Supersymmetric localization and black holes microstates

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YITP (Kyoto), August 19-23

Strings and Fields 2019

Introduction

Black holes have more lessons in store for us!

$$\text{Bekenstein-Hawking entropy:} \qquad S_{\rm BH} = \frac{\text{Area}}{4G_{\rm N}} \,.$$

The number of black hole microstates $d_{\rm micro}$ should then be given by

$$d_{\rm micro} = \mathrm{e}^{S_{\rm BH}} \,.$$

But where are the microstates accounting for the black hole entropy?

String theory provides a precise statistical mechanical interpretation of $S_{\rm BH}$ for a class of asymptotically flat black holes. [Strominger, Vafa'96]

Black holes $\rightarrow$ bound states of D-branes!

Introduction

No similar results for $\mathrm{AdS}_{d+1>4}$ black holes was known until recently!

[Benini, Hristov, Zaffaroni'15]

Holography + supersymmetric localization

Black hole entropy → counting states in the dual CFT

This talk

I will review recent progress for AdS_{d+1} BHs in diverse dimensions.

Basics

Stringy BPS black holes

I KN-AdS black holes $\leftrightarrow$ SCFT$_d$ on $S^{d-1} \times \mathbb{R}_t$

II *magnetic* AdS black holes $\leftrightarrow$ SCFT$_d$ on $\mathcal{M}^{d-1} \times \mathbb{R}_t$

- Case I has to *rotate*.
- Case II is *topologically twisted* and can be static.
  - Characterized by nonzero magnetic fluxes for the graviphoton/R-symmetry:

$$\int_{\mathcal{C} \subset \mathcal{M}_{d-1}} F \in 2\pi\mathbb{Z}\,.$$

Most manifest in AdS$_4$ black holes w/ horizon AdS$_2 \times S^2$. [Romans'92]

Counting microstates

BPS partition function

$$Z(\Delta_I, \omega_i) = \mathrm{Tr}_{\mathcal{Q}=0}\, e^{\mathrm{i}(\Delta_I Q_I + \omega_i J_i)} = \sum_{Q_I, J_i} d_{\text{micro}}(Q_I, J_i) e^{\mathrm{i}(\Delta_I Q_I + \omega_i J_i)} \,.$$

- It counts states w/ the same susy, charges, and angular momenta.
- $S_{\text{BH}}(Q_I, J_i) = \log d_{\text{micro}}(Q_I, J_i)\,,$

$$d_{\text{micro}}(Q_I, J_i) = e^{S_{\text{BH}}(Q_I, J_i)} = \int_{\Delta_I,\, \omega_i} Z(\Delta_I, \omega_i) e^{-\mathrm{i}(\Delta_I Q_I + \omega_i J_i)} \,.$$

Saddle point approximation (large charges)

$$S_{\text{BH}}(Q_I, J_i) \equiv \mathcal{I}(\Delta_I, \omega_i) = \log Z(\Delta_I, \omega_i) - \mathrm{i}(\Delta_I Q_I + \omega_i J_i) \,.$$

- $\dfrac{\partial \mathcal{I}(\Delta_I, \omega_i)}{\partial \Delta_I} = \dfrac{\partial \mathcal{I}(\Delta_I, \omega_i)}{\partial \omega_i} = 0 \,.$

Counting microstates

Problem

AdS BHs preserve *only* two real supercharges while we have efficient tools for counting states preserving four..

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Witten index (supersymmetric partition function)

$$Z^{\text{susy}}_{\mathcal{M}^{d-1}\times S^1}(\Delta_I,\omega_i) = \text{Tr}_{\mathcal{H}_{\mathcal{M}^{d-1}}}(-1)^F e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}} e^{\text{i}(\Delta_I Q_I + \omega_i J_i)} \,.$$

- Superconformal index for SCFTs on $S^{d-1}\times S^1$

  [Romelsberger'05; Kinney, Maldacena, Minwalla, Raju'05]

- Topologically twisted index for SCFTs on twisted $\mathcal{M}^{d-1}\times S^1$

  [Okuda, Yoshida'12; Nekrasov, Shatashvili'14; Gukov, Pei'15; Benini, Zaffaroni'15]

Lower bound on entropy. Index = entropy *if* there are *no* large cancellations between bosonic and fermionic ground states.

