Holographic OPE Coefficients from AdS Black Holes with Matters

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- Gauged supergravity examples

Abstract

We study the OPE coefficients $c_{\Delta,J}$ for heavy-light scalar four-point functions, which can be obtained holographically from two point function of a light scalar for some non-integer conformal dimension Δ_L is AdS black hole.
We verify that the OPE coefficient $c_{d,0}$=0 for pure gravity, satisfying the consistency of holographic energy-momentum tensor T^{μ}_{μ}=0.

We then study the OPE coefficients from black holes involving matter fields. First we consider the charged black hole and give some explicit example of OPE coefficients and then the recursion formula for the lowest-twist OPE coefficients with at most two current operators.

Finally we consider the charged AdS black hols in gauged supergravity derive the linear perturbation of such a scalar dual to the operators with $\Delta_L = d - 2$ and obtain the OPE coefficients $c_{d-2,0}$

Motivation

- The AdS/CFT correspondence establishes an insightful routine to investigate a strongly coupled conformal field theory(CFT) by using appropriate weakly coupled gravity in AdS space [J.M. Maldacena, hep-th/9711200]. Typically, even though the structures are same, different gravity theories may lead to different CFT data. Thus gravities can be served as effective CFTs
- The simplest as well as nontrivial structures of holographic CFTs is four point functions, which can be decomposed into conformal blocks determined by conformal symmetry with theory dependent OPE coefficient. [J.D. Qualls, 1511.04074; S.Rychkov, 1601.05000]. However, although the holographic conformal blocks as the geodesic Witten diagram were studied extensively [E. Hijano 1508.00501; K.B. Alkalaev 1510.06685], explicitly computing them for quite general classes of higher-derivative gravities is rather challenging.
- On the other hand, higher-point correlation functions, such as four-point functions can be studied further than the structures without referring to any specific theory by bootstrap program [D. Poland, 1805. 04405]. The consistency: the unitarity, the crossing symmetry and ANEC can universally constrain the spectrum and CFT data beyond higher-point functions.
- Follow this motivation and the formalism set up by literatures [A.L. Fitzpatrick, 1903.05306], we study the holographic OPE coefficients for general black holes involving matter fields.

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Four-point functions from holography

In this section, we will take a brief review about the holographic technique to compute the heavy-light four point functions in heavy limit, developed in [A.L. Fitzpatrick, 1903.05306].
In s-channel in the conformal frame, the four-point function can be decomposed as

$$\langle \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \mathcal{O}_H \rangle = (z\bar{z})^{-\frac{\Delta_H+\Delta_L}{2}} \sum_{\mathcal{O}} c_{\Delta,J}\, G_{\Delta,J}^{\Delta_{HL},-\Delta_{HL}}(z,\bar{z})\,, \tag{1}$$

In the holographic picture, the AdS black hole can be interpreted as the excited states $|\mathrm{BH}\rangle \simeq \mathcal{O}_H|0\rangle$. In heavy limit, we can treat the heavy-light four point function like

$$\langle \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \mathcal{O}_H \rangle \simeq \langle \mathcal{O}_L \mathcal{O}_L \rangle_{\mathrm{BH}}\,. \tag{2}$$

In other word, we can use the standard holographic dictionary to derive the two point function in the AdS black hole background instead of computing complicated four-point function.
According to the crossing symmetry, it is advantageous to compute such as four point function t-channel as

$$\langle \mathcal{O}_L \mathcal{O}_L \rangle_{\mathrm{BH}} = ((1-z)(1-\bar{z}))^{-\Delta_L} \sum_{\tilde{\mathcal{O}}} \tilde{c}_{\Delta,J}\, G_{\Delta,J}^{0,0}(1-z,1-\bar{z})\,, \tag{3}$$

Four-point functions from holography

For convenience, we replace z by $1 - z$ and take the light cone limit $z \to 0$:

$$\langle \mathcal{O}_L \mathcal{O}_L \rangle_{\rm BH} = (z\bar{z})^{-\Delta_L} \sum_{\mathcal{O}} c_{\Delta,J}\, G^{0,0}_{\Delta,J}(z,\bar{z})\,. \tag{4}$$

