YITP Workshop
Strings and Fields 2018

The Operator Product Expansions in the $\mathcal{N}=4$ Orthogonal Wolf Space Coset Model

Changhyun Ahn, Man Hea Kim and **Jinsub Paeng**
Department of Physics, Kyungpook National University, Taegu 41566, Korea

Eur. Phys. J. C79 (2019) no.7, 578 (arXiv : 1904.06855)

The $\mathcal{N}$= 4 Orthogonal Coset model (Scheme)

$\mathcal{N}$ =1 Kac-Moody Algebra

$\widehat{\mathfrak{so}}(N+4)_k$ Affine Kac-Moody algebra

$$\begin{aligned} V^a(z)\,V^b(w) &= \frac{1}{(z-w)^2}\,k\,g^{ab} - \frac{1}{(z-w)}\,f^{ab}{}_c\,V^c(w) + \cdots, \\ Q^a(z)\,Q^b(w) &= -\frac{1}{(z-w)}\,(k+N+2)\,g^{ab} + \cdots, \\ V^a(z)\,Q^b(w) &= +\cdots. \end{aligned}$$

where $f^{ab}{}_c$ (g_{ab}) are structural constant (dual Coxeter number) of the Lie algebra so(N+4).

$$\begin{aligned} [T_a, T_b] &= f_{ab}{}^c T_c \\ a, b, \cdots &= 1, 2, \cdots, (N+2)^2 - 1 \\ g_{ab} &= \tfrac{1}{2c_G} f_{ac}{}^d f_{bd}{}^c \end{aligned}$$

$$\left(\begin{array}{c|c} & \begin{matrix} * & * & * & * \\ * & * & * & * \\ \vdots & \vdots & \vdots & \vdots \\ * & * & * & * \\ * & * & * & * \end{matrix} \\ \hline \begin{matrix} * & * & \cdots & * & * \\ * & * & \cdots & * & * \\ * & * & \cdots & * & * \\ * & * & \cdots & * & * \end{matrix} & \end{array}\right)_{(N+4)\times(N+4)}$$

Generator T_a

G indices : $a, b, c, \cdots = 1, 2, \cdots, \frac{1}{4}(N+4)(N+3), 1^*, 2^*, \cdots, \left(\frac{1}{4}(N+4)(N+3)\right)^*$,

$\frac{G}{H}$ indices : $\bar{a}, \bar{b}, \bar{c}, \cdots = 1, 2, \cdots, 2N, 1^*, 2^*, \cdots, 2N^*$.

$$\text{Wolf} = \frac{G}{H} = \frac{SO(N+4)}{SO(N)\times SO(4)}.$$

$\mathcal{N}$ =4 Superconformal Algebra

The Stress Momentum Tensor :

$$T(z) = \frac{1}{2(k+N+2)^2}\left[(k+N+2)\,V_{\bar{a}}V^{\bar{a}} + k\,Q_{\bar{a}}\,\partial Q^{\bar{a}} + f_{\bar{a}\bar{b}c}\,Q^{\bar{a}}\,Q^{\bar{b}}\,V^c\right](z) - \frac{1}{(k+N+2)}\sum_{i=1}^{3}((-1)^i\hat{A}_i + \hat{B}_i)^2(z). \qquad i = 1, 2, 3,$$

Super Charge Currents :

$$\begin{aligned} G^0(z) &= \frac{i}{(k+N+2)}\,Q_{\bar{a}}\,V^{\bar{a}}(z), \\ G^i(z) &= \frac{i}{(k+N+2)}\,h^i_{\bar{a}\bar{b}}\,Q^{\bar{a}}\,V^{\bar{b}}(z), \end{aligned}$$

Spin1 currents of $SU(2)_k \times SU(2)_N$:

$$\begin{aligned} A^{+i}(z) &= -\frac{1}{4N}\,f^{\bar{a}\bar{b}}{}_c\,h^i_{\bar{a}\bar{b}}\,V^c(z), \\ A^{-i}(z) &= -\frac{1}{4(k+N+2)}\,h^i_{\bar{a}\bar{b}}\,Q^{\bar{a}}\,Q^{\bar{b}}(z), \end{aligned}$$

$$h^1_{\bar{a}\bar{b}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad h^2_{\bar{a}\bar{b}} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad h^3_{\bar{a}\bar{b}} = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix},$$

