Nuclear states and spectra in holographic QCD

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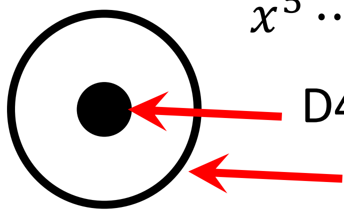
Introduction

Baryons in Sakai-Sugimoto model

⇨ D-branes (D4’) wrapping color D-branes (D4)

$x^5 \cdots x^9$ ⇨ U (radial direction) and S^4 ($\theta^1 \cdots \theta^4$)



D4 (at $U = 0$)

D4’ on S^4

	x^0	x^1	x^2	x^3	x^4	U	θ^1	θ^2	θ^3	θ^4
D4	✓	✓	✓	✓	✓					
D8	✓	✓	✓	✓		✓	✓	✓	✓	✓
D4’	✓						✓	✓	✓	✓

D4 ⇨ Background geometry (holography)

S^4 ⇨ Integrate out

Effective theory on D8 ⇨ Effective theory of mesons

D4’ in D8 effective theory ⇨ Instanton on D8 ⇨ Skyrmion

Effective fields on D4’ ⇨ ADHM data of instantons

Introduction

Baryons in holographic QCD

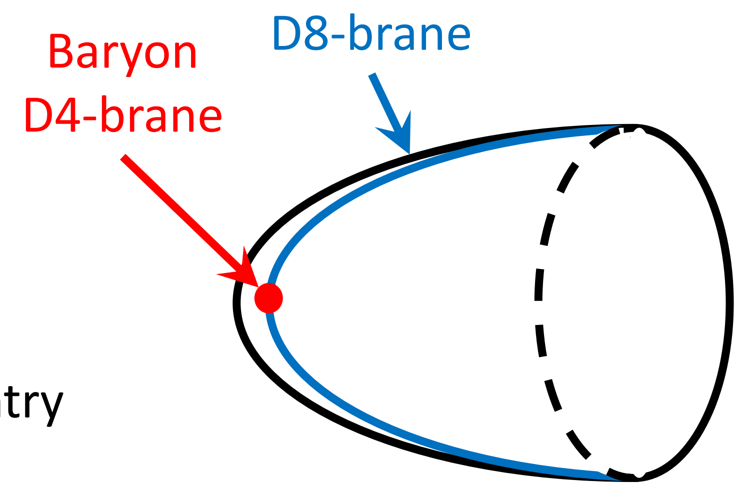
solitonic D4-brane geometry

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}}(-dt^2 + dx^2 + f(U)dx_4^2) + \left(\frac{R}{U}\right)^{\frac{3}{2}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

Anti-periodic b.c. for x^4

A similar factor to BH $f(U)$

Geometry ends at some U



Baryon is located near the tip of geomtry

Nuclear matrix model

Matrix model of Baryon vertex (D4-brane)

with bosonic field of D4-D8 open string

near the tip of solitonic (color) D4-brane background

Nuclear matrix model

Action for A baryons [Hashimoto-Iizuka-Yi,'10]

$$S = S_0 + N_c \int dt\, \mathrm{tr} A_t$$

$$S_0 = \int dt\, \mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 + \frac{1}{2}\left(D_t \overline{w}^{\dot{\alpha}i}\right)(D_t w_{\dot{\alpha}i}) - \frac{1}{2}M^2 \overline{w}^{\dot{\alpha}i} w_{\dot{\alpha}i} + \frac{1}{4\lambda}(D^I)^2 + D^I\left(2i\epsilon^{IJK}X^J X^K + \overline{w}^{\dot{\alpha}i}(\tau^I)_{\dot{\alpha}}{}^{\dot{\beta}} w_{\dot{\beta}i}\right)\right]$$

$D_t = \partial_t - iA_t$: covariant derivative

X^I: D4-D4 scalar ⇨ $A \times A$ matrix

Diagonal comp.: position of D-brane (= baryon)

Off-diagonal: interaction between D-branes

w ($\overline{w}$): D4-D8 scalar

carries charges of quarks (spin, flavor, baryon number)

A_t: gauge field (baryon $SU(A)$)

Non-dynamical field ⇨ EOM gives constraints

$$0 = \frac{\delta S}{\delta A_t} = \frac{\delta S_0}{\delta A_t} - N_c \mathbb{I} = Q_{U(A)} - N_c \mathbb{I}$$

