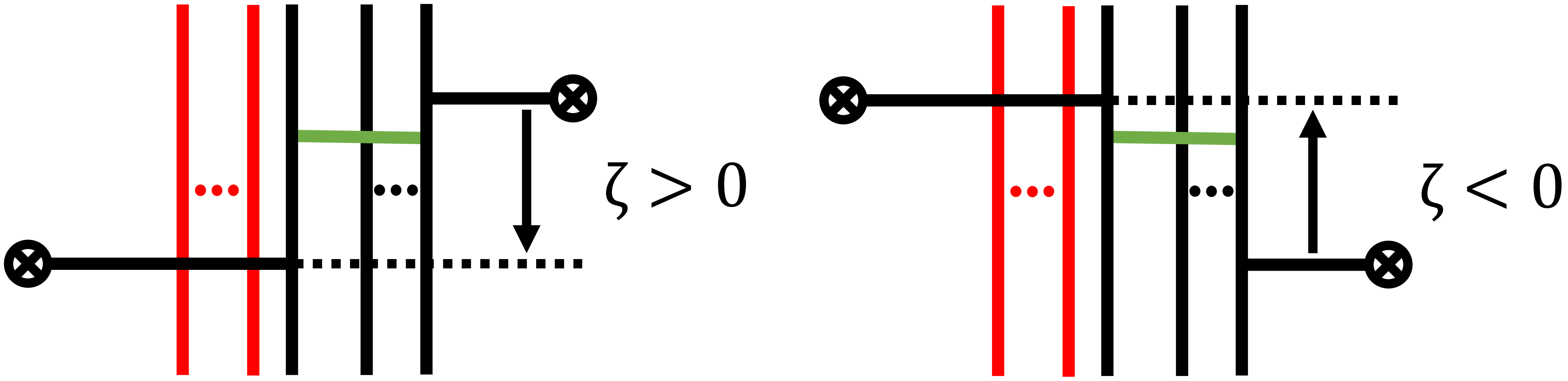
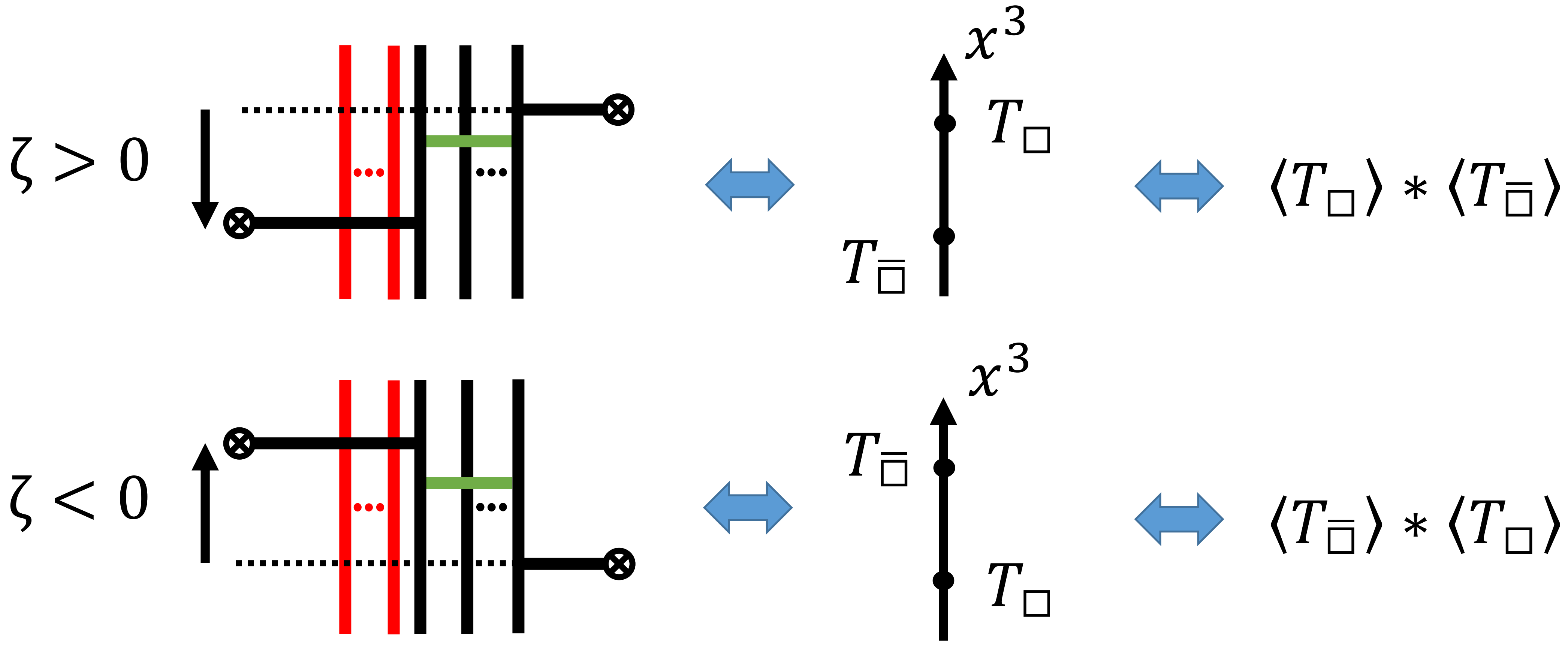
Brennan 18



