LIGHT CONE BOOTSTRAP IN 2D CFTS

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Based on [1905.02191] & [1810.01335]

Main Interest

2d CFT with $c > 1$ & no chiral primaries
(attracts attentions in holography)

My question: What is universal?

CFT is only specified by

central charge
spectrum
OPE coefficient

$\rightarrow$Which data is universal?

Warm up

Consistency condition in CFT is very clear!

Example: **Modular invariance** on partition function

$$\int \mathrm{d}h\,\mathrm{d}\bar{h}\;\rho(h,\bar{h})\mathrm{e}^{-\beta\left(h-\frac{c}{24}\right)-\bar{\beta}\left(\bar{h}-\frac{c}{24}\right)}$$

$$=\int \mathrm{d}h\,\mathrm{d}\bar{h}\;\rho(h,\bar{h})\mathrm{e}^{-\frac{(2\pi)^2}{\beta}\left(h-\frac{c}{24}\right)-\frac{(2\pi)^2}{\bar{\beta}}\left(\bar{h}-\frac{c}{24}\right)}$$

Constraints on spectrum density at E & J:

$$h=\frac{E+J}{2}\geq 0,\qquad \bar{h}=\frac{E-J}{2}\geq 0$$

Warm up

In $\beta, \bar{\beta} \to \infty$ limit, **vacuum** $(h, \bar{h} = 0)$ is dominant,

$$\underbrace{e^{\frac{\beta c}{24} + \frac{\bar{\beta} c}{24}}}_{\text{vacuum}} \sim \int dh\, d\bar{h}\ \rho(h, \bar{h}) e^{-\frac{(2\pi)^2}{\beta}\left(h - \frac{c}{24}\right) - \frac{(2\pi)^2}{\bar{\beta}}\left(\bar{h} - \frac{c}{24}\right)}$$

Inverse Laplace trans.

$$\rho(h, \bar{h}) \sim \int d\beta\, d\bar{\beta}\ e^{\frac{\beta c}{24} + \frac{(2\pi)^2}{\beta}\left(h - \frac{c}{24}\right) + \frac{\bar{\beta} c}{24} + \frac{(2\pi)^2}{\bar{\beta}}\left(\bar{h} - \frac{c}{24}\right)}$$

SPA gives **Cardy formula**

$$\log \rho(h, \bar{h}) \sim 2\pi \sqrt{\frac{c}{6}\left(h - \frac{c}{24}\right)} + 2\pi \sqrt{\frac{c}{6}\left(\bar{h} - \frac{c}{24}\right)}$$

at high energy $h, \bar{h} \gg c$

(because the saddle point should be $\beta, \bar{\beta} \sim \infty$).

Warm up

$$\int \mathrm{d}h\,\mathrm{d}\bar{h}\ \rho(h,\bar{h})\mathrm{e}^{-\beta\left(h-\frac{c}{24}\right)-\bar{\beta}\left(\bar{h}-\frac{c}{24}\right)} = \int \mathrm{d}h\,\mathrm{d}\bar{h}\ \rho(h,\bar{h})\mathrm{e}^{-\frac{(2\pi)^2}{\beta}\left(h-\frac{c}{24}\right)-\frac{(2\pi)^2}{\bar{\beta}}\left(\bar{h}-\frac{c}{24}\right)}$$

Primaries: h.w.s. of conformal algebra

→Info. of others is determined by primaries

Better to replace sum over all states by one over primaries

$$\underset{\text{in primaries}}{\int} \mathrm{d}h\mathrm{d}\bar{h}\ \rho(h,\bar{h})\chi_h(\beta)\chi_{\bar{h}}(\bar{\beta}) = \underset{\text{in primaries}}{\int} \mathrm{d}h\mathrm{d}\bar{h}\ \rho(h,\bar{h})\chi_h\left(\frac{(2\pi)^2}{\beta}\right)\chi_{\bar{h}}\left(\frac{(2\pi)^2}{\bar{\beta}}\right)$$

Possible to solve it in $\beta, \bar{\beta} \to \infty$ in a similar way

→Cardy formula for **primary states**

$$\log\rho(h,\bar{h}) \sim 2\pi\sqrt{\frac{c-1}{6}\left(h-\frac{c-1}{24}\right)} + 2\pi\sqrt{\frac{c-1}{6}\left(\bar{h}-\frac{c-1}{24}\right)}$$

New bootstrap approach

In $\beta, \bar{\beta} \to \infty$ limit,

$$\chi_{\mathbb{I}}(\beta)\chi_{\mathbb{I}}(\bar{\beta}) = \int \mathrm{d}h\mathrm{d}\bar{h}\ \rho(h,\bar{h})\chi_h\left(\frac{(2\pi)^2}{\beta}\right)\chi_{\bar{h}}\left(\frac{(2\pi)^2}{\bar{\beta}}\right)$$

where the saddle point is $(h,\bar{h}) \sim (\infty,\infty)$.

