Strings and Fields 2019 @ YITP, Kyoto 20 Aug 2019

Thermodynamics of AdS_3 gravity: extremal CFTs vs. semi-classical gravity

Yasunari Kurita
Kanagawa Inst. Tech.

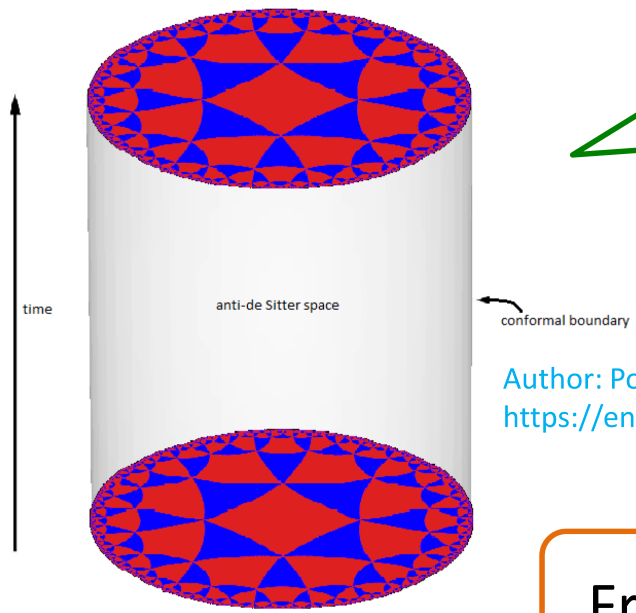
AdS$_3$ pure gravity

Asymptotic symmetry is Virasoro sym. with central charge (Brown-Henneaux 1986)

$$c_L = c_R = \frac{3\ell}{2G}$$

In terms of AdS/CFT, quantum gravity is boundary CFT.

$\Rightarrow$ Find the CFT!



Author: Polytope24
https://en.wikipedia.org/wiki/AdS/CFT_correspondence

At T=0, the boundary is cylinder.
For $T > 0$, it is 2-dim. Torus.

From now on, we consider CFT on torus

Witten(2007)

- Assumption： holomorphic factorization $c_L = c_R = \frac{3\ell}{2G}$

- Witten's conjecture (2007) :

quantum theory of AdS_3 pure gravity is extremal CFTs with
$c = 24k \ (k = 1,2,3,\ldots)$ and it's anti-holomorphic pair

Note 1： $\frac{\ell}{G} = 16k$ is quantized. For each k, one CFT possibly exists.

Note 2： 「large k」～「small G」（semi-classical）
「large k」～「large c」（thermodynamic limit）

This has nice microscopic description for BTZ entropy!

BTZ entropy

Witten 2007

$$S = \pi \left(\frac{\ell}{2G}\right)^{1/2} \left(\sqrt{M\ell - J} + \sqrt{M\ell + J}\right) = 4\pi\sqrt{k}\left(\sqrt{L_0} + \sqrt{\bar{L}_0}\right)$$

$$M\ell = L_0 + \bar{L}_0, \quad J = L_0 - \bar{L}_0, \quad c = \frac{3\ell}{2G} = 24k$$

- In extremal CFTs, for large k

$$\ln W \sim 4\pi\sqrt{kL_0} + \frac{1}{4}\ln k - \frac{3}{4}\ln L_0 - \frac{1}{2}\ln 2 + \cdots$$

correction ?

# of primaries which create BTZ

Bekenstein-Hawking

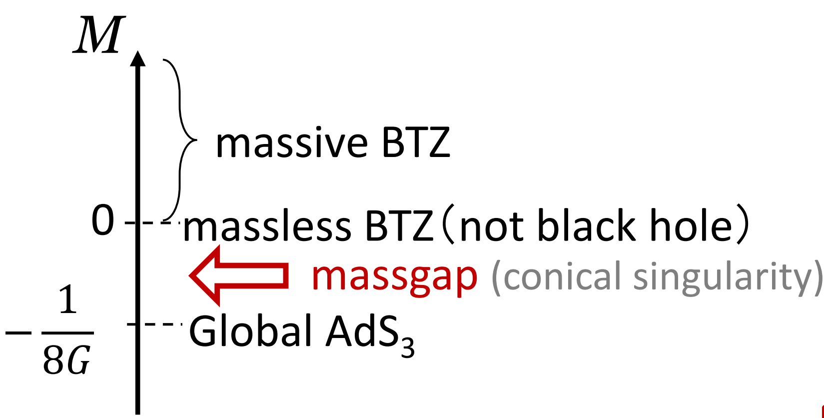
Appendix: Extremal CFT

For c=24k, extremal CFT is a CFT whose lowest dimension of nontrivial primary is k+1 (its maximum).

