Selection Rules for Schur Multiplets in 4D N=2 Superconformal Field Theories

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This poster is base on [1].

1.Introduction

In recent year, many 4D N=2 superconformal field theories (SCFT), which do not have any Lagrangian description, are discovered such as Argyres-Douglas type SCFT [4], Minahan Nemeschansky E_6, E_7, E_8 SCFT [5] and class S theories [3]. To research such no Lagrangian SCFT, we need to develop a technique which relies only on the symmetry and unitarity.

In this poster, we talk about our research of selection rules of BPS multiplet in 4D N=2 SCFT called Schur multiplets. Each Schur multiplets contain important operators which contribute to the associated chiral algebra of 4D N=2 SCFT [2], called Schur operators. In [7], using selection rule and superconformal indices, they determine some OPE coefficients of SCFT. It means that selection rules are important information of SCFT.

2.Review of Schur multiplet

Schur multiplets are defined as following superconformal (semi-) shortening conditions.

short

$$\mathcal{B}^1 : \; Q^{(i}_{\alpha} \, |\Delta\rangle^{i_1\cdots i_{2R})}_{(\dot\alpha_1\cdots\dot\alpha_{2\bar{j}})} = 0\,, \quad \text{for} \quad \alpha = \pm.$$

$$\bar{\mathcal{B}}^2 : \widetilde{Q}^{(i}_{\dot\alpha} \, |\Delta\rangle^{i_1\cdots i_{2R})}_{(\alpha_1\cdots\alpha_{2j})} = 0\,, \quad \text{for} \quad \dot\alpha = \pm.$$

Semi-short

$$\mathcal{C}^1 : \begin{cases} \epsilon^{\alpha\beta} Q^{(i}_{\alpha} \, |\Delta\rangle^{i_1\cdots i_{2R})}_{(\beta\alpha_2\cdots\alpha_{2j})} = 0\,, & \text{for} \quad j > 0\,, \\ \epsilon^{\alpha\beta} Q^{(i}_{\alpha} Q^{i'}_{\beta} \, |\Delta\rangle^{i_1\cdots i_{2R})} = 0\,, & \text{for} \quad j = 0\,. \end{cases}$$

$$\bar{\mathcal{C}}^2 : \begin{cases} \epsilon^{\dot\alpha\dot\beta} \widetilde{Q}^{(i}_{\dot\alpha} \, |\Delta\rangle^{i_1\cdots i_{2R})}_{(\dot\beta\dot\alpha_2\cdots\dot\alpha_{2\bar{j}})} = 0\,, & \text{for} \quad \bar{j} > 0\,, \\ \epsilon^{\dot\alpha\dot\beta} \widetilde{Q}^{(i}_{\dot\alpha} \widetilde{Q}^{i'}_{\dot\beta} \, |\Delta\rangle^{i_1\cdots i_{2R})} = 0\,, & \text{for} \quad \bar{j} = 0. \end{cases}$$

Operators $Q^i_\alpha, \widetilde{Q}^i_{\dot\alpha}$ are Poincaré supercharges. The Schur multiplets are defined as multiples, satisfying a shortening condition or a semi-shortening condition for each of the chiralities, and there are four types of such multiplet, denoted as $\widehat{\mathcal{B}}_R \, \widehat{\mathcal{C}}_{R(j,\bar{j})} \, \bar{\mathcal{D}}_{R(j,0)} \, \mathcal{D}_{R(0,\bar{j})}$ [6].
In superspace, above BPS conditions are interpreted as differential equations of superfield or correlation functions. For example, when a correlation function contain $\widehat{\mathcal{B}}_R$ multiplet (correspond superfield denoted as $\mathcal{L}_{(i_1\cdots i_{2R})}(z)$), it has to satisfy the following condition.

$$D^{\alpha}{}_{(i} \left\langle \mathcal{L}_{i_1\cdots i_{2R})}(z) \mathcal{O}_1(z_1) \mathcal{O}_2(z_2) \cdots \right\rangle = 0$$

$$\bar{D}^{\dot\alpha}{}_{(i} \left\langle \mathcal{L}_{i_1\cdots i_{2R})}(z) \mathcal{O}_1(z_1) \mathcal{O}_2(z_2) \cdots \right\rangle = 0$$

$D^{\alpha}{}_i, \bar{D}^{\dot\alpha}{}_i$ are ordinal (anti-)chiral superderibatives correspond to $Q^i_\alpha, \bar{Q}^i_{\dot\alpha}$. To determine the selection rules for the Schur multiplets, which are consistent with superconformal algebra, we must solve BPS conditions constraining three-point functions.

Multiplet	Condition	Dimension, $U(1)_r$ charge
$\widehat{\mathcal{B}}_R$	$\mathcal{B}^1 \cap \bar{\mathcal{B}}^2$	$\Delta = 2R, r = 0$
$\mathcal{D}_{R(0,\bar{j})}$	$\mathcal{B}^1 \cap \bar{\mathcal{C}}^2$	$\Delta = 2R + \bar{j} + 1, r = \bar{j} + 1$
$\bar{\mathcal{D}}_{R(j,0)}$	$\mathcal{C}^1 \cap \bar{\mathcal{B}}^2$	$\Delta = 2R + j + 1, r = -j - 1$
$\widehat{\mathcal{C}}_{R(j,\bar{j})}$	$\mathcal{C}^1 \cap \bar{\mathcal{C}}^2$	$\Delta = 2R + j + \bar{j} + 1, r = \bar{j} - j$