- Hence the different orderings of the Moyal product are directly related to the different signs of the FI parameter in the SQM through the brane configuration.



- It is possible to generalize the configuration into products of three minimal ’t Hooft line operators.

- For example we consider the products of $T_{\square}, T_{\square}, T_{\overline{\square}}$.

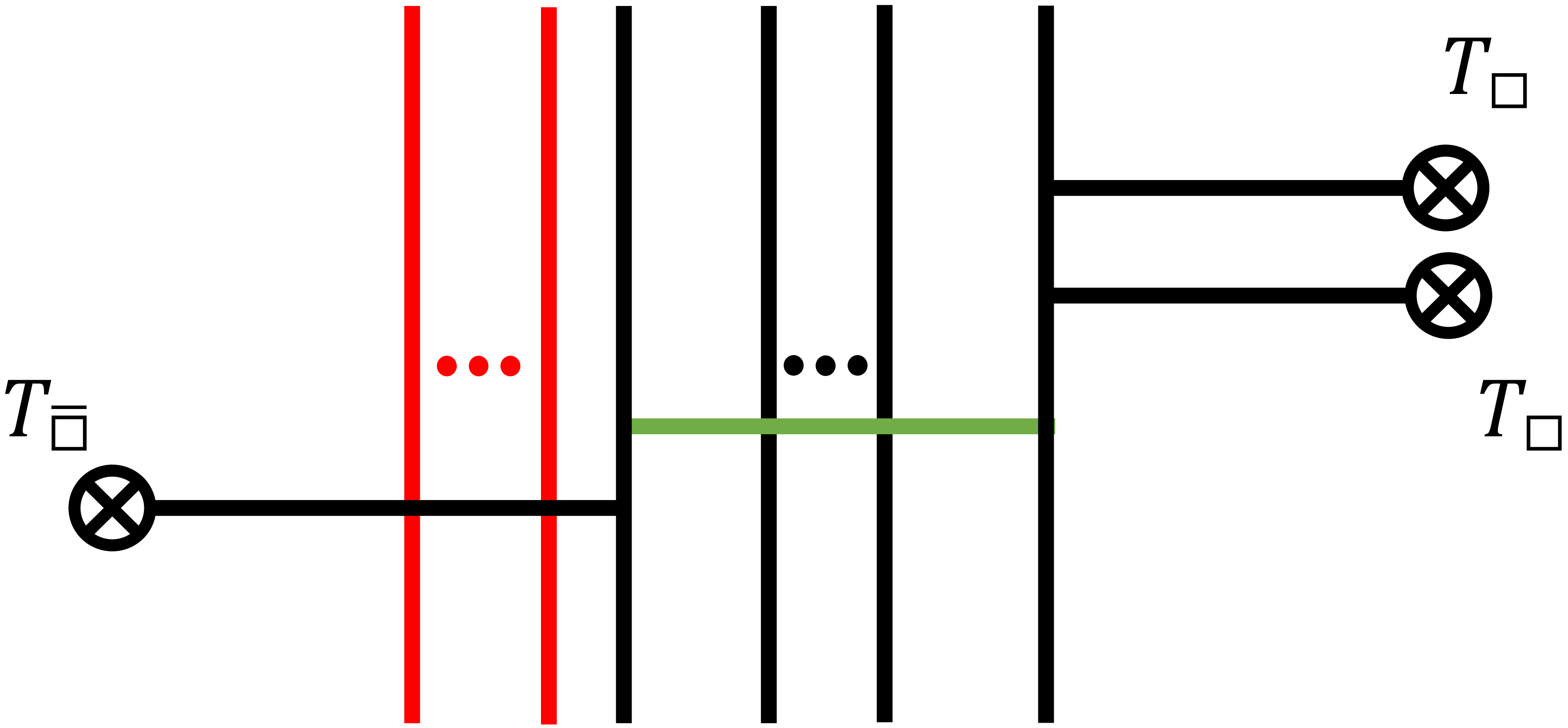
- There are three orderings for the Moyal product:

$$\langle T_{\square} \rangle * \langle T_{\square} \rangle * \langle T_{\overline{\square}} \rangle$$

$$\langle T_{\square} \rangle * \langle T_{\overline{\square}} \rangle * \langle T_{\square} \rangle$$

$$\langle T_{\overline{\square}} \rangle * \langle T_{\square} \rangle * \langle T_{\square} \rangle$$

- We focus on the $\boldsymbol{v} = (0, \ldots, 0,1)$ sector of the product.
- The corresponding brane configuration is given as follows.



- We focus on the $\boldsymbol{v} = (0, \dots, 0,1)$ sector of the product.
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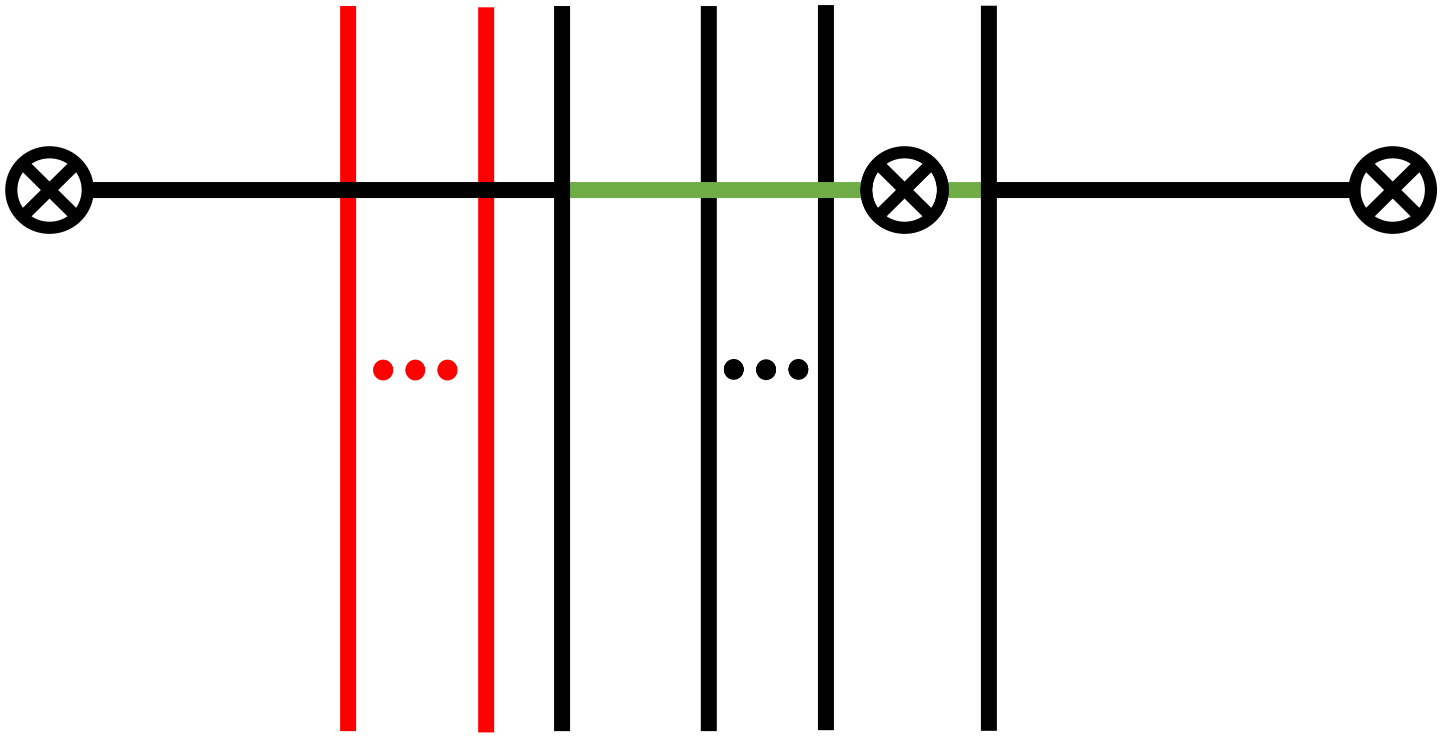
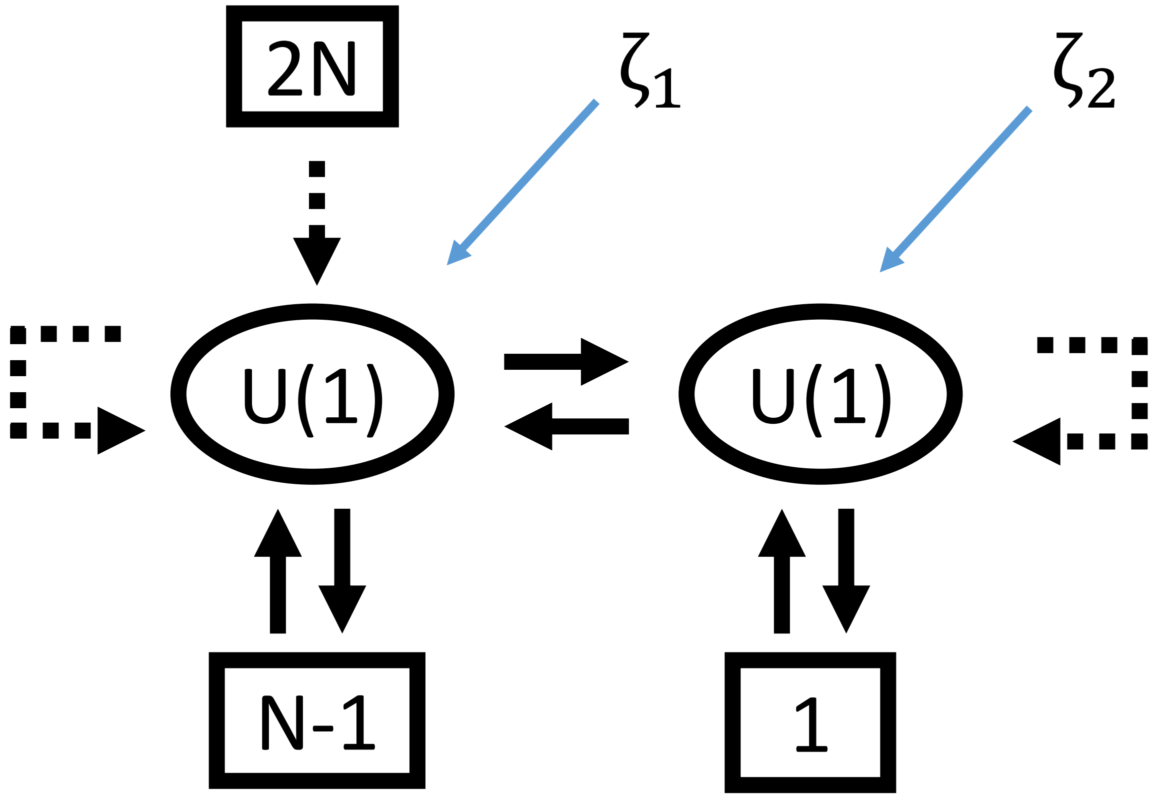


- The SQM which describes the monopole screening contribution can be read off from the worldvolume theory on the D1-brane.