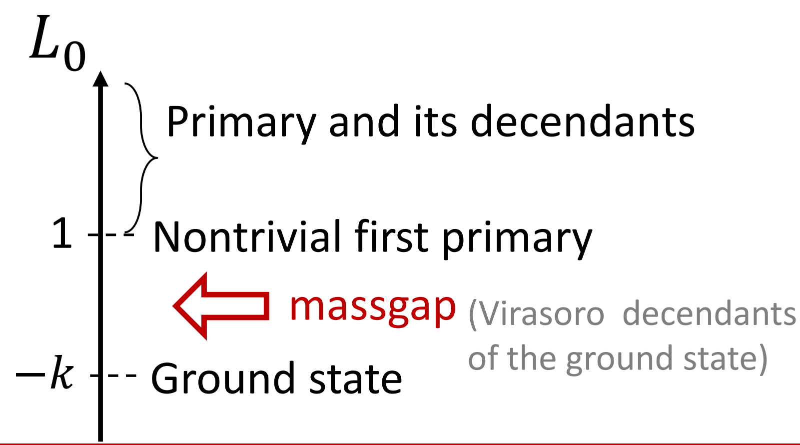
For c=24k, it is known that the lowest dimension of nontrivial primary is equal to or less than k+1.

Note 1: large mass gap

AdS$_3$ classical gravity



Extremal CFT spectrum



Interpretation: nontrivial primary fields make BTZ black hole

Appendix2: Extremal CFT

Note 2： the partition function is uniquely determined!

$$
\begin{aligned}
Z &= \mathrm{Tr}\left[e^{-\beta(H+i\Omega_E J)}\right] \\
&= \mathrm{Tr}\left[q^{L_0}\bar{q}^{\bar{L}_0}\right] \\
&= \left| q^{-k}\left[\prod_{n=2}^{\infty}\frac{1}{1-q^n} + \mathcal{O}(q^{k+1})\right]\right|^2
\end{aligned}
$$

$$q = e^{2\pi i\tau}, \quad \tau = \frac{\beta}{2\pi}\left(\Omega_E + \frac{i}{\ell}\right)$$

$$H = M = \frac{L_0 + \bar{L}_0}{\ell}$$

$$J = L_0 - \bar{L}_0$$

τ is moduli of the boundary torus

Ground state and its Virasoro descendants (these determine the pole structure at q=0.)

Contributions from primaries ($L_0 \geq 1$)

Mathematical fact： holomorphic & modular inv.
$\Rightarrow$ “$Z(\tau)$ is a polynomial of J-function”

$$J = 1728j(\tau) - 744 = q^{-1} + 196884q + \mathcal{O}(q^2)$$

Klein’s j-invariant

Determine the polynomial of J to have the same pole structure with Z, then the partition function is uniquely determined.

This work

- investigates thermodynamic quantities obtained from extremal CFT partition functions Z_k for several k.

→ We find usual Hawking-Page transition (AdS_3 ↔ BTZ)

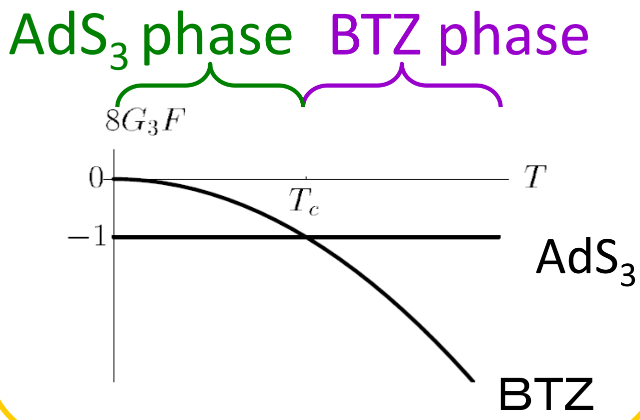
- For rapidly rotating spacetime, we also find several new phases which do not appear in the usual Hawking-Page transition. This is consistent with the phase diagram obtained by Maloney-Witten 2007.

- compare these (quantum) results with those of semi-classical gravity.