[Arguments for some asymptotically flat black holes by Sen'09]

Magnetic AdS black holes

Black holes in M-theory on $\mathrm{AdS}_4 \times S^7$:

[Cacciatori, Klemm'08; Dall'Agata, Gnecchi'10; Hristov, Vandoren'10; Halmagyi14; Hristov, Katmadas, Toldo'18]

- Preserve two real supercharges (1/16 BPS)
- Four electric and magnetic charges $(\mathfrak{p}^a, q_a)$ under $\mathrm{U}(1)^4 \subset \mathrm{SO}(8)$, one angular momentum $\mathcal{J}$ in AdS_4.
- *Only* seven independent parameters:

$$\textit{twisting condition:} \qquad \sum_{a=1}^{4} \mathfrak{p}^a = 2 - 2\mathfrak{g}\,.$$

  together with a charge constraint for having a regular horizon.
- $S_{\mathrm{BH}} = \mathcal{O}(N^{3/2})\,.$
- We focus on $\mathcal{J} = 0$.
- Near horizon $\mathrm{AdS}_2 \times \Sigma_{\mathfrak{g}}\,.$

Magnetic AdS black holes

Setting all $q_a = 0$

$$S_{\mathrm{BH}}(\mathfrak{p}) = \frac{2\pi}{3} N^{3/2} \sqrt{F_2 + \sqrt{\Theta}}\,,$$

$$F_2 \equiv \frac{1}{2} \sum_{a<b} \mathfrak{p}_a \mathfrak{p}_b - \frac{1}{4} \sum_{a=1}^{4} \mathfrak{p}_a^2\,, \qquad \Theta \equiv (F_2)^2 - 4\mathfrak{p}_1\mathfrak{p}_2\mathfrak{p}_3\mathfrak{p}_4\,.$$

- Attractor mechanism:

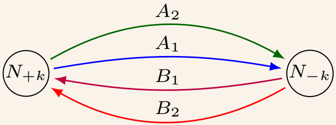
$$S_{\mathrm{BH}}(\mathfrak{p}^a, q_a) = \mathrm{i}\mathfrak{p}^a \frac{\partial \widetilde{\mathcal{W}}(\Delta_a)}{\partial \Delta_a} - \mathrm{i}\Delta_a q_a \Big|_{\text{crit.}}\,.$$

  - g-sugra prepotential: $\widetilde{\mathcal{W}}(\Delta_a) = -2\mathrm{i}\sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}\,.$
  - $\sum_a \Delta_a = 2\pi$: scalar fields at the horizon.

[Ferrara, Kallosh, Strominger' 06; Cacciatori, Klemm'08; Dall'Agata, Gnecchi'10]

Holographic setup

ABJM on $S^2 \times \mathbb{R}$ w/ a twist on S^2



$$W = \mathrm{Tr}\left(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right) ,$$

$$\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 2\pi ,$$

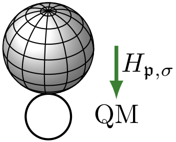
$$\mathrm{U}(1)_R \times \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_{\text{top}} .$$

- Magnetic background for global symmetries: Landau levels on S^2.
- Twisting condition: $\sum_{a=1}^4 \mathfrak{p}^a = 2\,.$

$$D_\mu \epsilon = \partial_\mu \epsilon + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon + \mathrm{i} \underbrace{V_\mu}_{\frac{\mathrm{i}}{4}\omega_\mu^{ab}\gamma_{ab}} \epsilon = \partial_\mu \epsilon$$

$\epsilon =$ constant on S^2.

Holographic microstates counting



A topologically twisted index

$$Z_{S^2\times S^1_\beta}(v_a,\mathfrak{p}^a) = \mathrm{Tr}_{\mathcal{H}_{S^2}} (-1)^F e^{-\beta H} e^{i\sum_{a=1}^4 \Delta_a Q_a}\,.$$

[Benini, Zaffaroni; 1504.03698]

- Δ_a : chemical potentials for flavor symmetry charges Q_a.
- σ_a : real masses.
- only states with $0 = H - \sigma_a J_a$ contribute.
- electric charges q_a can be introduced using Δ_a.
- can be computed using *supersymmetric localization*.