3. Conclusion and Discussion

In this poster, we show the results of selection rules and some conjectural rules instead of technical details. We only considered the $\widehat{\mathcal{B}}_{R_1} \times \widehat{\mathcal{B}}_{R_2}$, $\widehat{\mathcal{C}}_{0(0,0)} \times \mathcal{O}^{\text{Schur}}$ selection rules. Especially, the latter is more important, since $\widehat{\mathcal{C}}_{0(0,0)}$ contains stress-tensor of 4D SCFT and $SU(2)_R$ current, which corresponds to stress-tensor in corresponding 2D chiral algebra.

$$\widehat{\mathcal{B}}_{R_1} \times \widehat{\mathcal{B}}_{R_2} \sim \sum_{R=|R_1-R_2|>0}^{R_1+R_2} \widehat{\mathcal{B}}_R + \sum_{R=|R_1-R_2|}^{R_1+R_2-1} \sum_{l=0}^{\infty} \widehat{\mathcal{C}}_{R(\frac{l}{2},\frac{l}{2})}$$

$$\widehat{\mathcal{C}}_{0(0,0)} \times \widehat{\mathcal{B}}_R \sim \widehat{\mathcal{B}}_R + \sum_{l=0}^{\infty} \left[\widehat{\mathcal{C}}_{R(\frac{l}{2},\frac{l}{2})} + \widehat{\mathcal{C}}_{R-1(\frac{l}{2},\frac{l}{2})} \right]$$

$$\widehat{\mathcal{C}}_{0(0,0)} \times \bar{\mathcal{D}}_{R(j,0)} \sim \bar{\mathcal{D}}_{R(j,0)} + \sum_{l=0}^{\infty} \left[\widehat{\mathcal{C}}_{R+\frac{1}{2}(j+\frac{l}{2}+\frac{1}{2},\frac{l}{2})} + \widehat{\mathcal{C}}_{R-\frac{1}{2}(j+\frac{l}{2}+\frac{1}{2},\frac{l}{2})} \right]$$

$$\begin{aligned} \widehat{\mathcal{C}}_{0(0,0)} \times \widehat{\mathcal{C}}_{R(\bar{j}+l_1,\bar{j})} \sim\; & \bar{\mathcal{D}}_{R+\frac{1}{2}(l_1-\frac{1}{2},0)} + \bar{\mathcal{D}}_{R-\frac{1}{2}(l_1-\frac{1}{2},0)} \\ & + \sum_{\bar{j}+\frac{l}{2}=0}^{\infty} \left[\widehat{\mathcal{C}}_{R(\bar{j}+\frac{l}{2}+l_1,\bar{j}+\frac{l}{2})} \right] + \sum_{l=1}^{\infty} \left[\widehat{\mathcal{C}}_{R+1(\bar{j}+\frac{l}{2}+l_1,\bar{j}+\frac{l}{2})} \right] + \sum_{l=1}^{2\bar{j}} \left[\widehat{\mathcal{C}}_{R-1(\bar{j}-\frac{l}{2}+l_1,\bar{j}-\frac{l}{2})} \right] \end{aligned}$$

$$\widehat{\mathcal{C}}_{0(0,0)} \times \widehat{\mathcal{C}}_{R(j,j)} \sim \widehat{\mathcal{B}}_R + \widehat{\mathcal{B}}_{R+1} + \sum_{j+\frac{l}{2}=0}^{\infty} \left[\widehat{\mathcal{C}}_{R(j+\frac{l}{2},j+\frac{l}{2})} \right] + \sum_{l=1}^{\infty} \left[\widehat{\mathcal{C}}_{R+1(j+\frac{l}{2},j+\frac{l}{2})} \right] + \sum_{l=1}^{2j} \left[\widehat{\mathcal{C}}_{R-1(j-\frac{l}{2},j-\frac{l}{2})} \right]$$

These selection rules only depend on superconformal algebra, so any 4D N=2 SCFTs (including no Lagrangian case) satisfy them.

Although our results are only two types of selection rules, we guess some general rules in 4D N=2 SCFT. Let us consider the fusion of two Schur $\mathcal{O}_1^{\text{Schur}} \times \mathcal{O}_2^{\text{Schur}}$ multiplets, and when a Schur multiplet $\mathcal{O}_3^{\text{Schur}}$ exists in it, the fusion may satisfy the following conditions.

- **Schur operator in Schur multiplets must conserve $U(1)_r$ charge in selection rule.**
  $$r_1^{(s)} + r_2^{(s)} = r_3^{(s)}$$
- **$SU(2)_R$ Cartan eigenvalue of Schur operators must satisfy triangle inequality**
  $$\left| R_1^{(s)} - R_2^{(s)} \right| \leq R_3^{(s)} \leq R_1^{(s)} + R_2^{(s)}$$
- **When $\mathcal{O}_1^{\text{Schur}} = \widehat{\mathcal{C}}_{0(0,0)}$ case, there are following inequality**
  $$R_2^{(s)} < R_3^{(s)} \Rightarrow h_2 < h_3, \qquad R_3^{(s)} < R_2^{(s)} \Rightarrow h_3 < h_2$$

$r_i^{(s)}$ and $R_i^{(s)}$ are $U(1)_r$ charge and $SU(2)_R$ Cartan eigenvalue of Schur operators in Schur multiplets $\mathcal{O}_i^{\text{Schur}}$, and h_i is holomorphic dimension of 2D operators associated with the Schur operator.
Above three conjectural rules reproduce all of our selection rules and we can see that Schur operators are also central role in selection rules 4D N=2 SCFTs.

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