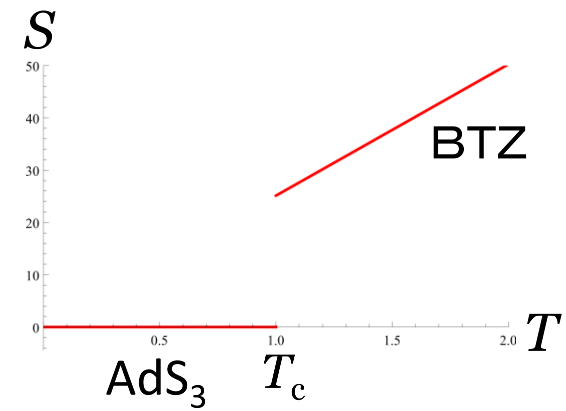
3-dim. Hawking-Page

Semi-classical

Free energy



entropy



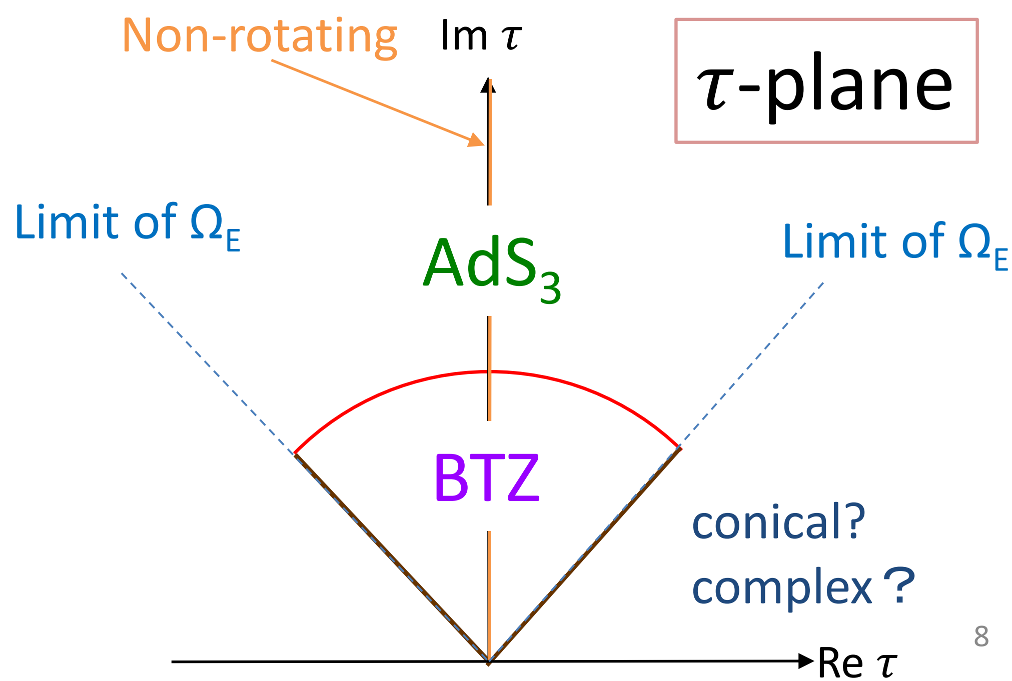
HP critical temperature

$$T_{\rm HP} = \frac{\sqrt{1+\Omega_E^2\ell^2}}{2\pi\ell}$$

$$\Leftrightarrow \quad |\tau| = 1$$

$$\tau = \frac{\beta}{2\pi}\left(\Omega_E + \frac{i}{\ell}\right)$$

Moduli parameter



Partition functions of extremal CFTs

- For first several k,

$$k = \frac{\ell}{16G}$$

Indices are k

$$Z_1(q) = |J(q)|^2 = \left| \frac{41E_4(\tau)^3 + 31E_6(\tau)^2}{72\eta(\tau)^{24}} \right|^2$$

obtained by FLM('84) having Monster symmetry

$$Z_2(q) = |J(q)^2 - 393767|^2$$

$$Z_3(q) = |J(q)^3 - 590651J(q) - 64481279|^2$$

$$Z_4(q) = |J(q)^4 - 787535J(q)^2 - 85975039J(q) + 74069025266|^2$$

Given k, the partition function is computable!