The index is a holomorphic function of v_a with $v_a = \Delta_a + i\beta\sigma_a$.

$$\sigma_a = 0\,.$$

Supersymmetric localization

Consider a supersymmetric gauge theory on a compact manifold $\mathcal{M}$.

Partition function

$$Z_{\mathcal{M}} \equiv \text{Euclidean Feynman path integral} = \int \mathcal{D}\phi \, e^{-S[\phi]} \, .$$

- ϕ: the set of fields in the theory.
- $S[\phi]$: the action functional.

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Localization argument [Witten'88; Pestun'06]

- Let δ be a Grassmann-odd symmetry of our theories, *i.e.* $\delta S = 0$.
- Deform the theories by a δ-exact term.

$$Z_{\mathcal{M}}(t) = \int \mathcal{D}\phi\, e^{-S[\phi]-t\delta V}\,, \qquad t \in \mathbb{R}_{>0}\,.$$

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$$Z_{\mathcal{M}}(t) = \int \mathcal{D}\phi \, e^{-S[\phi] - t\delta V} \, , \qquad t \in \mathbb{R}_{>0} \, .$$

The partition function is *independent* of t!

$$\frac{\partial Z_{\mathcal{M}}(t)}{\partial t} = -\int \mathcal{D}\phi \, e^{-S[\phi] - t\delta V} \delta V = -\int \mathcal{D}\phi \, \delta \left(e^{-S[\phi] - t\delta V} V \right) = 0 \, .$$

Hence we can evaluate $Z_{\mathcal{M}}(t)$ as $t \to \infty$.

Supersymmetric localization

Localization locus

If $(\delta V)|_{\text{even}} \geq 0 \implies$ the integral *localizes* to $(\delta V)|_{\text{even}}(\phi_0) = 0\,.$

- Let's parameterize the fields around the localization locus by

$$\phi = \phi_0 + t^{-1/2}\hat{\phi}\,.$$

- For large t, we can Taylor expand the action around ϕ_0:

$$S + \delta V = S[\phi_0] + (\delta V)^{(2)}[\hat{\phi}] + \mathcal{O}(t^{-1/2})\,.$$

- Gaussian integration!

Localization formula

$$Z_{\mathcal{M}} = \int_{(\delta V)|_{\text{even}}=0} \mathcal{D}\phi_0\, e^{-S[\phi_0]}\, Z_{\text{1-loop}}[\phi_0]\,.$$

- $Z_{\text{1-loop}}[\phi_0]$: the ratio of fermionic and bosonic determinants.

A topologically twisted index

Localization formula [Benini, Zaffaroni'15; Closset, Kim, Willett'16]

$$Z_{S^2\times S^1}(\mathfrak{p},y) = \frac{1}{|\mathfrak{W}|}\sum_{\mathfrak{m}\,\in\,\Gamma_{\mathfrak{h}}}\oint_{\mathcal{C}} Z_{\text{int}}\left(\mathfrak{m},x;\mathfrak{p},y\right)\,,$$

- $x = \mathrm{e}^{\mathrm{i}u}$, $y_a = \mathrm{e}^{\mathrm{i}\Delta_a}$.
- Classical piece:

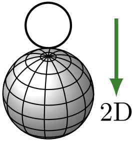
$$Z_{\text{cl}} = x^{k\mathfrak{m}}\,.$$

- One-loop contributions:

$$Z^{\chi}_{\text{1-loop}} = \prod_{\rho\in\mathfrak{R}}\left(\frac{\sqrt{x^\rho y_a}}{1-x^\rho y_a}\right)^{\rho(\mathfrak{m})-\mathfrak{p}^a+1}\,, \qquad Z^{\text{V}}_{\text{1-loop}} = \prod_{\alpha\in G}(1-x^\alpha)\,.$$

We are interested in the large N limit of the matrix integral.

