Black resonators and geons in AdS_5

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and work in progress

with Keiju Murata, Jorge Santos, Benson Way

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Introduction

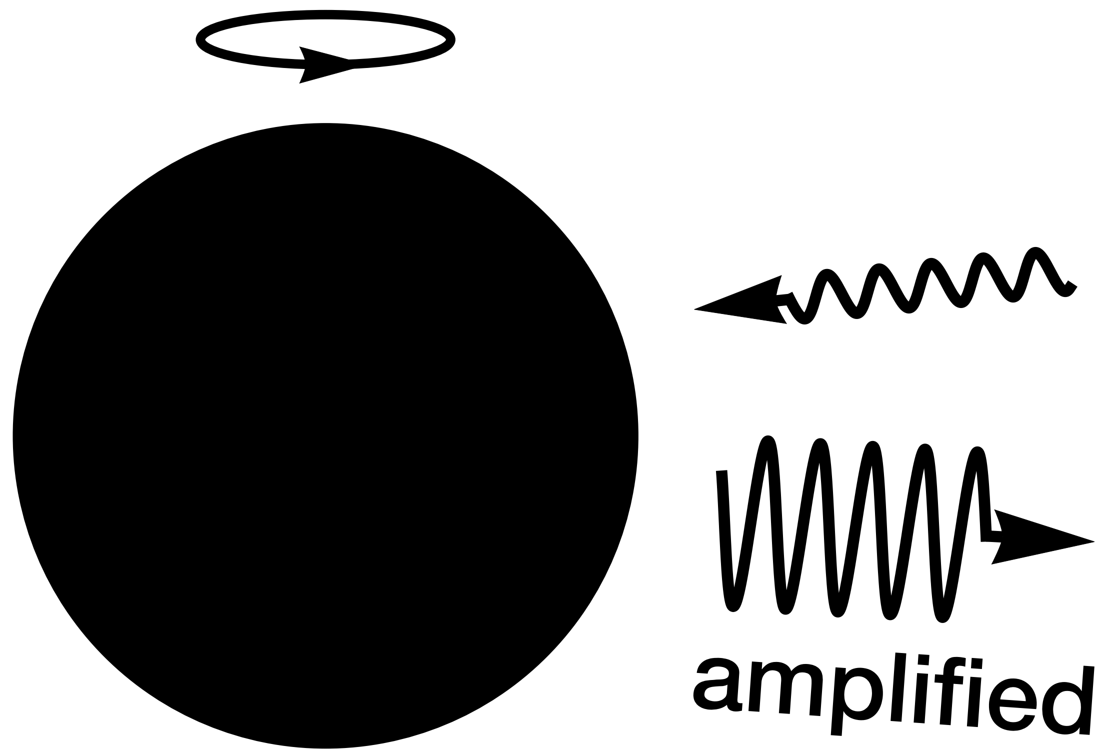
Motivations

Gravity in **higher dimensions** and **AdS spacetime**

Non-uniqueness and various black holes

Instabilities and **dynamics** of such black holes