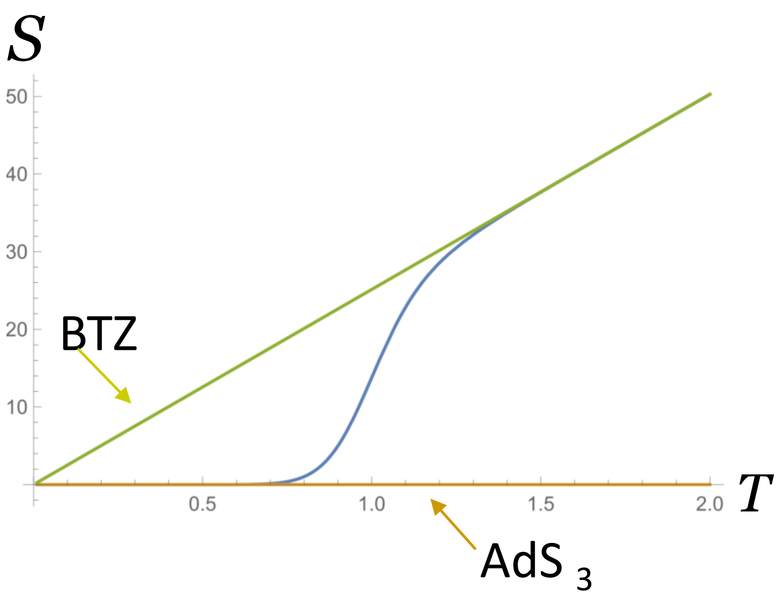
For example : $k = 10$

$$\begin{aligned} Z_{10} \quad = \quad & \big| J^{10} - 1968839J^8 - 214937599J^7 + 1348071256190J^6 \\ & +253704014739574J^5 - 361538450036076764J^4 \\ & -824143081027930253330J^3 + 30123373072315438416085J^2 \\ & +621970556517352063759223 6J - 2643904925535517177481002 92\big|^2 \end{aligned}$$

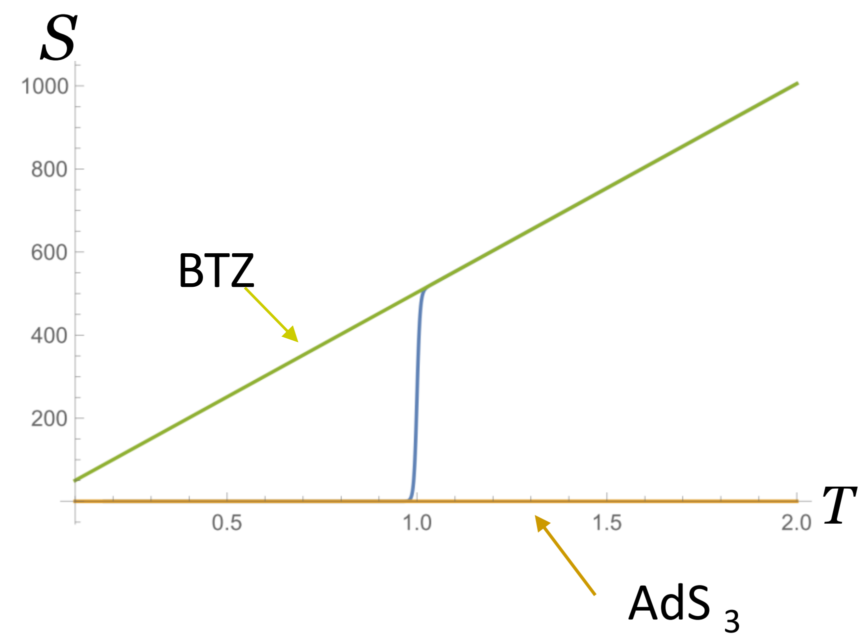
Non-rotating case

Extremal CFT

- k=1 case



- k=20 case



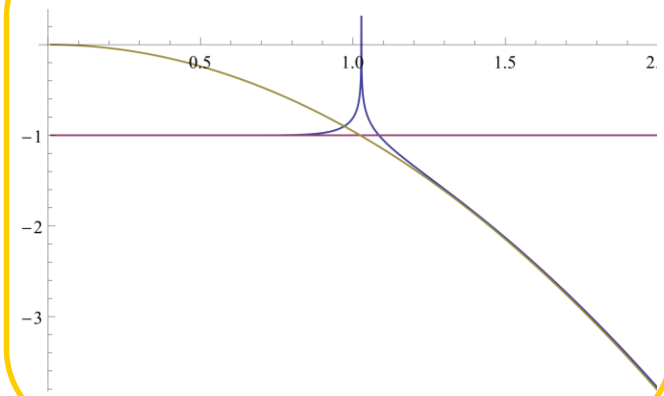
For large k, the transition (at T_{HP}) becomes sharp!
The sharp transition appears in the semi-classical limit.
One can see that quantum theory for AdS_3 gravity might be a sequence of extremal CFTs.