TQFT and Bethe vacua



Reduction to two-dimensional theory w/ all KK modes on S^1

[Witten'92; Nekrasov, Shatashvili'09]

- Massive theory w/ a set of discrete vacua (Bethe vacua),

$$\exp\left(\mathrm{i}\frac{\partial\mathcal{W}(x)}{\partial x}\right)\Bigg|_{x=x^*} = 1\,, \qquad \mathcal{W}(x,y_a) = \sum_{\rho\in\mathfrak{R}} \mathrm{Li}_2(x^\rho y_a) + \ldots\,.$$

Many 3D and 4D supersymmetric partition functions can be written as a sum over Bethe vacua. [Closset, Kim, Willett'17'18]

A topologically twisted index

Bethe sum formula:

$$Z_{S^2\times S^1}(\mathfrak{p}, y) = \frac{(-1)^{\text{rk}(G)}}{|\mathfrak{W}|} \sum_{x^*} Z_{\text{int}}\left(\mathfrak{m}=0, x^*; \mathfrak{p}, y\right) \left(\det_{ij} \partial_i \partial_j \mathcal{W}(x)\right)^{-1} .$$

[Okuda, Yoshida'12; Nekrasov, Shatashvili'14; Gukov, Pei'15; Benini, Zaffaroni'15; Closset, Kim, Willett'17]

A topologically twisted index

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[Okuda, Yoshida'12; Nekrasov, Shatashvili'14; Gukov, Pei'15; Benini, Zaffaroni'15; Closset, Kim, Willett'17]

For ABJM:

$$\mathcal{W} = \frac{k}{2}\sum_{i=1}^{N}(\tilde{u}_i^2 - u_i^2) + \sum_{i,j=1}^{N}\left[\sum_{b=3}^{4}\text{Li}_2\left(e^{\mathrm{i}(\tilde{u}_j-u_i+\Delta_b)}\right) - \sum_{a=1}^{2}\text{Li}_2\left(e^{\mathrm{i}(\tilde{u}_j-u_i-\Delta_a)}\right)\right] .$$

- At large N one Bethe vacuum dominates the partition function.

$$u_i = \mathrm{i}N^{1/2}t_i + v_i\,, \qquad \tilde{u}_i = \mathrm{i}N^{1/2}t_i + \tilde{v}_i\,.$$

$\mathcal{I}$-extremization principle

In the large N limit [Benini, Hristov, Zaffaroni'15]

$$\mathcal{I}(\Delta_a, \mathfrak{p}^a) \equiv \log Z_{S^2 \times S^1}(\Delta_a, \mathfrak{p}^a) - \mathrm{i} \sum_{a=1}^{4} \Delta_a q_a \Big|_{\text{crit.}}$$

$$= \sum_{a=1}^{4} \mathrm{i}\mathfrak{p}^a \frac{\partial \widetilde{\mathcal{W}}(\Delta_a)}{\partial \Delta_a} - \mathrm{i}\Delta_a q_a \Big|_{\text{crit.}} .$$

- $\mathcal{W}(x^*) \equiv \widetilde{\mathcal{W}}(\Delta_a) = \frac{2\mathrm{i}}{3} N^{3/2} \sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\,.$
- $\sum_{a=1}^{4} \Delta_a = 2\pi$ with $\mathbb{R}\mathrm{e}\,\Delta_a \in [0, 2\pi]\,.$

Localization meets holography:

$$\mathcal{W}(x^*) \leftrightarrow \text{ prepotential of 4D } \mathcal{N} = 2 \text{ g-sugra}\,.$$

$$\mathcal{I}\text{-extremization } \leftrightarrow \text{ attractor mechanism}\,.$$

Generalizations

- Other AdS_4 black holes in M-theory or massive type IIA.

  [SMH, Hristov, Passias'17; Benini, Khachatryan, Milan'17; Azzurli, Bobev, Crichigno, Min, Zaffaroni'17; Bobev, Min, Pilch'18; Gauntlett, Martelli, Sparks'19; SMH, Zaffaroni'19]

  An index theorem: $$\log Z_{S^2 \times S^1}(\Delta_a, \mathfrak{p}_a) = -\frac{1}{2} \sum_a \mathfrak{p}^a \frac{\partial F_{S^3}(\Delta_a)}{\partial \Delta_a} \, .$$

  [SMH, Zaffaroni'16; SMH, Mekareeya'16]

- Subleading corrections in N.

  [Liu, Pando Zayas, Rathee, Zhao'17; Liu, Pando Zayas, Zhou'18; SMH'18; Gang, Kim, Pando Zayas'19; Bae, Gang, Lee'19]

- Localization in supergravity. [Hristov, Lodato, Reys'17]

- Black holes and black strings in higher dimensions.