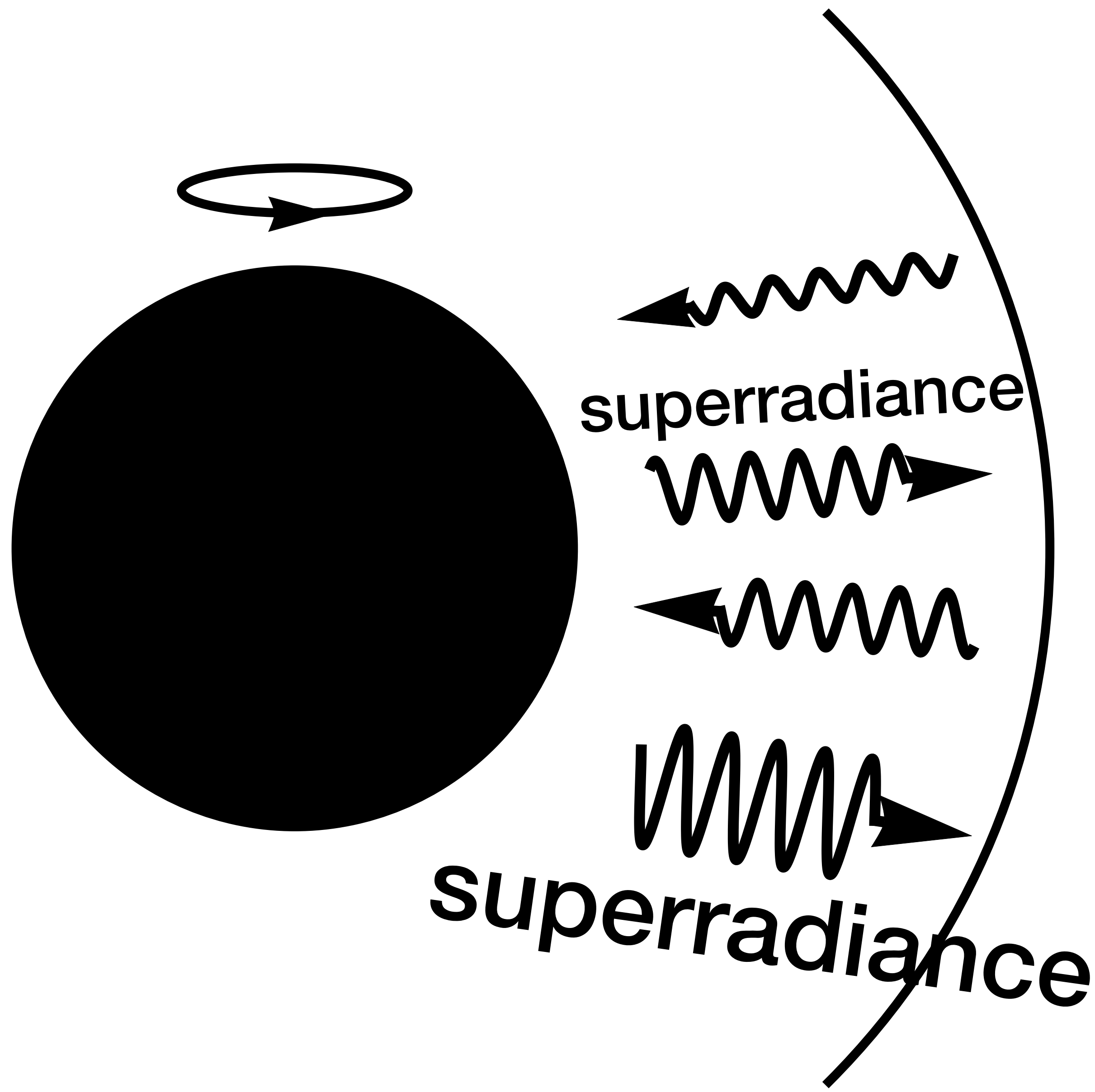
Superradiance



Rotational superradiance: Waves can be amplified by a rotating BH.

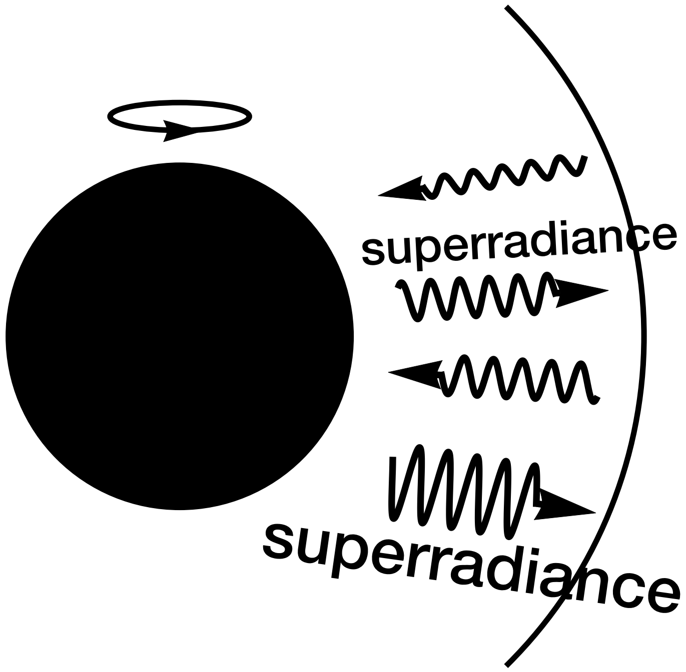
(cf. charged superradiance by a charged BH)

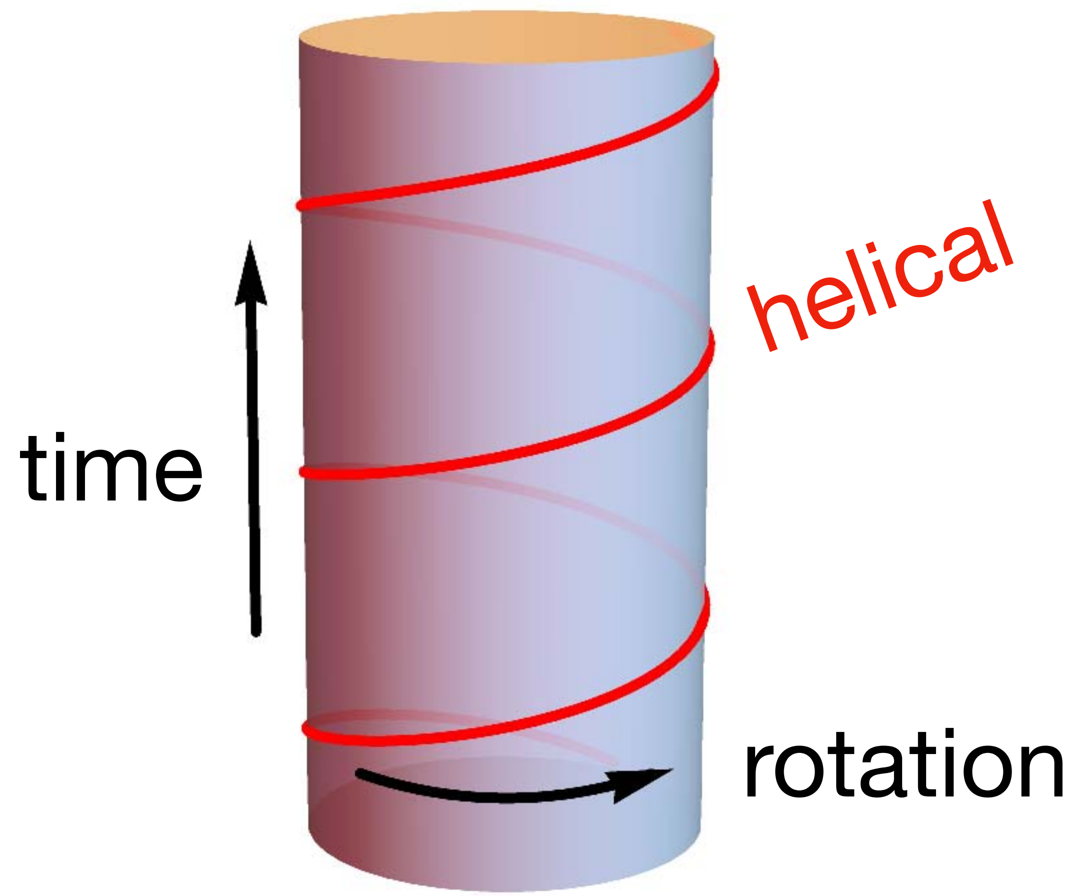
Superradiant instability



In AdS, superradiance repeats, and the growth of the wave gives rise to an **instability.** [Kunduri-Lucietti-Reall]