Nonlinear version algebra

$$\begin{aligned} T(z)\,T(w) &= \frac{1}{(z-w)^4}\frac{1}{2}\left[\frac{3(k+N+2kN)}{(k+N+2)}\right] + \frac{1}{(z-w)^2}\,2T(w) + \frac{1}{(z-w)}\,\partial T(w) + \cdots, \\ T(z)\begin{pmatrix} G^{\mu} \\ A^{\pm i}\end{pmatrix}(w) &= \frac{1}{(z-w)^2}\begin{pmatrix} \frac{3}{2}G^{\mu} \\ A^{\pm i}\end{pmatrix}(w) + \frac{1}{(z-w)}\begin{pmatrix} \partial G^{\mu} \\ \partial A^{\pm i}\end{pmatrix}(w) + \cdots, \\ G^{\mu}(z)\,G^{\nu}(w) &= \frac{1}{(z-w)^3}\,\frac{2}{3}\,\delta^{\mu\nu}\left[\frac{6kN}{(2+k+N)}\right] \\ &\quad - \frac{1}{(z-w)^2}\,\frac{8}{(k+N+2)}(N\,\alpha^{+i}_{\mu\nu}\,A^+_i + k\,\alpha^{-i}_{\mu\nu}\,A^-_i)(w) \\ &\quad + \frac{1}{(z-w)}\left[2\delta^{\mu\nu}T - \frac{4}{(k+N+2)}\partial(N\,\alpha^{+i}_{\mu\nu}\,A^+_i + k\,\alpha^{-i}_{\mu\nu}\,A^-_i) \right. \\ &\quad \left. - \frac{8}{(k+N+2)}(\alpha^{+i}A^+_i - \alpha^{-i}A^-_i)_{\rho(\mu}(\alpha^{+j}A^+_j - \alpha^{-j}A^-_j)_{\nu)}{}^{\rho}\right](w) + \cdots, \\ A^{\pm i}(z)\,G^{\mu}(w) &= \frac{1}{(z-w)}\,\alpha^{\pm i}_{\mu\nu}\,G^{\nu}(w) + \cdots, \\ A^{\pm i}(z)\,A^{\pm j}(w) &= -\frac{1}{(z-w)^2}\,\frac{1}{2}\,\delta^{ij}\begin{pmatrix} k \\ N \end{pmatrix} + \frac{1}{(z-w)}\,\epsilon^{ijk}A^{\pm k}(w) + \cdots, \end{aligned}$$

We can also construct linear superconformal generators by adding 4 (spin $\frac{1}{2}$) and 1 (spin 1)currents.

Extend to Higher Spin fields

$$\begin{aligned} \Phi^{(2)}_0(z) = {} & c_1\,V_{\bar{a}}V^{\bar{a}}(z) + c_2 \sum_{a':so(N)} V_{a'}V^{a'}(z) + c_3 \sum_{a'':so(4)} V_{a''}V^{a''}(z) \\ & + c_4 \sum_{i=1}^{3} \hat{A}_i\hat{A}_i(z) + c_5 \sum_{i=1}^{3} \hat{B}_i\hat{B}_i(z) \\ & + c_6\,Q_{\bar{a}}\partial Q^{\bar{a}}(z) + c_7 \sum_{\mu=0}^{3} h^{\mu}_{\bar{a}\bar{b}} h^{\mu}_{\bar{c}\bar{d}} f^{\bar{a}\bar{c}}{}_e Q^{\bar{b}} Q^{\bar{d}} V^e(z), \end{aligned}$$

This higher spin field satisfy the superconformal primary condition. (lowest spin2 field)

We can get super-spin2 multiplet include in sixteen Higher spin element fields by acting super charge currents G sequentially.