  [SMH, Nedelin, Zaffaroni'16; Hong, Liu'16; SMH, Yaakov, Zaffaroni'18; Crichigno, Jain, Willett'18; SMH, Hristov, Passias, Zaffaroni'18; Suh'18; Fluder, SMH, Uhlemann'19; Bae, Gang, Lee'19]

- Black hole thermodynamics: $\log Z_{\text{SCFT}} = \mathcal{I}_{\text{sugra}}\Big|_{\text{on-shell}}$.

  [Azzurli, Bobev, Crichigno, Min, Zaffaroni'17; Halmagyi, Lal'17; Cabo-Bizet, Kol, Pando Zayas, Papadimitriou, Rathee'17]

KN-AdS$_5$ black holes

Solutions of 5D, $\mathcal{N} = 1$ U$(1)^3$ gauged supergravity

BPS black holes in AdS$_5 \times S^5$ (w/ boundary $S^3 \times \mathbb{R}_t$ — no twist)

$$\begin{cases} \text{Two angular momenta } J_i \text{ in AdS}_5 & \mathrm{U}(1)^2 \subset \mathrm{SO}(4)\,, \\ \text{Three electric charges } Q_I \text{ in } S^5 & \mathrm{U}(1)^3 \subset \mathrm{SO}(6)\,. \end{cases}$$

- $F(Q_I, J_i) = 0 \Rightarrow$ *four* independent conserved charges.
- They *must* rotate.
- Asymptotically global AdS$_5$ $\to$ near horizon AdS$_2 \times_w S^3$.

[Gutowski, Reall'04; Chong, Cvetic, Lu, Pope'05; Kunduri, Lucietti, Reall'06]

$$S_{\text{BH}} = 2\pi \sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3 - \frac{\pi}{4 G_{\text{N}}} \left(J_1 + J_2 \right)} = \mathcal{O}(N^2)\,.$$

[Kim, Lee'06]

- d_{micro} = states of given J_i and Q_I in $\mathcal{N} = 4$ super Yang-Mills.

[Hairy black hols by Markeviciute, Santos'16'18]

Entropy function for AdS$_5$ black holes

BPS entropy function

$$S_{\rm BH}(Q_I, J_i) = -\pi \mathrm{i}(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} - 2\pi\mathrm{i}\left(\sum_{I=1}^{3}\Delta_I Q_I - \sum_{i=1}^{2}\omega_i J_i\right)\Bigg|_{\rm crit.} .$$

- $\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = \pm 1\,.$
- Complex critical points but $S_{\rm BH}(Q_I, J_i)$ is *real* at the extremum!

[SMH, Hristov, Zaffaroni'17]

Black hole thermodynamics:

- The critical points can be obtained by taking an appropriate zero temperature limit of a family of supersymmetric Euclidean BHs.

[Cabo-Bizet, Cassani, Martelli, Murthy'18]

$$-\pi\mathrm{i}(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} = \mathcal{I}_{\rm sugra}\Big|_{\text{on-shell}} .$$

A puzzle!

Superconformal index on $S^3 \times S^1$ [Romelsberger'05; Kinney, Maldacena, Minwalla, Raju'05]

$$Z(\Delta_I, \omega_i) = \mathrm{Tr}_{\mathcal{H}_{S^3}} (-1)^F e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}} e^{2\pi\mathrm{i}(\sum_I \Delta_I Q_I + \sum_i \omega_i J_i)} \,.$$

- $\#$ of fugacities $=$ $\#$ of conserved charges,

$$p = e^{2\pi\mathrm{i}\omega_1}\,, \quad q = e^{2\pi\mathrm{i}\omega_2}\,, \quad y_I = e^{2\pi\mathrm{i}\Delta_I}\,, \qquad \prod_{I=1}^{3} y_I = pq\,.$$

- For real fugacities $\log Z(\Delta_I, \omega_i) = \mathcal{O}(1)$. [Kinney, Maldacena, Minwalla, Raju'05]

Localization formula [*e.g.* Spiridonov, Vartanov'10]

$$Z(\Delta_I, \omega_i) = \mathcal{A} \oint \prod_{i=1}^{N-1} \frac{\mathrm{d}z_i}{2\pi\mathrm{i}z_i} \prod_{1\leq j<j\leq N} \frac{\prod_{I=1}^{3} \Gamma_e\big(y_I (z_i/z_j)^{\pm 1}; p, q\big)}{\Gamma_e\big((z_i/z_j)^{\pm 1}; p, q\big)}\,,$$

$$\mathcal{A} \equiv \frac{\big((p;p)_\infty (q;q)_\infty\big)^{N-1}}{N!} \prod_{I=1}^{3} \Gamma_e^{N-1}(y_I; p, q)\,.$$

A puzzle!