Remark on conformal block:
by the virtual of the conformal symmetry the four-point function can be written in a compact form

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \frac{g(u,v)}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2}{2}}(x_{34}^2)^{\frac{\Delta_3+\Delta_4}{2}}}\left(\frac{x_{24}^2}{x_{14}^2}\right)^{\frac{\Delta_{12}}{2}}\left(\frac{x_{14}^2}{x_{13}^2}\right)^{\frac{\Delta_{34}}{2}}\,, \tag{5}$$

where $x_{ij} = x_i - x_j$, $\Delta_{ij} = \Delta_i - \Delta_j$ and $g(u,v)$ is a function of the cross ratios (u,v):

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}\,, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})\,. \tag{6}$$

To study the four-point functions, it is standard and convenient to use the conformal symmetry to take the conformal frame, namely

$$x_1 = (0,0,\cdots)\,, \qquad x_2 = (x,y,0,\cdots)\,, \qquad x_3 = (1,0,\cdots)\,, \qquad x_4 \to \infty\,. \tag{7}$$

Defining $z = x + \mathrm{i}y$ and $\bar{z} = x - \mathrm{i}y$, we have

$$u = z\bar{z}\,, \qquad v = (1-z)(1-\bar{z})\,. \tag{8}$$

Four-point functions from holography

By applying the OPE expansion, $g(u, v)$ is expected to be decomposed into conformal blocks characterized by conformal dimension Δ and spin J

$$g(u, v) = \sum_{\Delta, J} \lambda^J_{12\Delta} \lambda^J_{34\Delta} G^{\Delta_{12}, \Delta_{34}}_{\Delta, J}(z, \bar{z}) , \tag{9}$$

where $\lambda_{ij\Delta}$'s are the coefficients in OPE expansions and hence the three-point functions are $\langle O_i O_j O_{\Delta,J} \rangle \propto \lambda^J_{ij\Delta}$.

In general even d dimension, we consider the OPE limit ($z \ll 1$, $\bar{z} \ll 1$), for which the leading OPE tells us the conformal block can take the simple form

$$G^{00}_{\Delta,J} = (z\bar{z})^{\frac{\Delta}{2}} \frac{J!}{(\frac{d}{2} - 1)_J} C^{\frac{d}{2}-1}_J \left(\frac{z + \bar{z}}{2\sqrt{z\bar{z}}}\right) + \cdots , \tag{10}$$

where $C^{\frac{d}{2}-1}_J$ is the Gegenbauer polynomial.

The construction in pure gravity backgrounds

we consider gravity minimally coupled to a free massive scalar:

$$\mathcal{L} = \sqrt{|g|}\Big(R - 2\Lambda + L(R_{\mu\nu\rho\sigma}) - \tfrac{1}{2}(\partial\phi)^2 - m^2\phi^2\Big), \qquad \Lambda = \frac{d(d-1)}{2\ell_0^2}, \tag{11}$$

Maintaining the spherical symmetry, one can construct Euclidean AdS planar black holes (with ϕ remaining zero):

$$ds^2 = r^2 f(r) dt^2 + \frac{1}{r^2 h(r)} dr^2 + r^2(du^2 + u^2 d\Omega_{d-2}^2). \tag{12}$$

For Einstein gravity, we have the solution of Schwarzschild-AdS planar black hole:

$$f = h = 1 - \frac{f_0}{r^d}. \tag{13}$$

For general $\mathcal{L}(R_{\mu\nu\rho\sigma})$, we could take the massless mode and h and f can have such asymptotic form

$$f(r) = 1 - \frac{f_0}{r^d} - \frac{f_d}{r^{2d}} - \cdots, \qquad h(r) = 1 - \frac{h_0}{r^d} - \frac{h_d}{r^{2d}} - \cdots. \tag{14}$$

The equation of motion of the free scalar field ϕ in the AdS black hole background is

$$(\Box - m^2)\phi = 0, \qquad m^2 = \Delta_L(\Delta_L - d) \geq m_{\rm BF}^2 = \tfrac{1}{4}d^2, \tag{15}$$