New solution with a helical Killing vector





New solutions with less isometries will bifurcate from the onset of the instability. [Kunduri-Lucietti-Reall]

Black resonators

Black holes with a single Killing vector field: black resonators

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We numerically construct asymptotically anti-de Sitter (AdS) black holes in four dimensions that contain only a single Killing vector field. These solutions, which we coin *black resonators*, link

arXiv:1505.04793 [hep-th]

Time-periodic black holes were constructed in AdS_4 and named **black resonators**.

This talk

The first black resonators were obtained by solving PDEs in 4D AdS.

[Dias-Santos-Way]

$$\mathrm{d}s^2 = \frac{L^2}{(1-y^2)^2}\Bigg[-y^2 q_1 \Delta(y)\,(\mathrm{d}\tau + y\,q_6 \mathrm{d}y)^2 + \frac{4y_+^2\,q_2 \mathrm{d}y^2}{\Delta(y)} + \frac{4y_+^2\,q_3}{2-x^2}\left(\mathrm{d}x + yx\sqrt{2-x^2}\,q_7 \mathrm{d}y + y^2 x\sqrt{2-x^2}\,q_8 \mathrm{d}\tau\right)^2 + (1-x^2)^2 y_+^2 q_4 \left(\mathrm{d}\phi - y^2 q_5 \mathrm{d}\tau + \frac{x\sqrt{2-x^2}q_9 \mathrm{d}x}{1-x^2} + y\,q_{10}\mathrm{d}y\right)^2 \Bigg], \quad (6)$$

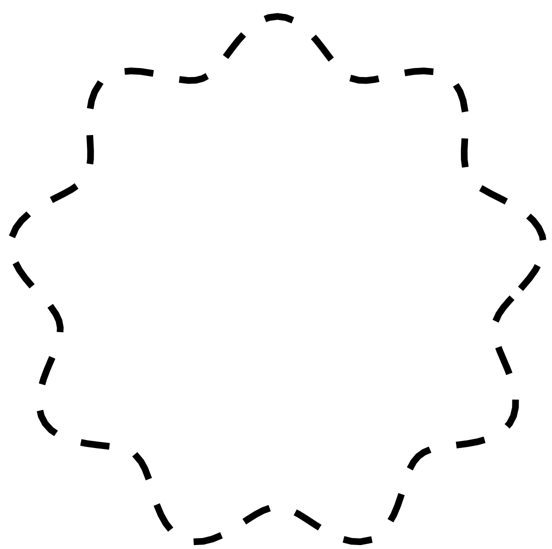
In **5D,** we can write a simple metric and obtain a class of black resonators by solving ODEs.

[TI-Murata]

Geons

This term was coined by Wheeler as "gravitational and electromagnetic entities."

Geons are self-gravitating horizonless geometries.



In the limit of zero horizon size, **black resonators smoothly reduce to geons.**

Contents

Myers-Perry AdS BH with equal angular momenta

Setup

5D pure Einstein gravity (AdS radius L=1)

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\,(R-2\Lambda)\,, \quad \Lambda=-6$$

Asymptotically global AdS (RxS³ boundary at r=∞)

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_3^2$$

Isometries of 5D black holes

Schwarzschild: $R_t \times SO(4) = R_t \times SU(2) \times SU(2)$

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega_3^2$$

General Myers-Perry: $R_t \times U(1) \times U(1)$

$$ds^2 = -\frac{\Delta(1+r^2)}{\Sigma_1\Sigma_2}dt^2 + \frac{\rho^2}{V-2M}d\tilde{r}^2 + \frac{2M}{\rho^2}\left(\frac{\Delta}{\Sigma_1\Sigma_2}dt - \frac{a_1\sin^2\tilde{\theta}}{\Sigma_1}d\varphi_1 - \frac{a_2\cos^2\tilde{\theta}}{\Sigma_2}d\varphi_2\right)^2$$
$$+\frac{\rho^2}{\Delta}d\tilde{\theta}^2 + \frac{r^2+a_1^2}{\Sigma_1}\sin^2\tilde{\theta}d\varphi_1^2 + \frac{r^2+a_2^2}{\Sigma_2}\cos^2\tilde{\theta}d\varphi_2^2$$

Myers-Perry with equal angular momenta:

$$R_t \times U(2) = \underline{R_t \times U(1)} \times SU(2)$$

$\Rightarrow$ broken to a helical Killing vector

MPAdS$_5$ with equal angular momenta

$$ds^2 = -F(r)d\tau^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left[\underbrace{\sigma_1^2 + \sigma_2^2}_{S^2} + B(r)\underbrace{(\sigma_3 + 2H(r)d\tau)^2}_{S^1 \text{ fiber}}\right]$$

SU(2) invariant 1-forms (θ,ϕ,χ: Euler angles of S^3)

$$\sigma_1 = -\sin\chi d\theta + \cos\chi \sin\theta d\phi$$
$$\sigma_2 = \cos\chi d\theta + \sin\chi \sin\theta d\phi$$
$$\sigma_3 = d\chi + \cos\theta d\phi$$

U(1) isometry: $\chi \to \chi + c$

(No χ-dependence in $d\Omega_2^2 = \sigma_1^2 + \sigma_2^2 = d\theta^2 + \sin^2\theta d\phi^2$)

Superradiant instability

SU(2)-preserving U(1)-breaking perturbation

To break the U(1), we unbalance $\sigma_1^2 + \sigma_2^2$.