Problem

Large cancellations between bosonic and fermionic states.

- The critical points of the BPS entropy function are complex.
- Phases may obstruct the cancellations in the index.
- Stokes phenomena.

[Cardy limit by Choi, Kim, Kim, Nahmgoong'18]

[Modified index by Cabo-Bizet, Cassani, Martelli, Murthy'18]

[Large N using Bethe sum formula by Benini, Milan'18]

Final result:

$$\log Z(\Delta_I, \omega_i) \sim -\pi \mathrm{i} N^2 \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_1 \omega_2}\,, \qquad \sum_{I=1}^{2} \Delta_I - \sum_{i=1}^{2} \omega_i = \pm 1\,.$$

Generalizations

- 4D $\mathcal{N} = 1$ gauge theories (equal charges)

$$\log Z \sim 2\pi \mathrm{i} \frac{\Delta^3}{\omega_1 \omega_2} (3c - 2a) + 2\pi \mathrm{i} \frac{\Delta}{\omega_1 \omega_2} (a - c) + \mathcal{O}(1) \,,$$

$$3\Delta - \omega_1 - \omega_2 = \pm 1 \,.$$

[Generalize Di Pietro, Komargodski'14][Kim, Kim, Song'19; Cabo-Bizet, Cassani, Martelli, Murthy'19; Amariti, Garozzo, Lo Monaco'19][Large N by González Lezcano, Pando Zayas; Lanir, Nedelin, Sela'19]

- BPS entropy functions for AdS_7, AdS_6, and AdS_4 black holes.

[SMH, Hristov, Zaffaroni'18, Choi, Hwang, Kim, Nahmgoong'18; Cassani, Papini'19]

- Similar computations of the SCI in various dimensions.

[Choi, Kim, Kim, Nahmgoong'18; Choi, Kim'19; Kántor, Papageorgakis, Richmond'19; Choi, Hwang, Kim'19]

- Near BPS entropy function. [Larsen, Nian, Zeng'19]

What we have learned by now?

- A *unique* function, $\mathcal{F}(\Delta_a)$, controls the entropy of both KN-AdS$_{d+1}$ and mAdS$_{d+1}$ black holes/strings.

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- A *unique* function, $\mathcal{F}(\Delta_a)$, controls the entropy of both KN-AdS$_{d+1}$ and mAdS$_{d+1}$ black holes/strings.

4D $\mathcal{N} = 2$ g-sugra

$$\mathcal{F}(\Delta_a) \propto F_{S^3}(\Delta_a)\,, \qquad F^{\text{ABJM}}_{S^3}(\Delta_a) \propto \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}\,.$$

- $\mathcal{I}^{e\text{KN-AdS}_4}(\Delta_a,\omega) \propto \dfrac{\mathcal{F}(\Delta_a)}{\omega}\,, \quad$ w/ $\sum_a \Delta_a - \omega = 2.$
- $\mathcal{I}^{m\text{AdS}_4}(\Delta_a,\mathfrak{p}^a) \propto \displaystyle\sum_a \mathfrak{p}^a \frac{\partial\mathcal{F}(\Delta_a)}{\partial\Delta_a}\,, \quad$ w/ $\sum_a \Delta_a = 2.$

[See “Generalization” slides for references.]

What we have learned by now?

