Exploring multipartite steering effect using Bell operators

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Abstract:

Einstein-Podolsky-Rosen (EPR) steering represents quantum correlation intermediate between entanglement and Bell nonlocality. We evaluate the steering ability of the superposition of two non-commuting Bell operators and explore the strong connection between the EPR steering and joint measurability based on the nonlinear steering inequalities. The necessary and sufficient criteria of unsteerability are investigated and demonstrated in the qutrit case.

1. Introduction

Quantum mechanics demonstrates richer correlation structure than classical system.

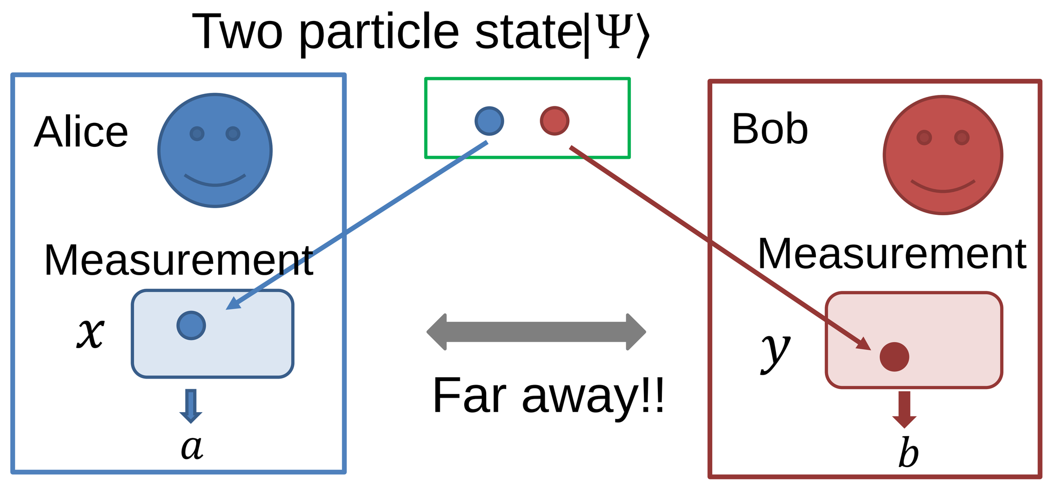
Separable (product or disentanglement) states: $|0\rangle = |z;+\rangle$, $|1\rangle = |z;-\rangle$

$$|\Psi_S\rangle = \frac{1}{2}(|0\rangle_A + |1\rangle_A) \otimes (|0\rangle_B + |1\rangle_B)$$

Non-separable state: Entangled state (a Bell state)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

Nonlocality



How two outcomes a and b are correlated?

If Alice knows her result, a, then she may make a statement on b with some certainty.

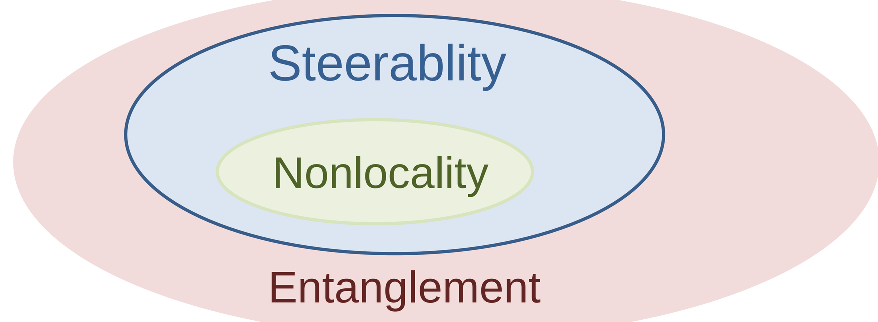
Separable: No correlation.

The results of Alice and Bob are independent. $P(a,b) = P_A(a)P_B(b)$

Bell state: 100% correlation.

If Alice observes 0, she is sure that Bob's is 0 too.

Not all entangle states show nonlocality/steering.



EPR Steering

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B)$$
$$= \frac{1}{\sqrt{2}}(|x;+\rangle_A|x;+\rangle_B + |x;-\rangle_A|x;-\rangle_B)$$

Alice measures S_z ➡ Bob's ensemble $|\pm\rangle_B$

measures S_x ➡ ensemble $|x;\pm\rangle_B$

Through her measurement, Alice can *steer* Bob's ensemble!!