- The correspondence between the orderings and the FI-chambers:

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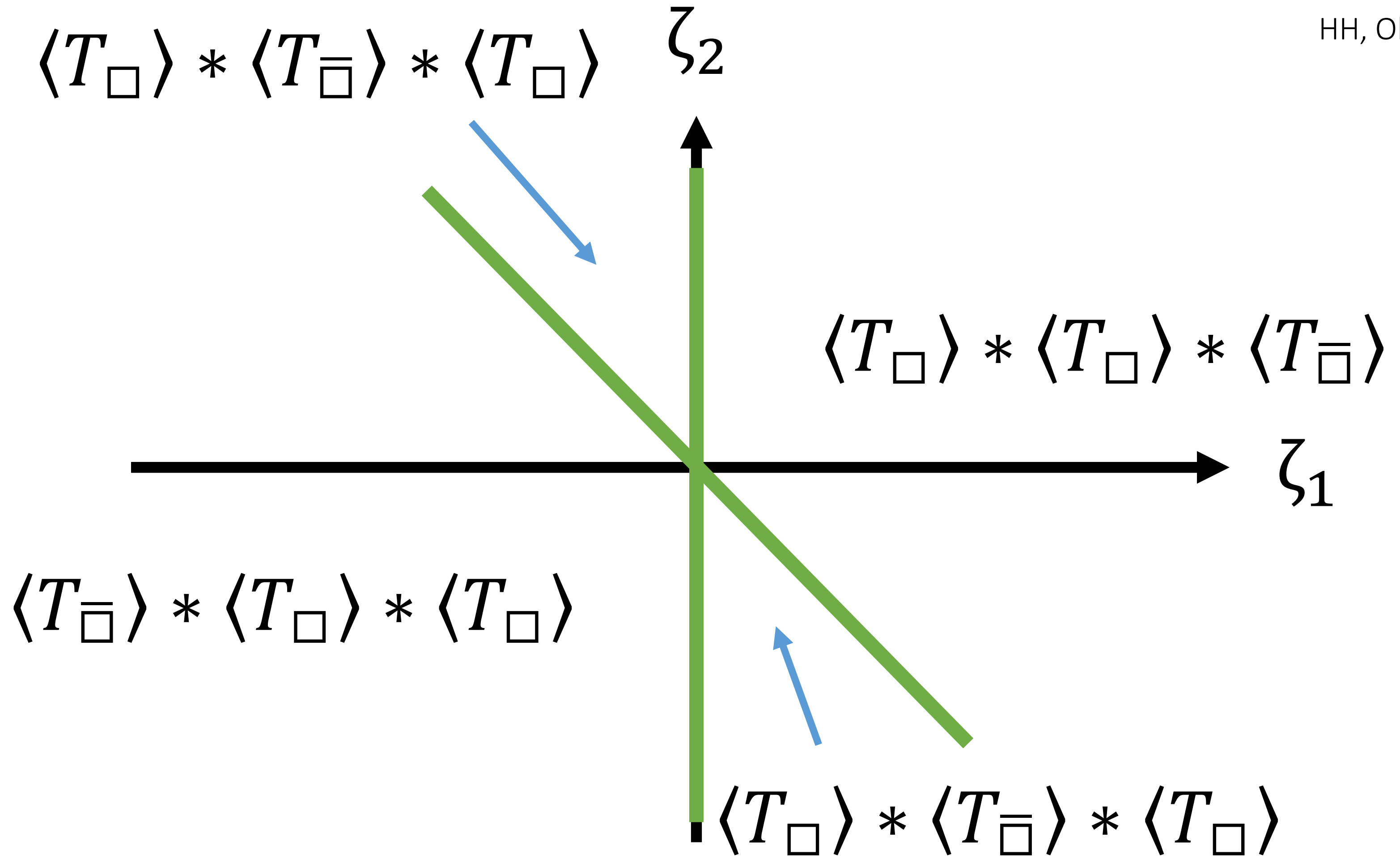

4. Conclusion

- The different orderings of the Moyal products correspond to different FI-chambers in SQMs.
- The correspondence can be seen explicitly seen from the brane configuration.
- We checked the correspondence explicitly by comparing the monopole screening contributions in the Moyal products and those in the Witten index of SQMs for the products of two and three minimal ’t Hooft line operators.