The construction in pure gravity backgrounds

According to the holographic dictionary, the solution of ϕ give rise to the bulk-to-boundary propagator $\Phi(r,t,u)$ in which the coefficient of $1/r^{\Delta_L}$ is the two point function. We making the coordinate transformation as

$$t = -\frac{1}{2}(z+\bar{z})\,, \qquad u = \frac{\mathrm{i}}{2}(z-\bar{z})$$
$$w^2 = 1 + r^2(t^2+u^2) = 1 + r^2 z\bar{z}\,, \qquad \hat{u} = ru = \frac{\mathrm{i}}{2}r(z-\bar{z})\,. \tag{16}$$

In this coordinate system ,the equation of motion of ϕ in AdS vacuum can be expressed simply as

$$\Phi_{\mathrm{AdS}} = (\frac{r}{w^2})^{\Delta_L} \sim \frac{(z\bar{z})^{-\Delta_L}}{r^{\Delta_L}} + \cdots\,, \qquad \text{for} \qquad r \to \infty\,. \tag{17}$$

Comparing to the four point function in heavy limit, it is natural to factorize the bulk to boundary propagator in general asymptotic form

$$\Phi(r,w,\hat{u}) = \Phi_{\mathrm{AdS}} G(r,w,\hat{u})\,. \tag{18}$$

Then the function $G(r,w,\hat{u})$ in the $r\to\infty$ limit is precisely the conformal block. In other words, the holographic dictionary now reduces to

$$\boxed{\sum_{\mathcal{O}} c_{\Delta,J}\, G^{0,0}_{\Delta,J}(z,\bar{z}) = \lim_{r\to\infty} G(r,w,\hat{u})\,.} \tag{19}$$

The construction in pure gravity backgrounds

There are two sets of operators that can exchange in the OPE expansions in scalar four-point function, one is multi-stress operator T^n, denoting n stress tensor multiplication and contributing $\Delta = nd$ in conformal block. like

$$T_{\mu_1\mu_2}\cdots T_{\mu_{n-1}\mu_n}. \tag{20}$$

The other set is double-trace operator with spin J and conformal dimension $\Delta = 2\Delta_L + 2n + J$:

$$[\mathcal{O}_L]^{\Delta}_J = \mathcal{O}_L \Box^n \partial_{\mu_1}\cdots\partial_{\mu_J}\mathcal{O}_L\,.. \tag{21}$$

According to this result, the near boundary expansion for $G(r, w, \hat{u})$ should be take the form

$$\begin{aligned} G(r,w,\hat{u}) = 1 \quad &+ \quad G^T(r,w,\hat{u}) + G^L(r,w,\hat{u})\,, \\ G^T(r,w,\hat{u}) = \frac{1}{r^d}\sum_{i\in\mathbb{N}}\frac{G_i^T(w,\hat{u})}{r^{id}}\,, \qquad &\qquad G^L(r,w,\hat{u}) = (\frac{w}{r})^{2\Delta_L}\sum_{i\in 2\mathbb{N}}\frac{G_i^L(w,\hat{u})}{r^i}\,, \end{aligned} \tag{22}$$

When Δ_L is not an integer, this two set is independent.

The construction in pure gravity backgrounds

Since G_i^T and G_i^L should be related to the conformal blocks with certain Δ, they must take the polynomials of $\hat{u}$,

$$G_i^T = \sum_{j\in 2\mathbb{N}}^{2(1+i)} a_{ij}(w)\hat{u}^j\,, \qquad G_i^L = \sum_{j\in 2\mathbb{N}}^{i} b_{ij}(w)\hat{u}^j\,. \tag{23}$$

The truncation is a little subtle. If there is no truncation, we will obtain

$$\frac{1}{r^\Delta}\sum_{m=-\infty}^{\infty} w^{\Delta-m}\hat{u}^m \sim \sum_{m=-\infty}^{\infty}(z\bar{z})^{\frac{\Delta-m}{2}}(z-\bar{z})^m\,. \tag{24}$$