Rotating case

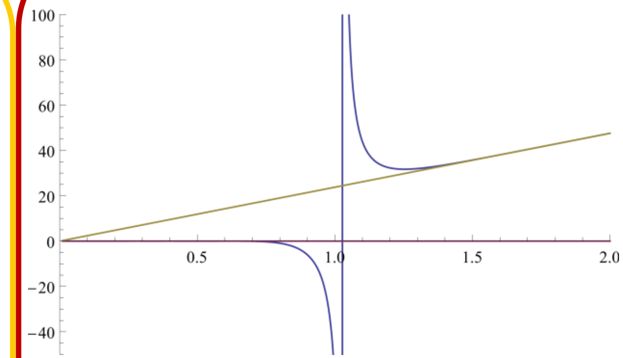
For small k, one finds smooth transition between AdS_3 and BTZ in many parameter region, as non-rotating case. However, at some points, singular behavior appears.

For example : k=1, $\Omega_E \ell = 0.237527\cdots$

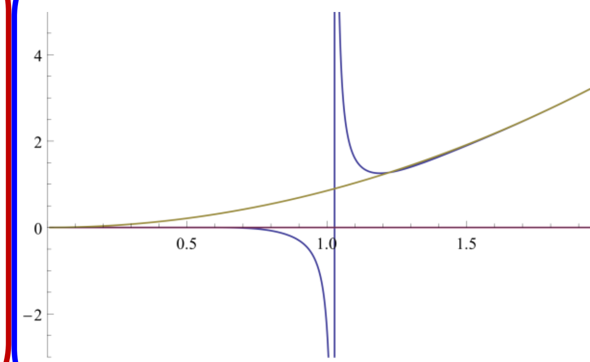
Free energy



entropy



Angular momentum



This singular transition appears at T_{HP}

The transition appears even for small k, that is, not in the thermodynamic limit.

Zeroes of Z_k

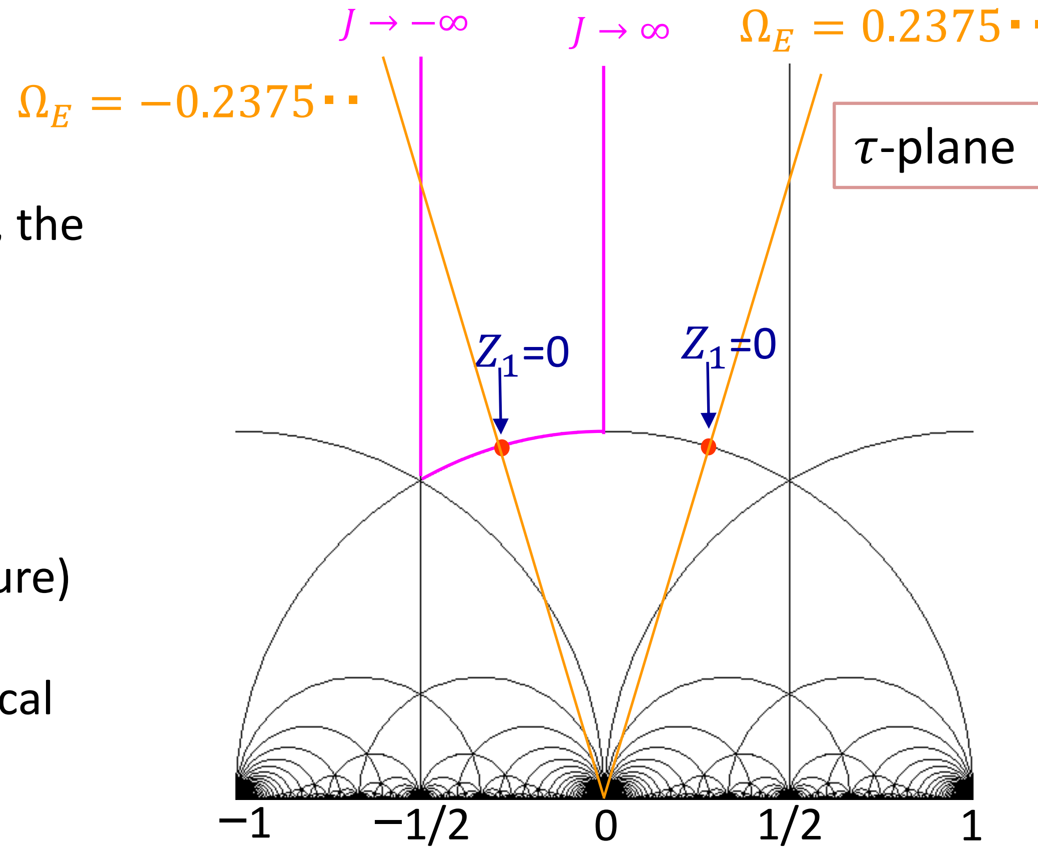
Zeroes of Z_1

At the zeroes of the partition function, the free energy diverges.