Can we relax the assumption, $\beta, \bar{\beta} \to \infty$?

Let us consider $\beta \to \infty$ limit with $\bar{\beta}$ fixed

$$\chi_{\mathbb{I}}(\bar{\beta}) = \int \mathrm{d}\bar{h}\,\rho(\infty,\bar{h})\chi_{\bar{h}}\left(\frac{(2\pi)^2}{\bar{\beta}}\right)$$

vacuum

Saddle point

$h \sim \infty$

New bootstrap approach

- Bootstrap in $\beta \to \infty$ limit

$$\chi_{\mathbb{I}}(\bar{\beta}) = \int \mathrm{d}\bar{h}\, \rho(\infty, \bar{h}) \chi_{\bar{h}}\left(\frac{(2\pi)^2}{\bar{\beta}}\right)$$

- **Modular S matrix:**

$$\chi_{\mathbb{I}}(\bar{\beta}) = \int_{\frac{c-1}{24}}^{\infty} \mathrm{d}\bar{h}\, S_{\mathbb{I}\bar{h}}\ \chi_{\bar{h}}\left(\frac{(2\pi)^2}{\bar{\beta}}\right)$$

which is perfectly determined by Virasoro alg.

→We can conclude $\quad \rho(\infty, \bar{h}) = S_{\mathbb{I}\bar{h}} \quad$ for any $\bar{h} > \frac{c-1}{24}$

In particular, $S_{\mathbb{I}\bar{h}} \sim \mathrm{e}^{2\pi\sqrt{\frac{c-1}{6}\left(\bar{h}-\frac{c-1}{24}\right)}}$ for $\bar{h} \gg c$ (i.e. Cardy formula).

New bootstrap approach

Operator product expansion

$$O_i(x)O_j(0) \sim \sum_k C_{ijk} \, |x|^{\Delta_k-\Delta_i-\Delta_j} O_k(0) \;\; (+\text{derivatives})$$

Another consistency: **OPE associativity**

$$\langle \phi_B(\infty)\phi_B(1)\underbrace{\phi_A(z,\bar{z})\phi_A(0)}_{\text{OPE}}\rangle = \langle \phi_B(\infty)\phi_A(0)\underbrace{\phi_A(1-z,1-\bar{z})\phi_B(1)}_{\text{OPE}}\rangle$$

$z \leftrightarrow 1-z$

in terms of Virasoro blocks (like $\chi_h(\tau)$ in modular bootstrap),

$$\sum_{p} C_{AAp}C_{pBB} \left|F^{AA}_{BB}(p|z)\right|^2 = \sum_{p} \left(C_{ABp}\right)^2 \left|F^{AB}_{AB}(p|1-z)\right|^2$$

in primaries   in primaries

New bootstrap approach

- Bootstrap in z → 0 limit with $\bar{z}$ fixed

$$\overline{F^{AA}_{BB}}(\mathbb{I}|\bar{z}) = \int \mathrm{d}\bar{h}\, \rho_{AB}(\infty, \bar{h}) \overline{F^{AB}_{AB}}(\bar{h}|1-\bar{z})$$

vacuum

$$\rho_{AB} = \sum_p (C_{ABp})^2 \delta(\bar{h} - \bar{h}_p)$$

- **Fusion matrix:**

$$\overline{F^{AA}_{BB}}(\mathbb{I}|\bar{z}) = \int \mathrm{d}\bar{h}\, F_{\mathbb{I}\bar{h}} \begin{bmatrix} A & A \\ B & B \end{bmatrix} \overline{F^{AB}_{AB}}(\bar{h}|1-\bar{z})$$

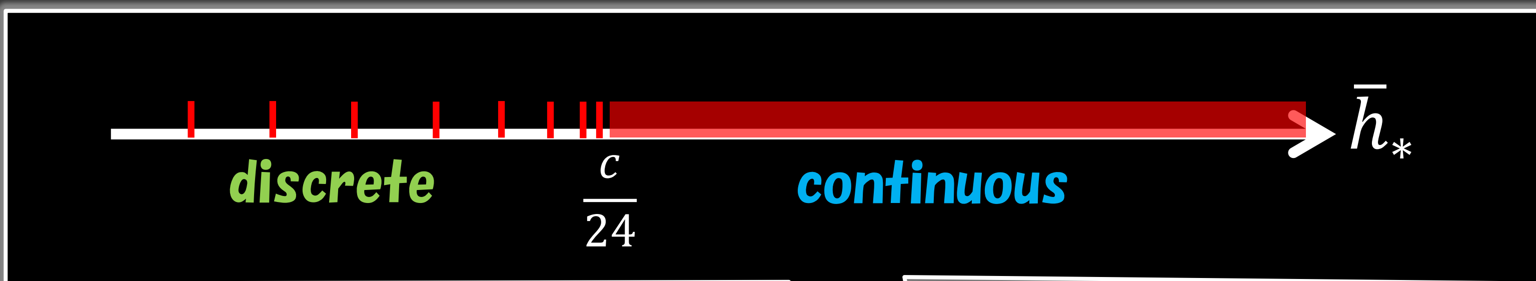
which is perfectly determined by Virasoro alg.