2. LHV and LHS

Bell nonlocality may be explained by a local model

$$p(ab|xy) = \int d\lambda\, q(\lambda) p(a|x,\lambda) p(b|y,\lambda)$$

x: Alice's measurement ($x = 0: S_z,\ x = 1: S_x$)
a: Outcome of x
$q(\lambda)$: Some probability distribution

→ Local Hidden Variable (LHV) model

Quantum Calculation: $p(ab|xy) = \mathrm{Tr}\left(\rho\, \Pi_{a|x} \Pi_{b|y}\right)$

Projector to $x = a$ state

Bell inequality (CHSH inequality[4])

$$S = \langle a_0 b_0\rangle + \langle a_1 b_0\rangle + \langle a_0 b_1\rangle - \langle a_1 b_1\rangle$$

$$\left(\langle a_x b_y\rangle = \sum_{a,b} ab\, p(ab|xy), \quad a, b = \pm 1\right)$$

LHV models: $S \leq 2$

QM for Bell state: $S = 2\sqrt{2}$

Steering and LHS

Bob assumes that he has local states σ_λ^B that depends on the local variable λ. (Local Hidden States: LHS)

Bob can explain the steering phenomenon by a local model if

$$p(ab|xy) = \int d\lambda\, q(\lambda) p(a|x,\lambda) \mathrm{Tr}(\Pi_{b|y}\sigma_\lambda^B)$$

(Alice arranges a set of σ_λ^B to convince Bob of steering.)

➡ No such σ_λ^B and λ: the initial state is *steerable*.
Otherwise, *unsteerable*. [2]

n-qubit Klyshko-Bell operators [5]

$$B_n = B_{n-1}B_1^+ + B'_{n-1}B_1^-$$
$$B'_n = B'_{n-1}B_1^+ - B_{n-1}B_1^-$$
$$B_1^\pm = \frac{B_1 \pm B'_1}{2}$$

(Choose $B_1 = \sigma_x$, $B'_1 = \sigma^y$)

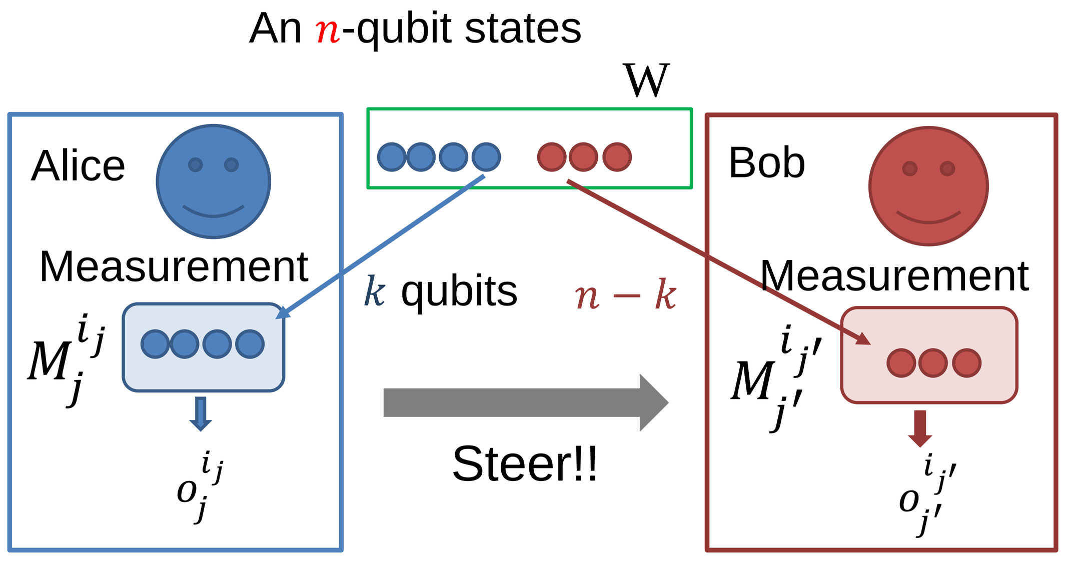
➡ Bell inequalities and multipartite nonlocality

$$B_2 = \frac{1}{2}(B_1B_1 + B_1B'_1 + B'_1B_1 - B'_1B'_1) = \frac{S}{2} \quad \text{(CHSH operator)}$$