For technical reasons, we work in the **rotating frame at infinity** in which $H(\infty) = \Omega$ and $H(r_h) = 0$

In this frame, the perturbation we consider is

$$\delta g_{\mu\nu} dx^\mu dx^\nu = \frac{r^2}{4} \delta\alpha(r)(\sigma_1^2 - \sigma_2^2) = \frac{r^2}{2} \delta\alpha(r)(\sigma_+^2 + \sigma_-^2)$$

$$\sigma_\pm \equiv (\sigma_1 \mp i\sigma_2)/2$$

View as a time periodic perturbation

We can go to the **non-rotating frame** by

$$dt = d\tau,\ d\bar{\chi} = d\chi + 2\Omega d\tau$$

so that $\bar{H}(r) \equiv H(r) - \Omega$ with $\bar{H}(\infty) = 0$.

σ_i transform as $\sigma_\pm = e^{\pm 2i\Omega t}\bar{\sigma}_\pm$. $\quad \sigma_\pm \equiv (\sigma_1 \mp i\sigma_2)/2$

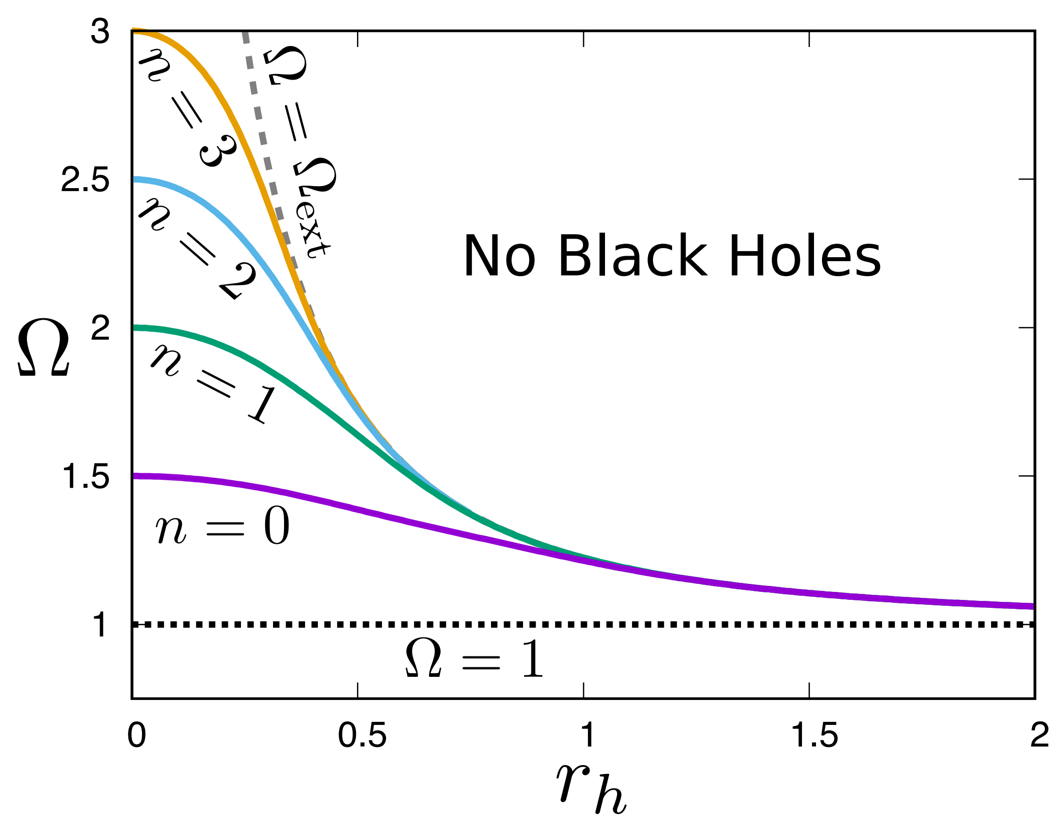
In this frame, hence, the perturbation is **time periodic**