→We can conclude $\rho_{AB}(\infty, \bar{h}) = F_{\mathbb{I}\bar{h}}$

CFT Physics

If $F_{\mathbb{I}\bar{h}}$ is non-zero at $\bar{h}_*$,

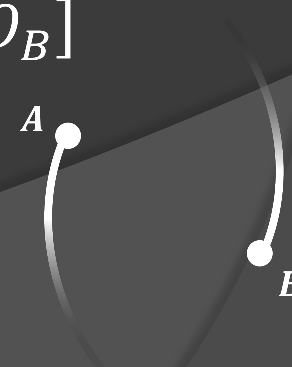
$\rightarrow$state composed of O_A & O_B exists at $\bar{h}_*$



Physical interpretation:

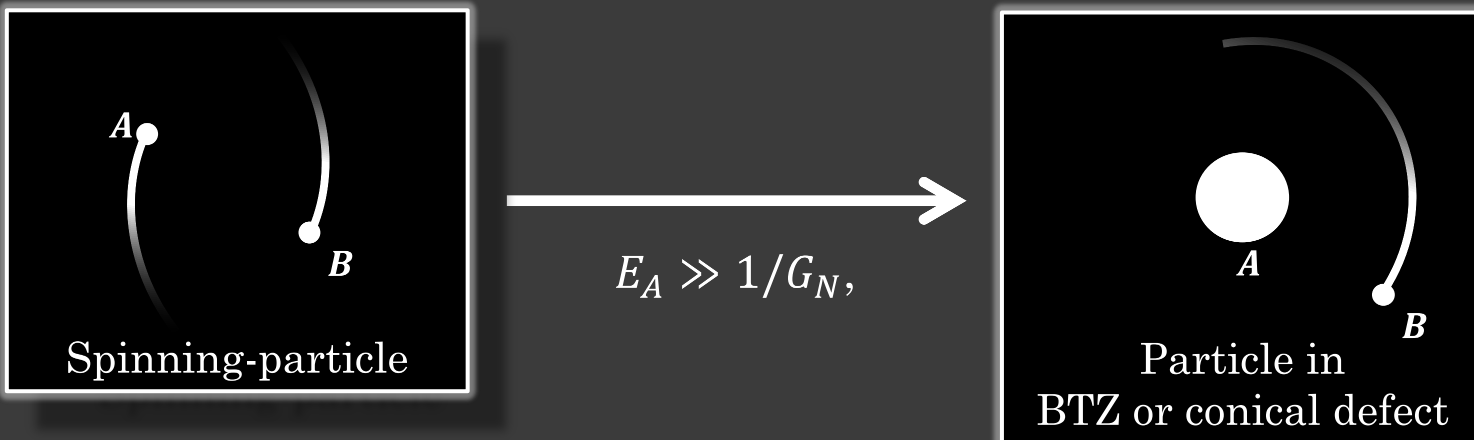
E, J: energy & spin of spinning particle $[O_A O_B]$

$E - J = 2\bar{h} \xrightarrow[J\sim h\gg\bar{h}]{}$ universal spectrum



Bulk side

$E - J$ is calculable if $E_A \sim \frac{1}{G_N}$ with E_B fixed [Fitzpatrick, Kaplan, Walters]



Our result perfectly matches bulk result in $c \to \infty$ with $\frac{h_A}{c}, h_B$ fixed.

Point: Our result is ***non-perturbative*** in c & holds for any h_A.

→Hint for understanding quantum gravity?

Discussion

Universal CFT data is extracted from **fusion matrix**

Fusion matrix approach is also useful

- Light cone singularity & Regge singularity
  →Dynamics of
  - EE [1810.01335] & [1905.02191]
  - OTOC [1905.02191]
  - EWCS [1907.06646]
  - Gravity Force [1908.03351]

Future work:

Can we extract other CFT data
from a combination of **fusion** & **monodromy** matrices?