$$F_1 = -T \ln Z_1$$

The zeroes (shown as • in the right figure) are on the unit circle $|\tau| = 1$, which corresponds to the Hawking-Page critical temperature.

$$|\tau| = 1 \Leftrightarrow T_{\rm HP} = \frac{\sqrt{1+\Omega_E^2 \ell^2}}{2\pi\ell}$$

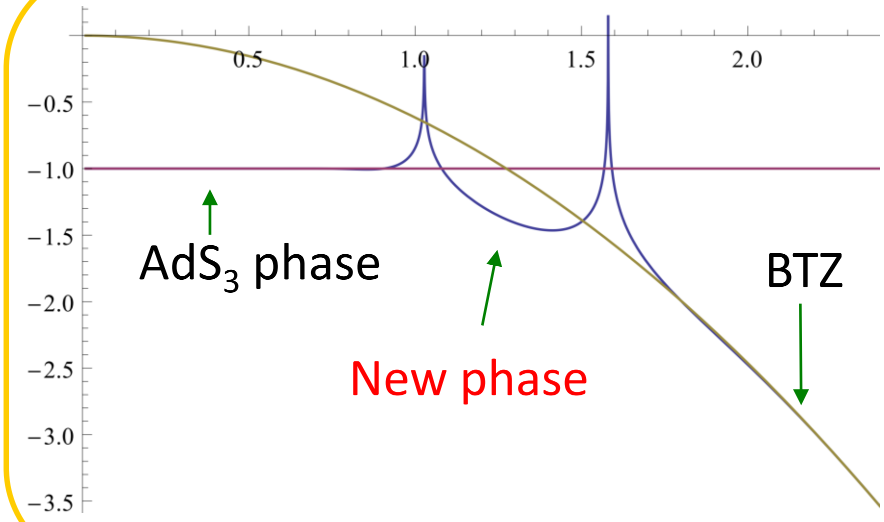


Along the pink line, J function takes real value.

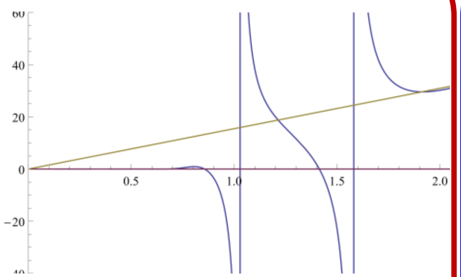
Spin up

- At $\Omega_E \ell \simeq 0.790295$ (for the case of k=1)

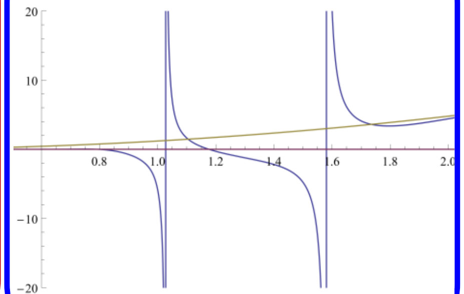
Free energy



entropy



J



These two critical temperatures are different from T_{HP}

Zeroes of Z_1 again (k=1 case)