$$\begin{aligned} G^i\,\Phi^{(s)}_0 &\longrightarrow \Phi^{(s),i}_{\frac{1}{2}}, \\ G^i\,\Phi^{(s),j}_{\frac{1}{2}} &\longrightarrow \varepsilon^{ijkl}\,\Phi^{(s),kl}_1, \\ G^i\,\Phi^{(s),jk}_1 &\longrightarrow -\delta^{ij}\,\Phi^{(s),k}_{\frac{3}{2}} + \cdots, \\ G^i\,\Phi^{(s),j}_{\frac{3}{2}} &\longrightarrow -\delta^{ij}\,\Phi^{(s),k}_2 + \cdots, \end{aligned}$$

Then we can calculate OPEs between Higher spin fields
(calculation of higher spin OPEs base on Kac-Moody algebra)

Superconformal Primary in SuperSpace

Superconformal transformation : $Z(z,\theta) \to \tilde{Z}(\tilde{z},\tilde{\theta}) \longleftrightarrow D^i\tilde{z} = \tilde{\theta}^j D^i\tilde{\theta}^j$ that is $D^i = (D^i\tilde{\theta}^j)\tilde{D}^j$,

Superconformal Jacobian : $\frac{D^i}{\tilde{D}^j} = (D\tilde{\theta})^i{}_j \longrightarrow \left|D\tilde{\theta}\right|^2 = \left|D\tilde{\theta}(D\tilde{\theta})^T\right| \simeq \left|(1 + \partial_z\delta z + \theta^j\partial_z\delta\theta^j)I\right|$ ...... (1)

Superconformal Primary field : $\mathbf{\Phi}^h(Z) = \tilde{\mathbf{\Phi}}^h(\tilde{Z})\left|D\tilde{\theta}\right|^{\frac{2h}{N}} \longleftarrow \phi(z) \mapsto \tilde{\phi}(z) = \left(\frac{\partial\tilde{z}}{\partial z}\right)^h \phi(\tilde{z})$
(h=conformal dimension)
Analogues of ordinary conformal primary

O(N) extended Superconformal Primary field : $\boxed{\mathbf{\Phi}^h(Z) = \tilde{\mathbf{\Phi}}^h(\tilde{Z})\left|D\tilde{\theta}\right|^{\frac{2h}{N}} M^{\alpha\beta}\left[\frac{D\tilde{\theta}}{\partial_z\tilde{z} + \tilde{\theta}^j\partial_z\tilde{\theta}^j}\right]}$

Superconformal infinitesimal variation :

$$\delta\mathbf{\Phi}^h = \mathbf{\Phi}^h(Z) - \tilde{\mathbf{\Phi}}^h(Z) = \tilde{\mathbf{\Phi}}^h(\tilde{Z})|D\tilde{\theta}|^{\frac{2h}{N}} - \tilde{\mathbf{\Phi}}^h(Z) = \boxed{\left[h(\partial_z v) + \frac{1}{2}(D^i v)D^i + v\partial_z\right]\mathbf{\Phi}} \quad \text{......... (2.1)}$$

$$\| $$

$$\boxed{\Big(1 + \delta\theta^i D^i + v(Z)\partial_z\Big)\Big(1 + h\,\partial_z v(Z)\Big)\tilde{\mathbf{\Phi}}(Z)}$$

From super taylor expansion+(1)

infinitesimal super distance : $v(Z) = \delta z + \theta\delta\theta$

conserved charge $Q = \int v(Z)\mathbf{J}^{(\mathcal{N})}$

Comparison of superconformal variation in the $\mathcal{N}$=4 Unitary with Orthogonal Coset model,

$$\delta_v\mathbf{\Phi}^h = \left[h(\partial_z v) + \frac{1}{2}(D^i v)D^i + v\partial_z\right]\mathbf{\Phi}^h \quad \text{......... (2.1) unitary case}$$

$$\delta_v\mathbf{\Phi}^{h,\alpha} = \left[h(\partial_z v) + \frac{1}{2}(D^i v)D^i + v\partial_z\right]\mathbf{\Phi}^{h,\alpha} + \boxed{\frac{i}{2}\,(D^i D^j v)(T^{ij})^{\alpha\beta}\,\mathbf{\Phi}^{h,\beta}} \quad \text{......... (2.2) orthogonal case}$$