3. Multipartite case

$$|W\rangle \propto |00100\cdots 1\rangle + |11101\cdots 0\rangle + \cdots$$

An n-qubit states



Multipartite criteria of unsteerability

Task function

$$F^{(n,k)}(W) = \sqrt{\langle B_k^+ B_{n-k}\rangle^2 + \langle B_k^+ B'_{n-k}\rangle^2} + \sqrt{\langle B_k^- B_{n-k}\rangle^2 + \langle B_k^+ B'_{n-k}\rangle^2}$$

LHV-LHS model

$$p(ab|\mathcal{M}\mathcal{M}') = \int d\lambda\, q(\lambda)\, p(a|\mathcal{M},\lambda) \mathrm{Tr}(\mathcal{M}'_b \sigma_\lambda)$$

Alice: $\mathcal{M} = \left\{M_1^{i_1}, \cdots, M_k^{i_k}\right\}$, $\quad a = \left\{o_1^{i_1}, \cdots, o_k^{i_k}\right\}$

Bob: $\mathcal{M}' = \left\{\Pi_{o|M}\right\}$: Set of projectors for Bob

W is *unsteerable*

$$\Longleftrightarrow \quad F^{(n,k)}(W) \leq R_{n-k} \equiv 2^{\frac{n-k-1}{2}}$$

(Also related to Joint Measurability)

A fully entangled state (GHZ state):

$$|\psi_n^\theta\rangle = \frac{1}{\sqrt{2}}\left(|00\cdots 0\rangle + e^{i\theta}|11\cdots 1\rangle\right)$$

The task function takes the maximum value, $F^{(n,k)}(\psi_n^\theta) = R_n$

For n −qutrit system $\quad |\psi\rangle \sim |201102\cdots 2\rangle$

An n-qutrit states

Bell operators for qutrits [6]

$$M_n^k = \frac{1}{3}[(X + \alpha^2 Y)^n + \omega^{2k}(X + \omega\alpha^2 Y)^n + \omega^k (X + \omega^2\alpha^2 Y)^n]$$

$$X = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 0 & \alpha^{-2} \\ \alpha & 0 & 0 \\ 0 & \alpha & 0 \end{pmatrix}, \quad \alpha = e^{\frac{2\pi i}{9}}, \quad \omega = \alpha^3$$

(Qutrit version of Mermin inequalities are derived.)

Task function

$$R^{(n,k)} = \frac{1}{3}\sum_{j=0}^{2}\sqrt{\sum_{l=0}^{2}\left\langle M_k^j M_{n-k}^l\right\rangle^2}$$

The upper bound for LHV-LHS model

$$R^{(n,k)} \leq \frac{1}{3}\left(\max_\lambda \sum_{j=0}^{2}\left|\left\langle M_k^j\right\rangle_{\mathrm{LHV}}\right|\right)\left(\max_{\sigma_\lambda}\sqrt{\sum_{l=0}^{2}\left\langle M_{n-k}^l\right\rangle_{\sigma_\lambda}^2}\right)$$

Numerical evaluation: l.h.s / r.h.s

n \ k	1	2	3
3	0.88 / 0.86	1.089 / 0.844	
4	1.440 / 1.213	1.761 / 1.721	2.177 / 1.477

l.h.s is evaluated for qutrit GHZ states

$$|\Psi_k\rangle = \frac{1}{\sqrt{3}}[|0\cdots 0\rangle + \alpha^k|1\cdots 1\rangle + \alpha^{2k}|2\cdots 2\rangle]$$

r.h.s (max.) is realized for a uniform distribution.

4.Conclusion

- We have considered EPR steering inequalities for multipartite case by using Bell-type operators.
- For a bipartition of multi-qubits, we find a rather simple task function that judges a given state is steerable or not.
- A generalization of multi-qutrit case is considered and the derived steering inequality is evaluated numerically.

References

[1] L.-Y. Hsu, S. Kawamoto "Exploring multipartite steering effect using Bell operators," in submission.
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[3] R. Uola, et al., "Quantum Steering," arXiv: 1903.06663
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[6] J. Lawrence, PRA95, 042123