5D $\mathcal{N}=2$ g-sugra

$$\mathcal{F}(\Delta_a) \propto a_{4\mathrm{D}}(\Delta_a)\,, \qquad a_{4\mathrm{D}}^{\mathcal{N}=4}(\Delta_a) \propto \Delta_1\Delta_2\Delta_3\,.$$

- $\mathcal{I}^{\text{KN-AdS}_5}(\Delta_a,\omega_i) \propto \dfrac{\mathcal{F}(\Delta_a)}{\omega_1\omega_2}\,, \quad \text{w/ } \sum_a \Delta_a - \omega_1 - \omega_2 = 2.$
- $\mathcal{I}^{\text{AdS}_5\,\text{BS}}(\Delta_a,\mathfrak{p}^a) \propto \displaystyle\sum_a \mathfrak{p}^a \frac{\partial\mathcal{F}(\Delta_a)}{\partial\Delta_a}\,, \quad \text{w/ } \sum_a \Delta_a = 2.$

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$$\mathcal{F}(\Delta_a) \propto a_{4\text{D}}(\Delta_a)\,, \qquad a_{4\text{D}}^{\mathcal{N}=4}(\Delta_a) \propto \Delta_1 \Delta_2 \Delta_3\,.$$

- $\mathcal{I}^{\text{KN-AdS}_5}(\Delta_a, \omega_i) \propto \dfrac{\mathcal{F}(\Delta_a)}{\omega_1 \omega_2}\,, \quad \text{w/ } \sum_a \Delta_a - \omega_1 - \omega_2 = 2.$
- $\mathcal{I}^{\text{AdS}_5\,\text{BS}}(\Delta_a, \mathfrak{p}^a) \propto \displaystyle\sum_a \mathfrak{p}^a \frac{\partial \mathcal{F}(\Delta_a)}{\partial \Delta_a}\,, \quad \text{w/ } \sum_a \Delta_a = 2.$

$F(4)$ g-sugra

$$\mathcal{F}(\Delta_a) \propto F_{S^5}(\Delta_a)\,, \qquad F_{S^5}^{\text{USp}(2N)}(\Delta_a) \propto (\Delta_1 \Delta_2)^{3/2}\,.$$

- $\mathcal{I}^{\text{KN-AdS}_6}(\Delta_a, \omega_i) \propto \dfrac{\mathcal{F}(\Delta_a)}{\omega_1 \omega_2}\,, \quad \text{w/ } \Delta_1 + \Delta_2 - \omega_1 - \omega_2 = 2.$
- $\mathcal{I}^{m\text{AdS}_6}(\Delta_a, \mathfrak{p}^a) \propto \displaystyle\sum_{a,b=1}^{2} \mathfrak{p}^a \mathfrak{s}^b \frac{\partial^2 \mathcal{F}(\Delta_a)}{\partial \Delta_a \partial \Delta_b}\,, \quad \text{w/ } \Delta_1 + \Delta_2 = 2.$

What we have learned by now?

7D $\mathcal{N}=2$ g-sugra

$$\mathcal{F}(\Delta_a) \propto a_{6\text{D}}(\Delta_a)\,, \qquad a_{6\text{D}}^{(2,0)}(\Delta_a) \propto (\Delta_1\Delta_2)^2\,.$$

- $\mathcal{I}^{\text{KN-AdS}_7}(\Delta_a,\omega_i) \propto \dfrac{\mathcal{F}(\Delta_a)}{\omega_1\omega_2\omega_3}\,, \quad \text{w/ } \Delta_1+\Delta_2-\omega_1-\omega_2-\omega_3=2.$
- $\mathcal{I}^{\text{AdS}_7\,\text{BS}}(\Delta_a,\mathfrak{p}^a) \propto \displaystyle\sum_{a,b=1}^{2} \mathfrak{p}^a\mathfrak{s}^b \frac{\partial^2\mathcal{F}(\Delta_a)}{\partial\Delta_a\partial\Delta_b}\,, \quad \text{w/ } \Delta_1+\Delta_2=2.$

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Food for thought

- Attractor mechanism for black objects in various dimensions.

[SMH, Hristov, Zaffaroni (work in progress)]

Outlook

- Other black holes in AdS_5?
- *Dyonic* KN-AdS_4 black holes. [Hristov, Katmadas, Toldo'19]
- Black holes microstates in $\mathrm{AdS}_4 \times \mathrm{SE}_7$. Problems w/ large N..
- Rotating *magnetic* AdS_4 black holes. [Hristov, Katmadas, Toldo'18]
- Finite N corrections.
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Thank you for your attention!