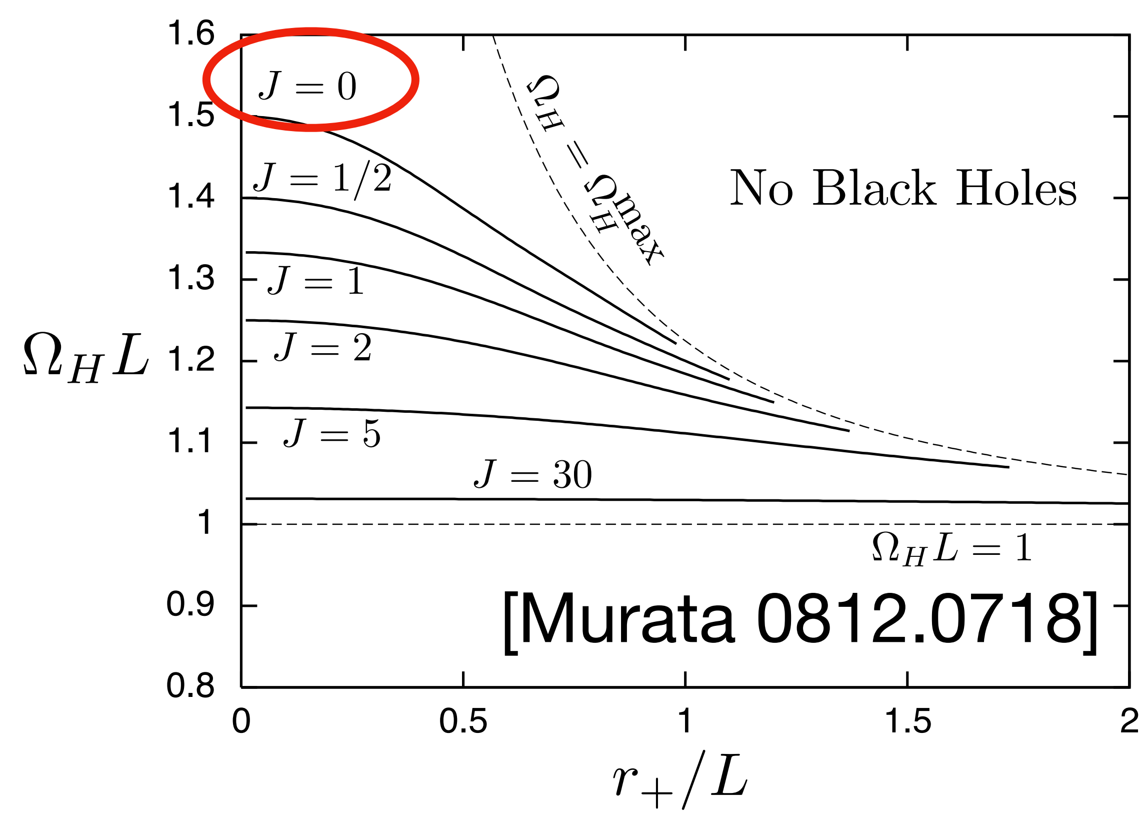
$$\delta g_{\mu\nu} dx^\mu dx^\nu = \frac{r^2}{2}\delta\alpha(r)(e^{4i\Omega t}\bar{\sigma}_+^2 + e^{-4i\Omega t}\bar{\sigma}_-^2)$$

Onset of superradiant instability

As Ω is increased, the perturbation induces an instability.

New solutions bifurcate from the onset of the instability.





Black resonators and geons

Cohomogeneity-1 metric ansatz

$$ds^2 = -(1+r^2)f(r)d\tau^2 + \frac{dr^2}{(1+r^2)g(r)} + \frac{r^2}{4}\left[\underline{\alpha(r)}\sigma_1^2 + \frac{1}{\underline{\alpha(r)}}\sigma_2^2 + \beta(r)(\sigma_3 + 2h(r)d\tau)^2\right]$$

In the non-rotating frame, the ansatz is time periodic.

$$\alpha\sigma_1^2 + \frac{1}{\alpha}\sigma_2^2 = 2\left(\alpha + \frac{1}{\alpha}\right)\bar{\sigma}_+\bar{\sigma}_- + \left(\alpha - \frac{1}{\alpha}\right)(e^{4i\Omega t}\bar{\sigma}_+^2 + e^{-4i\Omega t}\bar{\sigma}_-^2)$$

Isometries: $R_\tau \times SU(2)$

helical

Einstein equations

$$
\begin{aligned}
f' =& \frac{1}{r(1+r^2)^2 g\alpha^2(r\beta'+6\beta)}[4r^2h^2(\alpha^2-1)^2\beta \\
&+ r(r^2+1)g\{r(1+r^2)f\alpha'^2\beta - r^3h'^2\alpha^2\beta^2 - 2(2+3r^2)f\alpha^2\beta'\} \\
&- 4(1+r^2)f\{6r^2\alpha^2\beta(g-1) + 3g\alpha^2\beta + (\alpha^2-\alpha\beta+1)^2 - 4\alpha^2\}] \\
g' =& \frac{1}{6r(1+r^2)^2 f\alpha^2\beta}[-4r^2h^2(\alpha^2-1)^2\beta \\
&+ r(1+r^2)g\{-r(1+r^2)f\alpha'^2\beta + r^3h'^2\alpha^2\beta^2 \\
&- (-r(1+r^2)f' + 2f)\alpha^2\beta'\} + 4(1+r^2)f\{-6r^2\alpha^2\beta(g-1) - 3g\alpha^2\beta \\
&\qquad + \alpha^4 + 4\alpha^3\beta - 5\alpha^2\beta^2 - 2\alpha^2 + 4\alpha\beta + 1\}] \\
h'' =& \frac{1}{2r^2(1+r^2)\alpha^2\beta fg}[8fh(\alpha^2-1)^2 \\
&- r(1+r^2)h'\alpha^2\{r(fg'\beta - f'g\beta + 3fg\beta') + 10fg\beta\}] \\
\alpha'' =& \frac{1}{2r^2(1+r^2)^2 f\alpha g\beta}[2r^2(r^2+1)^2 fg\alpha'^2\beta \\
&- r(r^2+1)\alpha\alpha'\{r(1+r^2)(fg\beta)' + 2(3+5r^2)fg\beta\} \\
&- 8(\alpha^2-1)\{r^2h^2\beta(\alpha^2+1) - (1+r^2)f\alpha(\alpha-\beta) - (1+r^2)f\}] \\
\beta'' =& \frac{1}{(2r^2(1+r^2))fg\alpha^2\beta}[-2r^4gh'^2\alpha^2\beta^3 \\
&- r\alpha^2\beta'\{r(1+r^2)(f'g\beta + fg'\beta - fg\beta') + 2(3+5r^2)fg\beta\} \\
&- 8f\beta(\alpha^4 + \alpha^3\beta - 2\alpha^2\beta^2 - 2\alpha^2 + \alpha\beta + 1)]
\end{aligned}
$$