However, the lowest power for z in conformal blocks should be $\frac{1}{2}(\Delta-J)$, we then have $m\leq J$. Thus, for the multi-stress set T^n, we have the $m\leq 2(1+i)$ truncation and for the double-trace set (21) we have the truncation that the $m\leq J=i-2n\leq i$ and that the coefficients of the higher-n terms vanish.
As the result, it can turns out that in d-dimension $a_{ij}(w)$ can be the polynomial of w:

$$a_{ij}(w) = \sum_{k=-2(1+i)}^{(1+i)d-j} a_{ijk}w^k\,, \tag{25}$$

The consistency of $c_{d,0} = 0$

The OPE $c_{d,0}$ describe the exchange particle with $\Delta = d$ and spin-0 operator. The only operator contribute $c_{d,0}$ is the trace of the energy-momentum tensor

$$T^{\mu}_{\mu}. \tag{26}$$

To show this, we note that for $n = 1$, with maximum $J = 2$, so that $G(r, w, \hat{u})$ should take up to $\hat{u}^2$:

$$G(r, w, \hat{u}) = 1 + \frac{1}{r^d}\Big(\sum_{k=-2}^{\frac{d}{2}} a_k w^{2k} + \sum_{k=-2}^{\frac{d-2}{2}} b_k \hat{u}^2 w^{2k}\Big). \tag{27}$$

Plugging into the equation of motion of ϕ, a_n and b_n can be solved as

$$b_{-1} = -\frac{f_0 \Delta_L}{d+1}\,, \qquad b_k = \frac{(d-2k)}{2(d-k)} b_{k-1}\,, \qquad k = 2, 3, \ldots\,. \tag{28}$$

For even d, the series terminates at $k = d/2$ and hence we have

$$b_k = -\frac{(d-2) f_0 \Delta_L (2 - \frac{d}{2})_{k-1}}{4(d^2-1)(2-d)_{k-1}}\,, \qquad -1 \le k \le \tfrac{1}{2}d - 1\,, \tag{29}$$

where $(i)_j$ is the Pochhammer polynomial

$$(i)_j = \frac{\Gamma(i+j)}{\Gamma(i)}\,. \tag{30}$$

The consistency of $c_{d,0} = 0$

The $\hat{u}^0$-order terms give rise to the recursion relation for a_n:

$$\begin{aligned} a_{-1} &= -\frac{(f_0+h_0)\Delta_L}{d+1}\,, \qquad a_0 = \frac{(d-1)(f_0-h_0)+2(f_0+h_0)\Delta_L}{4(d+1)}\,, \\ a_1 &= \frac{(f_0+h_0(d-\Delta_L)+df_0(\Delta_L-2))\Delta_L}{4(d^2-1)(\Delta_L-1)}\,, \\ a_k &= \frac{(2+d-2k)(k-\Delta_L-1)a_{k-1}-(d-1)b_{k-1}}{2(d-k)(k-\Delta_L)}\,, \qquad k \geq 2\,. \end{aligned} \tag{31}$$

We thus end up with

$$a_k = \frac{\Delta_L(-dh_0 - f_0 k + df_0(k+1-\Delta_L) + h_0\Delta_L)(1-\frac{d}{2})_{k-1}}{4(d^2-1)(k-\Delta_L)(2-d)_{k-1}}\,, \qquad k \geq -1\,. \tag{32}$$

With the solution (32) and (29), both OPE coefficients $c_{d,0}$ can be read off. We find that the coefficient $c_{d,0}$ is

$$c_{d,0} = \frac{\sqrt{\pi}2^{-d-1}\Delta_L(d-\Delta_L)\Gamma\left(\frac{d}{2}\right)(f_0-h_0)}{(d-2\Delta_L)\Gamma\left(\frac{d+3}{2}\right)}\,. \tag{33}$$

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We consider a general class of theories of the following form

$$L = R - 2\Lambda - \tfrac{1}{4}F^2 + \mathcal{L}(R_{\mu\nu\rho\sigma}, F_{\mu\nu}) - \tfrac{1}{2}(\partial\phi^2 + m^2\phi^2)\,, \tag{34}$$

where $\mathcal{L}(R_{\mu\nu\rho\sigma}, F_{\mu\nu})$ represents the higher-order invariant polynomials of the curvature tensor and the strength $F_{\mu\nu}$ and hence matter and gravity can be generally non-minimally coupled.