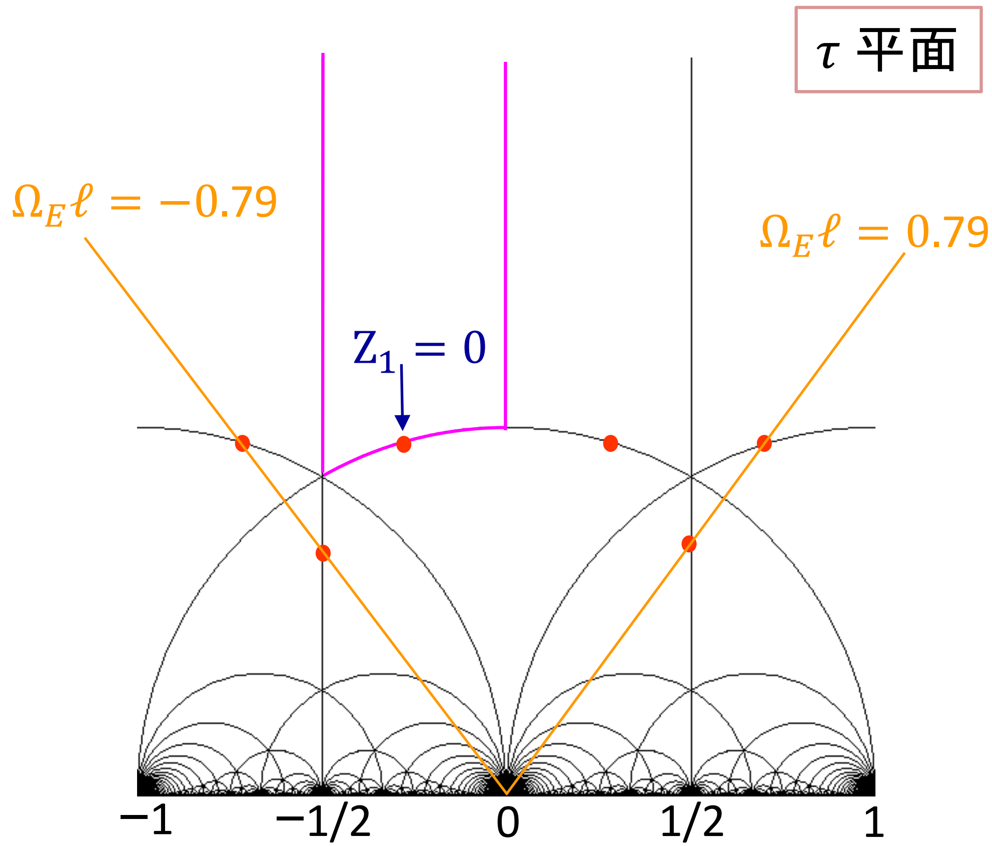
S and T transformations

$$\tau \to \tau + 1, \qquad \tau \to -\frac{1}{\tau}$$

and their combinations move zeroes to other points which are not on the circle $|\tau|$=1.

⇒ The transition occurs at

$$T \neq T_{HP}$$

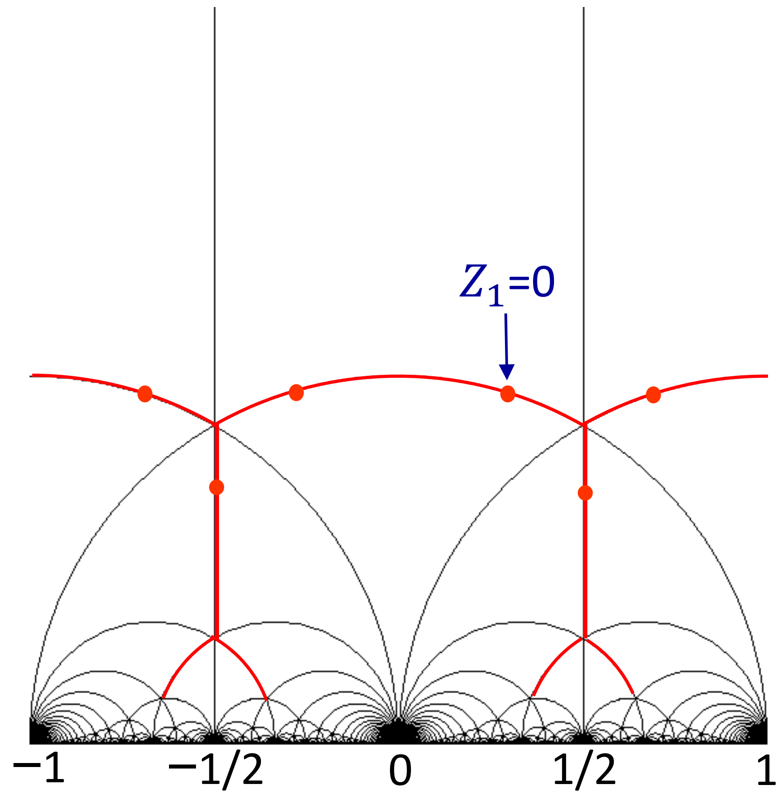


The appearance of new phase might be a prediction from 3-dim. Quantum gravity (extremal CFT).

Semi-classical limit ($k \to \infty$)

- For large k, the # of zeroes of Z_k increases and the zeroes condense into the red line.