O(N)extended

$$\delta_v\mathbf{\Phi}^h(Z_2) = \left[Q, \mathbf{\Phi}^h(Z_2)\right] = \frac{1}{2}\oint_{C_2}\frac{dz_1}{2\pi i}\int d^{\mathcal{N}}\theta_1\,v(Z_1)\boxed{\mathbf{J}^{(\mathcal{N})}(Z_1)\,\mathbf{\Phi}(Z_2)} \quad \text{......... (3)}$$

OPE between superconformal current and superconformal primary field.

(2.1,2)=(3) Superconformal primary condition SOPE (superspace operator product expansion),

$$\mathbf{J}^{(\mathcal{N})}(Z_1)\,\mathbf{\Phi}^{(s)}(Z_2) = \left[2\,s\,\frac{\theta^{\mathcal{N}}_{12}}{Z^2_{12}} + \frac{\theta^{\mathcal{N}-i}_{12}}{Z_{12}}\,D^i + \frac{\theta^{\mathcal{N}}_{12}}{Z_{12}}\,2\,\partial_{Z_2}\right]\mathbf{\Phi}^{(s)}(Z_2) + \cdots,$$

There are only singlet superfield in unitary coset.

$$\begin{aligned} \mathbf{J}^{(\mathcal{N})}(Z_1)\,\mathbf{\Phi}^{(s),\alpha}(Z_2) = {} & \left[2\,s\,\frac{\theta^{\mathcal{N}}_{12}}{Z^2_{12}} + \frac{\theta^{\mathcal{N}-i}_{12}}{Z_{12}}\,D^i + \frac{\theta^{\mathcal{N}}_{12}}{Z_{12}}\,2\,\partial_{Z_2}\right]\mathbf{\Phi}^{(s),\alpha}(Z_2) \\ & - \frac{i}{2}\frac{\theta^{\mathcal{N}-ij}_{12}}{Z_{12}}\boxed{(T^{ij})^{\alpha\beta}\,\mathbf{\Phi}^{(s),\beta}}(Z_2) + \cdots, \qquad \begin{aligned} &\alpha = 1, \cdots, 6, \\ &\alpha \to \mu = 1, \cdots, 4, \end{aligned} \end{aligned}$$

We observed quartet and sextet superfield in orthogonal coset model.

If N=5,

Six SO(4) Adjoint Super-spin 3 (Integer) superfield

$SO(4)$ adjoint representation

$$T^{12} = \begin{pmatrix} 0 & -i & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^{13} = \begin{pmatrix} 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 \end{pmatrix},$$

$$T^{14} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & i & 0 \end{pmatrix},$$

$$T^{24} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^{34} = \begin{pmatrix} 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \text{... (4.1)}$$

Four SO(4) vector Super-spin $\frac{7}{2}$, $\frac{9}{2}$(Half Integer) superfield

$SO(4)$ vector representation

$$T^{12} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^{13} = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T^{23} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^{14} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix},$$

$$T^{24} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \quad T^{34} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad \text{... (4.2)}$$

$$[T^{ij}, T^{kl}] = i(\delta^{ik}\,T^{jl} - \delta^{il}\,T^{jk} - \delta^{jk}\,T^{il} + \delta^{jl}\,T^{ik}).$$

Lowest Higher Spin OPE for N=5 group SO(N =5+4)

Schematically, we can present the below OPE.