The falling off of the metric function

$$f = 1 - \frac{f_0}{r^d} + \frac{\tilde{f}_0}{r^{2(d-1)}} - \frac{f_d}{r^{2d}} + \cdots\,, \qquad h = 1 - \frac{h_0}{r^d} + \frac{\tilde{h}_0}{r^{2(d-1)}} - \frac{h_d}{r^{2d}} + \cdots\,, \tag{35}$$

where $\tilde{f}_0$ is proportional to Q^2 (the charge squared) of black holes.

To be precise, we now have the power series

$$G(r, w, \hat{u}) = G^s(r, w, \hat{u}) = \sum_{i,j\in\mathbb{N}} \frac{G_{ij}(w, \hat{u})}{r^{id+2j(d-1)}}\,, \qquad G_{00} = 1\,, \tag{36}$$

In $d = 4$,

$$f = 1 - \frac{f_0}{r^4} + \frac{\tilde{f}_0}{r^6} - \frac{f_4}{r^8} + \frac{f_6}{r^{10}} + \cdots, \qquad h = 1 - \frac{h_0}{r^4} + \frac{\tilde{h}_0}{r^6} - \frac{h_4}{r^8} + \frac{h_6}{r^{10}} + \cdots. \tag{37}$$

The structure of G:

$$G(r, w, \hat{u}) = 1 + \frac{G_{10}(w, \hat{u})}{r^4} + \frac{G_{01}(w, \hat{u})}{r^6} + \frac{G_{20}(w, \hat{u})}{r^8} + \frac{G_{11}(w, \hat{u})}{r^{10}} + \cdots,$$

$$G_{10} = \sum_{j=-2}^{4-i} \sum_{i=0}^{2} \alpha_{ij}^{10} \hat{u}^i w^j, \qquad G_{10} = \sum_{j=-2}^{6-i} \sum_{i=0}^{2} \alpha_{ij}^{01} \hat{u}^i w^j,$$

$$G_{20} = \sum_{j=-4}^{8-i} \sum_{i=0}^{4} \alpha_{ij}^{20} \hat{u}^i w^j, \qquad G_{11} = \sum_{j=-4}^{10-i} \sum_{i=0}^{4} \alpha_{ij}^{11} \hat{u}^i w^j. \tag{38}$$

We obtain explicit OPE coefficients

<u>$\Delta = 6$</u>:

$$\mathcal{J}_\mu \mathcal{J}^\mu, \qquad c_{6,0} = -\frac{\Delta_L(\Delta_L^2 - 4\Delta_L + 9)(3\tilde{f}_0 - 2\tilde{h}_0)}{1680(\Delta_L - 3)(\Delta_L - 2)}$$

$$\mathcal{J}_\mu \mathcal{J}_\nu, \qquad c_{6,2} = -\frac{\tilde{f}_0 \Delta_L(1 + \Delta_L)}{560(\Delta_L - 2)}, \tag{39}$$

$\underline{\Delta = 8}$:

$$
\begin{aligned}
T_{\mu\nu}T^{\mu\nu}\,, \qquad & c_{8,0} = \frac{\Delta_L}{201600(\Delta_L-4)(\Delta_L-3)(\Delta_L-2)}\Big(2(\Delta_L(\Delta_L(\Delta_L(7\Delta_L-45)+100) \\
& +100)+228)f_0^2 - 2(\Delta_L(\Delta_L(\Delta_L(7\Delta_L-55)+130)+80)+168)f_0h_0 \\
& +40\Delta_L((\Delta_L-3)\Delta_L+20)(2f_4-h_4)+960(2f_4-h_4) \\
& +(\Delta_L-6)(\Delta_L(\Delta_L(7\Delta_L-23)+22)+12)h_0^2\Big)\,, \\
T_{\mu\rho}T^{\rho}_{\nu}\,, \qquad & c_{8,2} = \frac{\Delta_L}{201600(\Delta_L-3)(\Delta_L-2)}\Big((21\Delta_L^3-49\Delta_L^2+126\Delta_L+76)f_0^2 \\
& -2(7\Delta_L^3-13\Delta_L^2+52\Delta_L+32)f_0h_0+80(\Delta_L^2+3\Delta_L+2)f_4\Big)\,, \\
T_{\mu\nu}T_{\rho\sigma}\,, \qquad & c_{8,4} = \frac{\Delta_L(7\Delta_L^2+6\Delta_L+4)f_0^2}{201600(\Delta_L-2)}\,,
\end{aligned}
\tag{40}
$$