Einstein equations

$$f' = \frac{1}{r(1+r^2)^2 g\alpha^2(r\beta' + 6\beta)}[4r^2h^2(\alpha^2-1)^2\beta$$
$$+ r(r^2+1)g\{r(1+r^2)f\alpha'^2\beta - r^3h'^2\alpha^2\beta^2 - 2(2+3r^2)f\alpha^2\beta'\}$$

Coupled **ODEs** for (f',g',h'',α'',β'').

$$g' = \frac{1}{6r(1+r^2)^2 f\alpha^2\beta}[-4r^2h^2(\alpha^2-1)^2\beta$$
$$+ r(1+r^2)g\{-r(1+r^2)f\alpha'^2\beta + r^3h'^2\alpha^2\beta^2$$
$$- (-r(1+r^2)f' + 2f)\alpha^2\beta'\} + 4(1+r^2)f\{-6r^2\alpha^2\beta(g-1) - 3g\alpha^2\beta$$

Boundary conditions for the ODEs:

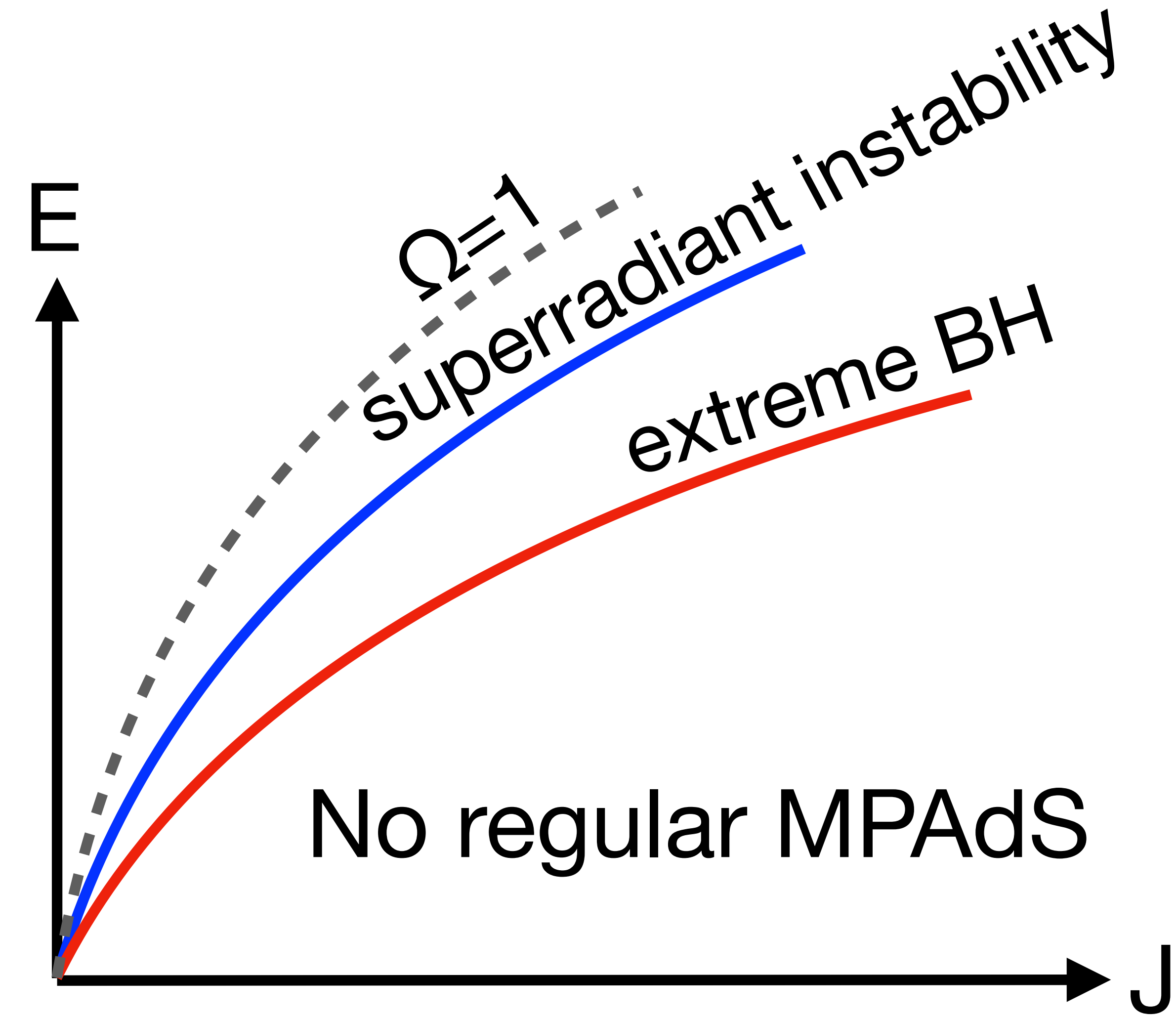
1) **Asymptotically AdS** with $h|_{r=\infty}=\Omega$