$$[\mathbf{\Phi}^{(2)}\cdot\mathbf{\Phi}^{(2)}] = [\mathbf{I}] + \theta^{4-0}\,([\mathbf{\Phi}^{(2)}] + [\mathbf{X}^{(2)}]) + \theta^{4-ij}\,\boxed{(M^{\alpha})^{ij}}\,[\mathbf{\Phi}^{(3),\alpha}] + \theta^{i}\,[\mathbf{\Phi}^{(\frac{7}{2}),i}] + \theta^{4-0}\,[\tilde{\mathbf{\Phi}}^{(4)}] + \theta^{4-i}\,[\mathbf{\Phi}^{(\frac{9}{2}),i}],$$

$$M^1 \equiv L_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, M^2 \equiv L_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, M^3 \equiv L_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M^4 \equiv K_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, M^5 \equiv K_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, M^6 \equiv K_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix},$$

$$\begin{aligned} [L_i, L_j] &= i\,\varepsilon_{ijk}\,L_k, \\ [K_i, K_j] &= i\,\varepsilon_{ijk}\,L_k, \\ [L_i, K_j] &= i\,\varepsilon_{ijk}\,K_k. \end{aligned}$$

$SO(4)$

List of superfields.

$\mathbf{\Phi}^{(2)}$, $\mathbf{\Phi}^{(4)}$, $\mathbf{X}^{(2)}$,
$\mathbf{\Phi}^{(3),\alpha}$, $\alpha = 1, \cdots, 6$,
$\mathbf{\Phi}^{(\frac{7}{2}),\mu=i}$, $\mathbf{\Phi}^{(\frac{9}{2}),\mu=i}$, $\mu = 1, \cdots, 4$,

Super OPE (SOPE)