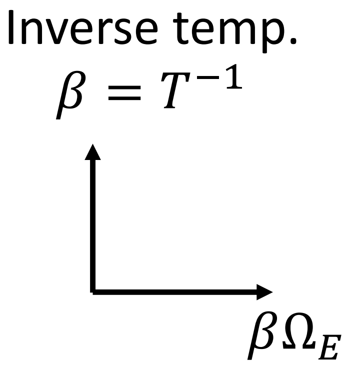
$\Rightarrow$ phase boundary



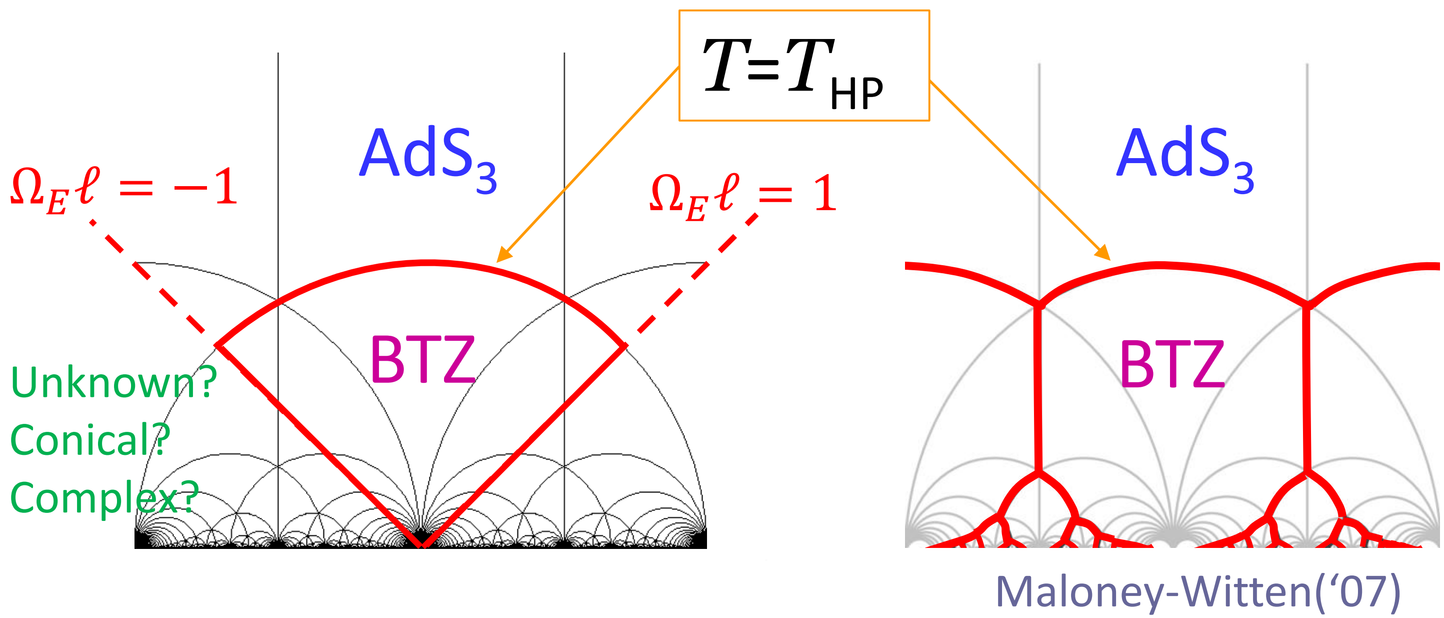
The condensation of zeroes has been proved by Maloney and Witten (2007).

Inconsistent with the semi-classical result!

Phase diagram



Semi-classical extremal CFT



Phase diagrams are different！

Discussion (summary)

- There is an inconsistency in the semi-classical limt ($k \to \infty$).

Possibility 1 : unknown classical solution that corresponds to the new phase?

Possibility 2 : Witten conjecture might get some correction at least for large k: the quantum theory for pure AdS_3 gravity might not be the sequence of extremal CFTs.

- conformal bootstrap : for $k \geq 20$, non exisistence of extremal CFTs!

  Bae, Lee, Lee 2016

- The new phase at k=1 (the FLM model does exist) might be a new prediction from quantum gravity.

In order to obtain semi-classical phase diagram, it seems that modular invariance has to be broken in large k limit. Is it correct? How?

（It also seems to be consistent with Honda-Iizuka-Tanaka-Terashima 2015 ）