2) Geon: **regular** at r=0

Black resonator: **horizon** at $r=r_h$

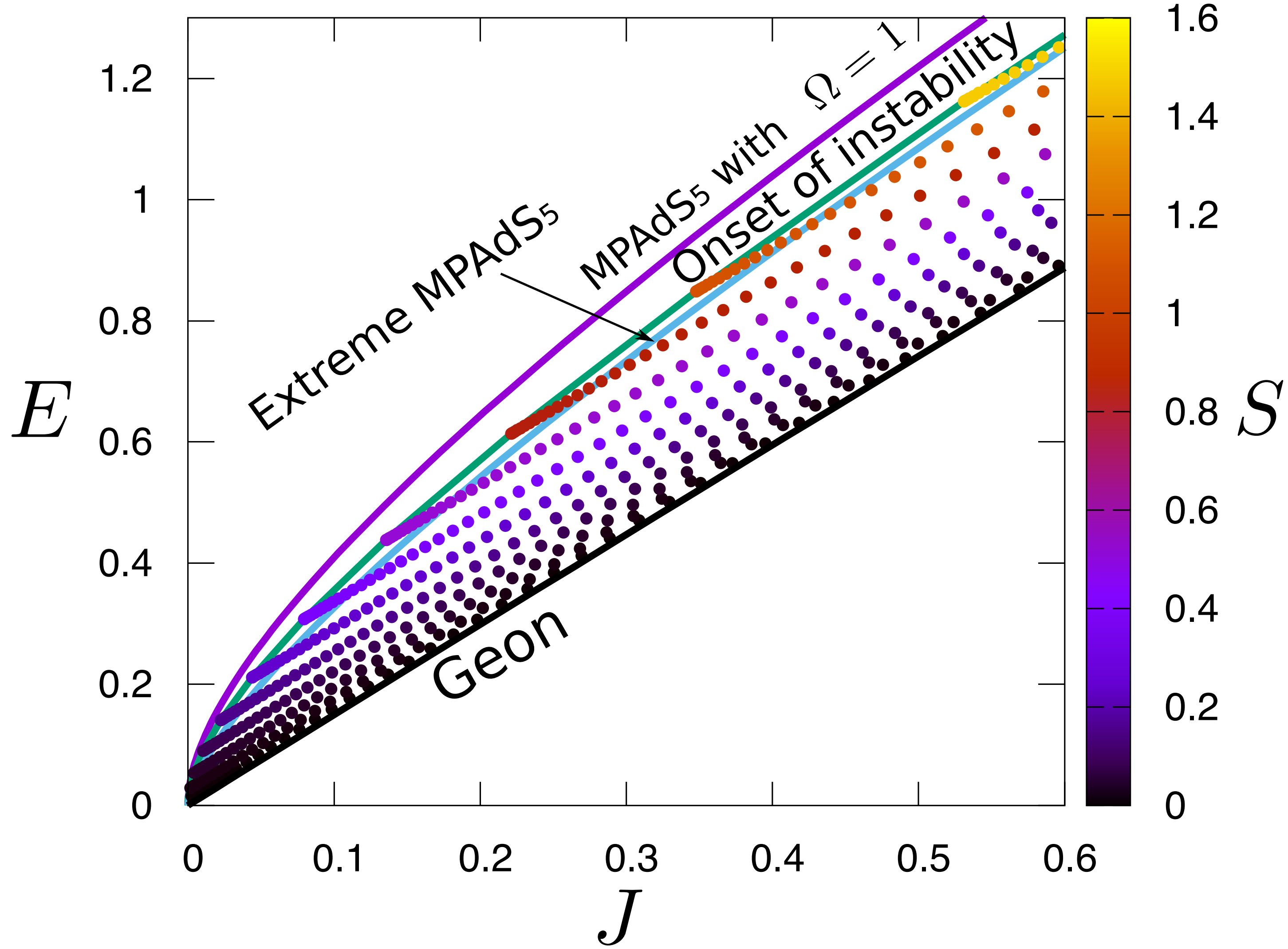
$$\beta'' = \frac{1}{(2r^2(1+r^2))fg\alpha^2\beta}[-2r^4gh'^2\alpha^2\beta^3$$
$$- r\alpha^2\beta'\{r(1+r^2)(f'g\beta + fg'\beta - fg\beta') + 2(3+5r^2)fg\beta\}$$
$$- 8f\beta(\alpha^4 + \alpha^3\beta - 2\alpha^2\beta^2 - 2\alpha^2 + \alpha\beta + 1)]$$

(E,J) diagram for MPAdS_5



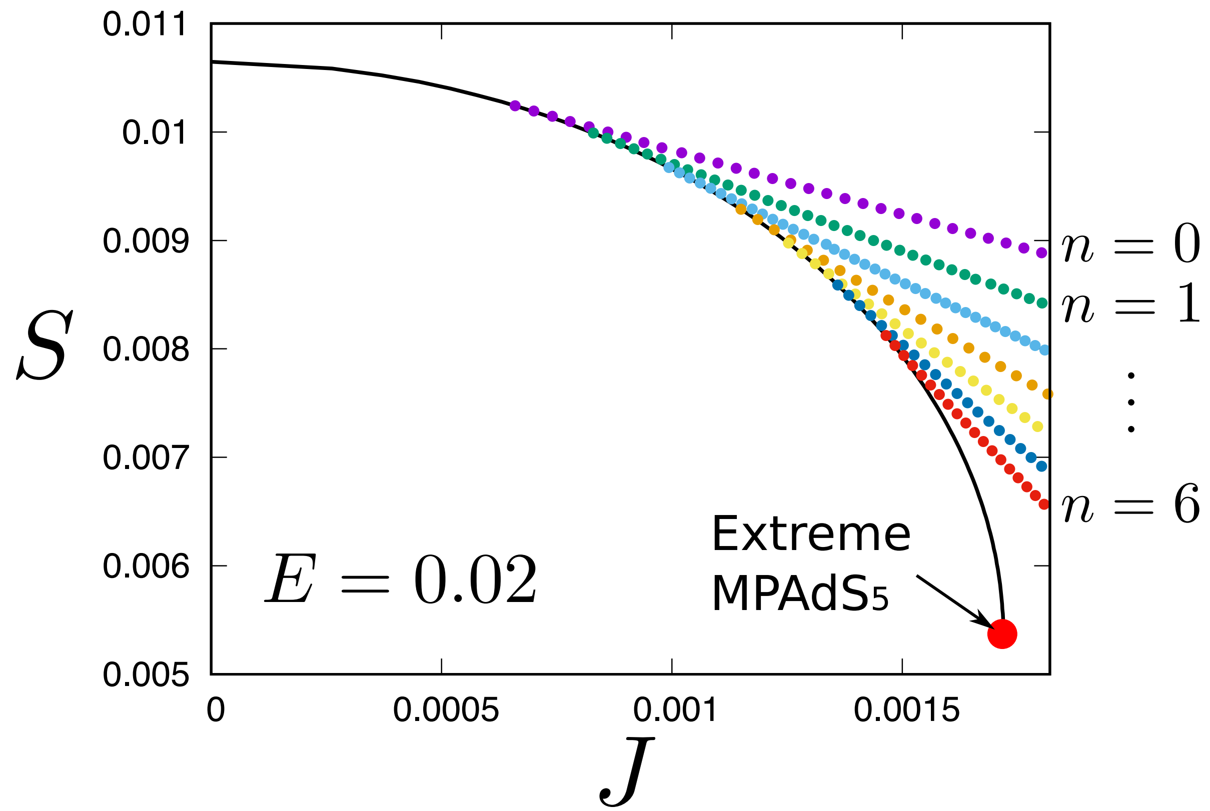
$$E = \int d\Omega_3 T_{tt} \qquad J = -\int d\Omega_3 T_{t(\bar{\chi}/2)}$$