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Guaged supergravity

Chow proposed a class of Einstein-Maxwell-Dilaton (STU) models in general dimensions, in which the Lagrangian is [D.D.K. Chow 1108.5319].

$$
\begin{aligned}
L &= R - V - \frac{1}{2}(\partial\varphi_1)^2 - \frac{1}{2}(\partial\varphi_2)^2 - \frac{1}{4}\sum_{i=1}^{2} X_i^{-2}(F^i)^2 \,, \qquad X_i = e^{-\frac{1}{2}\vec{a}_i\cdot\vec{\varphi}} \,, \\
V &= -(D-3)^2 X_1 X_2 - 2(D-3)(X_1X_2)^{-\frac{D-3}{2}}(X_1 + X_2) \\
&\quad +(D-5)(X_1X_2)^{-(D-3)} \,, \\
\vec{\varphi} &= (\varphi_1, \varphi_2) \,, \qquad \vec{a}_1 = \Big(\sqrt{\tfrac{2}{D-2}}, \sqrt{2}\Big) \,, \qquad \vec{a}_2 = \Big(\sqrt{\tfrac{2}{D-2}}, -\sqrt{2}\Big) \,. \qquad (41)
\end{aligned}
$$

Charged AdS black hole can be obtain:

$$
\begin{aligned}
ds_D^2 &= (H_1H_2)^{-\frac{D-3}{D-2}} \tilde{f} dt^2 + (H_1H_2)^{\frac{1}{D-2}} (\tilde{f}^{-1} d\rho^2 + \rho^2 dx^i dx^i) \,, \\
X_i &= H_i^{-1}(H_1H_2)^{\frac{D-3}{2(D-2)}} \,, \qquad A^i = \sqrt{\frac{\mu}{q_i}}(1 - H_i^{-1}) dt \,, \\
\tilde{f} &= -\frac{\mu}{\rho^{D-3}} + \rho^2 H_1 H_2 \,, \qquad H_i = 1 - \frac{q_i^2}{\rho^{D-3}} \,, \qquad i = 1, 2 \,. \qquad (42)
\end{aligned}
$$

We can consistently truncate $\varphi_2 = 0$ by requiring $F^1 = F^2 = F/\sqrt{2}$. The resulting theory admits the the black holes in (42) with $q_1 = q_2$. We then turn on the linear perturbation. The linear perturbation of the ϕ can be deduced as

$$(\Box - m^2(r))\phi = 0, \tag{43}$$

where

$$m^2(r) = -2(D-3)e^{\frac{D-4}{\sqrt{2(D-2)}}\varphi_1} + \frac{1}{2}e^{\sqrt{\frac{2}{D-2}}\varphi_1}F^2 . \tag{44}$$

Plugging the background solution in to the perturbation function, we find the fall off of $m^2(r)$ is

$$m^2(r) = -2(d-2) - \frac{m_0}{r^{d-2}} + \cdots , \qquad m_0 = -\frac{2(d-2)(d-3)q}{d-1} . \tag{45}$$

We could conclude that we have operators contribute $\Delta = d-2$ and we now turn to derive it. For the spin-0, the G can be taken

$$G(r, w, \hat{u}) = 1 + \frac{G^{(d-2)}(w)}{r^{d-2}} + \cdots . \tag{46}$$

Substituting the antantz into the E.O.M, the reduced equation is

$$\begin{aligned} &-2(d-2)(w^2(\Delta_L - 1) - 2\Delta_L)G^{(d-2)} + w((d(w^2-2) + (w^2-1)(2\Delta_L - 5))G^{(d-2)\prime} \\ &-2(m_0 + (w^2-1)G^{(d-2)\prime\prime})) = 0 . \end{aligned} \tag{47}$$