$$\begin{aligned} \mathbf{\Phi}^{(2)}(Z_1)\,\mathbf{\Phi}^{(2)}(Z_2) = {} & \frac{1}{z^4_{12}}\,c^{0,4}_0 + \frac{\theta^{4-0}_{12}}{z^6_{12}}\,8\,\alpha\,c^{0,4}_0 + \frac{\theta^{4-i}_{12}}{z^5_{12}}\,\mathbf{Q}^{(\frac{1}{2}),i}_{\frac{3}{2}}(Z_2) \\ & + \frac{\theta^{4-0}_{12}}{z^5_{12}}\,\mathbf{Q}^{(1)}_2(Z_2) + \frac{\theta^{4-ij}_{12}}{z^4_{12}}\,\mathbf{Q}^{(1),ij}_1(Z_2) + \frac{\theta^{4-i}_{12}}{z^4_{12}}\left[2\,\partial\mathbf{Q}^{(\frac{1}{2}),i}_{\frac{3}{2}} + \mathbf{Q}^{(\frac{3}{2}),i}_{\frac{3}{2}} + \mathbf{R}^{(\frac{3}{2}),i}_{\frac{3}{2}}\right](Z_2) \\ & + \frac{\theta^{4-0}_{12}}{z^4_{12}}\left[\frac{3}{2}\,\partial\mathbf{Q}^{(1)}_2 + \mathbf{Q}^{(2)}_2 + \mathbf{R}^{(2)}_2\right](Z_2) + \frac{\theta^{i}_{12}}{z^3_{12}}\,\mathbf{Q}^{(\frac{3}{2}),i}_{\frac{1}{2}}(Z_2) + \frac{\theta^{4-ij}_{12}}{z^3_{12}}\left[\partial\mathbf{Q}^{(1),ij}_1 + \mathbf{Q}^{(2),ij}_1\right](Z_2) \\ & + \frac{\theta^{4-i}_{12}}{z^3_{12}}\left[\frac{3}{2}\,\partial^2\mathbf{Q}^{(\frac{1}{2}),i}_{\frac{3}{2}} + \partial\mathbf{Q}^{(\frac{3}{2}),i}_{\frac{3}{2}} + \mathbf{Q}^{(\frac{5}{2}),i}_{\frac{3}{2}} + \mathbf{R}^{(\frac{5}{2}),i}_{\frac{3}{2}}\right](Z_2) \\ & + \frac{\theta^{4-0}_{12}}{z^3_{12}}\left[\partial^2\mathbf{Q}^{(1)}_2 + \partial\mathbf{Q}^{(2)}_2 + \mathbf{Q}^{(3)}_2 + \mathbf{R}^{(3)}_2\right](Z_2) + \frac{1}{z^2_{12}}\,\mathbf{Q}^{(2)}_0(Z_2) \\ & + \frac{\theta^{i}_{12}}{z^2_{12}}\left[\frac{2}{3}\,\partial\mathbf{Q}^{(\frac{3}{2}),i}_{\frac{1}{2}} + \mathbf{Q}^{(\frac{5}{2}),i}_{\frac{1}{2}}\right](Z_2) + \boxed{\frac{\theta^{4-ij}_{12}}{z^2_{12}}\left[\frac{1}{2}\,\partial^2\mathbf{Q}^{(1),ij}_1 + \frac{3}{4}\,\partial\mathbf{Q}^{(2),ij}_1 + \mathbf{Q}^{(3),ij}_1\right](Z_2)} \\ & + \frac{\theta^{4-i}_{12}}{z^2_{12}}\left[\frac{2}{3}\,\partial^3\mathbf{Q}^{(\frac{1}{2}),i}_{\frac{3}{2}} + \frac{1}{2}\,\partial^2\mathbf{Q}^{(\frac{3}{2}),i}_{\frac{3}{2}} + \frac{4}{5}\,\partial\mathbf{Q}^{(\frac{5}{2}),i}_{\frac{3}{2}} + \mathbf{Q}^{(\frac{7}{2}),i}_{\frac{3}{2}} + \mathbf{R}^{(\frac{7}{2}),i}_{\frac{3}{2}}\right](Z_2) \\ & + \frac{\theta^{4-0}_{12}}{z^2_{12}}\left[\frac{5}{12}\,\partial^3\mathbf{Q}^{(1)}_2 + \frac{1}{2}\,\partial\mathbf{Q}^{(2)}_2 + \frac{5}{6}\,\mathbf{Q}^{(3)}_2 + \mathbf{Q}^{(4)}_2 + \mathbf{R}^{(4)}_2\right](Z_2) \\ & + \frac{1}{z_{12}}\frac{1}{2}\partial\mathbf{Q}^{(2)}_0(Z_2) + \boxed{\frac{\theta^{i}_{12}}{z_{12}}\left[\frac{1}{4}\,\partial^2\mathbf{Q}^{(\frac{3}{2}),i}_{\frac{1}{2}} + \frac{3}{5}\,\partial\mathbf{Q}^{(\frac{5}{2}),i}_{\frac{1}{2}} + \mathbf{Q}^{(\frac{7}{2}),i}_{\frac{1}{2}}\right](Z_2)} \\ & + \frac{\theta^{4-ij}_{12}}{z_{12}}\left[\frac{1}{6}\,\partial^3\mathbf{Q}^{(1),ij}_1 + \frac{3}{10}\,\partial^2\mathbf{Q}^{(2),ij}_1 + \frac{2}{3}\,\partial\mathbf{Q}^{(3),ij}_1 + \mathbf{Q}^{(4),ij}_1\right](Z_2) \\ & + \frac{\theta^{4-i}_{12}}{z_{12}}\left[\frac{5}{24}\,\partial^4\mathbf{Q}^{(\frac{1}{2}),i}_{\frac{3}{2}} + \frac{1}{6}\,\partial^3\mathbf{Q}^{(\frac{3}{2}),i}_{\frac{3}{2}} + \frac{1}{3}\,\partial^2\mathbf{Q}^{(\frac{5}{2}),i}_{\frac{3}{2}} + \frac{5}{7}\,\partial\mathbf{Q}^{(\frac{7}{2}),i}_{\frac{3}{2}} + \mathbf{Q}^{(\frac{9}{2}),i}_{\frac{3}{2}}\right](Z_2) \\ & + \frac{\theta^{4-0}_{12}}{z_{12}}\left[\frac{1}{8}\,\partial^4\mathbf{Q}^{(1)}_2 + \frac{1}{6}\,\partial^3\mathbf{Q}^{(2)}_2 + \frac{5}{14}\,\partial^2\mathbf{Q}^{(3)}_2 + \frac{3}{4}\,\partial\mathbf{Q}^{(4)}_2 + \mathbf{Q}^{(5)}_2\right](Z_2) + \cdots. \end{aligned}$$

... (5)

$$\mathbf{Q}^{(2)}_2 = c^{2,4}_1\,\mathbf{\Phi}^{(2)} + c^{2,4}_2\,\mathbf{X}^{(2)} + \cdots,$$

A new primary higher spin field $X^{(2)}$ which cannot be written in terms of the known composite fields Of the currents and higher spin currents.