(E,J) diagram for black resonators



Black resonators extend to the (E,J) region where no regular MPAdS BHs exist.

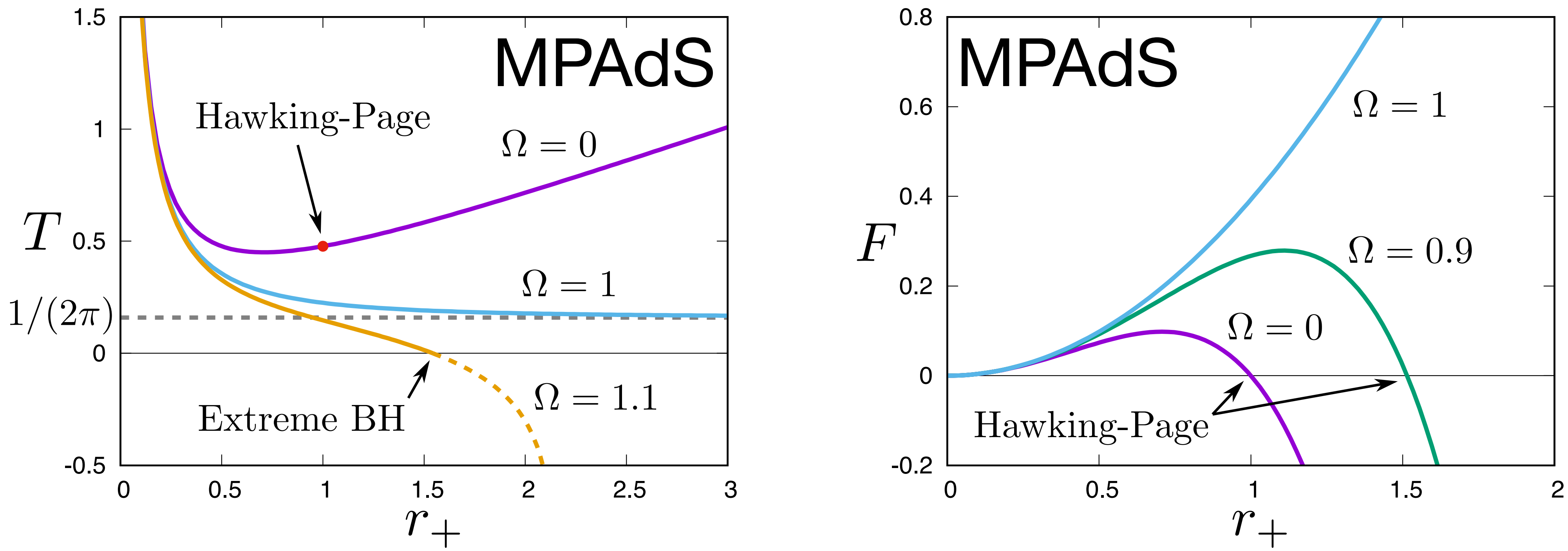
Entropy



$S_{BR} > S_{MPAdS}$ at the same (E,J). This means that an unstable MPAdS evolves into a black resonator.

Implications to AdS/CFT

The black resonators constructed so far have Ω>1, and in fact, BH with Ω>1 are small BH.



What are dual (unstable) states to black resonators?

Application

Instability of black resonators

There is a **theorem**: a BH with $\Omega>1$ is always unstable (against some perturbations).

[Green-Hollands-Ishibashi-Wald]

By using the cohomogeneity-1 metric, it is doable to study **perturbations of black resonators.**

We find instabilities against general perturbations which include **SU(2) breaking modes.**

[TI-Murata-Santos-Way, to appear]

Adding matter fields

We can add matter fields to the cohomogeneity-1 black resonators.

Coupling to a Maxwell field, we can obtain **black resonators dressed with photons.**

[TI-Murata, to appear]

Conclusion

We constructed **black resonators** and **geons** with a **cohomogeneity-1 metric** in 5D AdS

They bifurcate from the **superradiant instability** of **MPAdS BH with equal angular momenta** and have a **helical Killing vector** and a **SU(2) isometry.**

We can use of this metric to study properties of black resonators including instability.