It can be exactly solved that

$$G^{(d-2)} = \frac{1}{2(\Delta_L - 1)\Gamma(d-1)}\Big(m_0 w^{d-4}(w^2-1)^{1-\frac{d}{2}}(w^d\Gamma(\frac{d}{2}-1)\Gamma(\frac{d}{2}) \\ -w^2\Gamma(d-2))\,{}_2F_1[2-\tfrac{1}{2}d, \tfrac{1}{2}(d-2); \tfrac{1}{2}d; w^{-2}]\Big)\,. \tag{48}$$

We consider the even d in such that the polynomial antantz, we have

$$c_{d-2,0} = \frac{m_0\Gamma(\frac{d}{2}-1)\Gamma(\frac{d}{2})}{2(\Delta_L-1)\Gamma(d-1)} = -\frac{2q\Gamma(\frac{d}{2})^2}{\Gamma(d)}\,. \tag{49}$$

We take an example of $d = 4$, denoting that the gauged supergravity will reduce to $D = 5$ STU model. The metric function will have such fall off that

$$
\begin{aligned}
h &= -\frac{g_{tt}}{r^2} = 1 - \frac{\mu}{r^4} + \frac{\mu q^{(1)}}{3r^6} - \frac{\mu q^{(2)}}{9r^8} + \frac{\mu q^{(3)}}{81 r^{10}} + \cdots\,, \\
f &= \frac{r^2}{g_{\rho\rho}}\left(\frac{dr}{d\rho}\right)^2 = 1 - \frac{\mu - \frac{2}{9}q^{(2)}}{r^4} + \frac{\frac{1}{3}\mu q^{(1)} - \frac{4}{81}q^{(3)}}{r^6} - \frac{q^{(2)}(\frac{1}{3}\mu - \frac{1}{27}q^{(2)})}{r^8} \\
&\quad + \frac{\frac{1}{81}\mu(6q^{(1)}q^{(2)} + 5q^{(3)}) - \frac{4}{729}q^{(2)}q^{(3)}}{r^{10}} + \cdots\,,
\end{aligned}
\tag{50}
$$

where we denote

$$
\begin{aligned}
q^1 &= q_1 + q_2 + q_3\,, \qquad q^{(2)} = q_1^2 + q_2^2 + q_3^2 - q_1 q_2 - q_1 q_3 - q_2 q_3\,, \\
q^{(3)} &= (2q_1 - q_2 - q_3)(2q_2 - q_1 - q_3)(2q_3 - q_1 - q_2)\,.
\end{aligned}
\tag{51}
$$

Here we print some explicit result of the OPE coefficient:

$$
\begin{aligned}
c_{2,0} &= \frac{m_0}{4(\Delta_L - 1)}\,, \qquad c_{4,2} = \frac{f_0 \Delta_L}{120}\,, \qquad c_{6,2} = \frac{\Delta_L(7 f_0 m_0 - 6\tilde{f}_0(\Delta_L + 1))}{3360(\Delta_L - 2)}\,, \\
c_{4,0} &= \frac{4(\Delta_L - 1)((\Delta_L - 4)\Delta_L(f_0 - h_0) + 5m_2) + 15 m_0^2}{480(\Delta_L - 2)(\Delta_L - 1)}\,,
\end{aligned}
\tag{52}
$$

Conclusion

- We study the holographic OPE coefficients for heavy-light scalar four point function in heavy limit where the heavy operator can be treated as the excited state of black hole.We find that in pure gravity involving only massless graviton, i.e. $f_0 = h_0$, the OPE coefficient $c_{d,0}$=0.
- We also include the Maxwell field and study the holographic OPE coefficients in the charged AdS black hole background, where the current operator will be exchanged. However we found that the spin-1 current operator $\mathcal{J}$ that the scaling dimension is $\Delta = d - 1$ will not violate the consistency.
- Finally, we consider gauged supergravity theory in which the black holes involve a set of scalar fields. We find that the scalar are conformally massless and contribute the exchange scalar with $\Delta = d - 2$. We found that in $d = 4$, even though when the leading order $f_0 = h_0$, $c_{d,0} \neq 0$.