All (possible 136) component OPEs can be extracted from the SOPE (superspace expansion).

Example of Component OPEs.

$$\begin{aligned} \Phi^{(2),ij}_1(z)\,\Phi^{(2)}_0(w) = {} & \frac{1}{(z-w)^4}\,Q^{(1),ij}_1(w) + \frac{1}{(z-w)^3}\left[\partial Q^{(1),ij}_1 + Q^{(2),ij}_1\right](w) \\ & + \boxed{\frac{1}{(z-w)^2}\left[\frac{1}{2}\,\partial^2 Q^{(1),ij}_1 + \frac{3}{4}\,\partial Q^{(2),ij}_1 + Q^{(3),ij}_1\right](w)} \\ & + \frac{1}{(z-w)}\left[\frac{1}{6}\,\partial^3 Q^{(1),ij}_1 + \frac{3}{10}\,\partial^2 Q^{(2),ij}_1 + \frac{2}{3}\,\partial Q^{(3),ij}_1 + Q^{(4),ij}_1\right](w) \\ & - \sum_{n=1}^{4}\frac{1}{(z-w)^n}\,(i \leftrightarrow j) + \cdots. \end{aligned}$$

$$Q^{(3),ij}_1(w) = w_{0,3}\,(M^{\alpha})^{ij}\,\Phi^{(3),\alpha}_0(w) + \cdots,$$

... (6.1)

$$\begin{aligned} \Phi^{(2),i}_{\frac{1}{2}}(z)\,\Phi^{(2)}_0(w) = {} & \frac{1}{(z-w)^3}\,Q^{(\frac{3}{2}),i}_{\frac{1}{2}}(w) \\ & + \frac{1}{(z-w)^2}\left[\frac{2}{3}\,\partial Q^{(\frac{3}{2}),i}_{\frac{1}{2}} + Q^{(\frac{5}{2}),i}_{\frac{1}{2}}\right](w) \\ & + \boxed{\frac{1}{(z-w)}\left[\frac{1}{4}\,\partial^2 Q^{(\frac{3}{2}),i}_{\frac{1}{2}} + \frac{3}{5}\,\partial Q^{(\frac{5}{2}),i}_{\frac{1}{2}} + Q^{(\frac{7}{2}),i}_{\frac{1}{2}}\right](w)} + \cdots. \end{aligned}$$

$$Q^{(\frac{7}{2}),i}_{\frac{1}{2}}(w) = w_{0,\frac{7}{2}}\,\Phi^{(\frac{7}{2}),i}_0(w) + \cdots.$$

... (6.2)

We found SO(4) representations (4.1,2) from the OPEs like (6.1) and (6.2).

Conclusion and Outlook

We have described one single $\mathcal{N}=4$ super OPE (5) between the lowest higher spin-2 multiplet in the $\mathcal{N}=4$ superspace. In the process, we also found explicit representations for the SO(4) extended superconformal primary fields. (3, 4)

- Higher spin algebra in the bulk theory

The free field construction at $\lambda=0$ by using the bosons and fermions can be describe the full higher spin algebra. The final goal is to obtain the higher spin algebra at finite λ which will provide the corresponding algebra in the dual conformal field theory at the classical level. Contrary to the unitart case, the orthognal case needs to obtain the appropriate truncation on the matrix elements

- The large k limit

We can examine the behavior of large k limit from the results. We take the large k limit in the structure constants appearing in the OPEs. We expect to observe the realization of vanishing of 't Hooft-like coupling constant $\lambda = \frac{N+1}{N+k+2} \to 0$ for fixed N.