Eigenstates of Hamiltonian must be

$Q_{SU(A)} = 0$ ⇨ Singlet in baryon $SU(A)$ symmetry

$Q_{U(1)_B} = N_c A$ ⇨ Baryon (quark) number must be $N_c A$

Perturbation around harmonic potential V: perturbation

$$H = H_0 + V$$

$$H_0 = \frac{1}{2}\mathrm{tr}\left(\Pi^{\mathrm{I}}\right)^2 + \frac{1}{2}m^2\mathrm{tr}(X^I)^2 + \frac{1}{2}\bar{\pi}^a_{\dot{\alpha}i}\pi^{\dot{\alpha}i}_a + \frac{1}{2}M^2\bar{w}^{\dot{\alpha}i}_a w^a_{\dot{\alpha}i}$$

$$V = -\frac{1}{2}m^2(X^I)^2 - 2\lambda[X^I, X^J]^2$$
$$-4i\lambda\epsilon^{IJK}X^J_A X^K_B f^{AB}{}_C \bar{w}^{\dot{\alpha}i}_a (\tau^I)_{\dot{\alpha}}{}^{\dot{\beta}} (t^C)^a{}_b w^b_{\dot{\beta}i} + \lambda\left(\bar{w}^{\dot{\alpha}i}(\tau^I)_{\dot{\alpha}}{}^{\dot{\beta}} w_{\dot{\beta}i}\right)^2$$

Ground state and constraint

0-th order Hamiltonian H_0 ⇨ Harmonic oscillators of X^I, w and $\bar{w}$.

Constraint 1: baryon $U(1)$ charge must be $N_c A$

Baryon $U(1)$ charges $X^I: 0$ $w: 1$ $\bar{w}: -1$

Constraint for excitation of harmonic oscillators

$$(\text{Number of } w) - (\text{Number of } \bar{w}) = N_c A$$

Lowest energy state ⇨ Smallest number of excitations

$$(\text{Number of } w) = N_c A \qquad (\text{Number of } \bar{w}) = 0$$

Constraint 2: physical state must be singlet of baryon $SU(A)$

Physical ground state for $A \leq 2N_f$

$$|\psi_0\rangle = \underbrace{\left(\epsilon_{a_1 \cdots a_A} w^{a_1}_{\dot{\alpha}_1 i_1} \cdots w^{a_A}_{\dot{\alpha}_A i_A}\right) \times \cdots \times \left(\epsilon_{b_1 \cdots b_A} w^{b_1} \cdots w^{b_A}\right)}_{N_c \text{ of } \left(\epsilon_{a_1 \cdots a_A} w^{a_1} \cdots w^{a_A}\right)} |0\rangle$$

$|0\rangle$: ground state of harmonic oscillators w^a: raising operator of oscillator

Physical ground state for $A \leq 2N_f$

$$|\psi_0\rangle = \underbrace{\left(\epsilon_{a_1\cdots a_A} w^{a_1}_{\dot\alpha_1 i_1} \cdots w^{a_A}_{\dot\alpha_A i_A}\right) \times \cdots \times \left(\epsilon_{b_1\cdots b_A} w^{b_1} \cdots w^{b_A}\right)}|0\rangle$$

N_c of $\left(\epsilon_{a_1\cdots a_A} w^{a_1} \cdots w^{a_A}\right)$

Only $2 \times N_f$ of different $w_{\dot\alpha i}$: 2 different spins, N_f different flavors

⇨ A $(> 2N_f)$ of w^a cannot form antisymmetric combination

construct different operator by using X^I:

$$(X^I w)^a = (X^I)^a{}_b w^b, \qquad (X^I X^J w)^a, \qquad (X^I X^J X^K w)^a, \qquad \cdots$$

Physical ground state for $A > 2N_f$

$$|\psi_0\rangle = \underbrace{\left[\epsilon_{a_1\cdots a_A} w^{a_1} \cdots w^{a_{2N_f}} (X^I w) \cdots (X^J w) \cdots (X^K \cdots X^L w)^{a_A}\right] \times \cdots \times \left[\epsilon_{b_1\cdots b_A} w^{b_1} \cdots w^{b_{2N_f}} (X w) \cdots (X \cdots X w)^{b_A}\right]}|0\rangle$$

N_c of $\left[\epsilon_{a_1\cdots a_A} w^{a_1} \cdots (X^I \cdots X^J) w^{a_A}\right]$

Magic numbers

Nuclei are stable (small energy) for some specific numbers of proton (neutron)

Magic number for u (or d) quarks ($N_c = 1$ case for simplicity)

$w = u^{\dot{\alpha}} \quad \dot{\alpha} = 1, 2 \quad (i = 1)$ ⇨ 2 of u (spin ↑ and ↓)

$A = 1 \qquad |\psi_0\rangle = \epsilon u_\uparrow |0\rangle$

Magic number → $A = 2 \qquad |\psi_0\rangle = \epsilon u_\uparrow u_\downarrow |0\rangle$

$A = 3 \qquad |\psi_0\rangle = \epsilon u_\uparrow u_\downarrow u_\uparrow |0\rangle = 0$ ⇨ $|\psi_0\rangle = \epsilon uu(Xu)|0\rangle$ (Additional energy of X^I)

$A = 4 \qquad |\psi_0\rangle = \epsilon uu(uX)(uX)|0\rangle$

$\vdots$

Magic number → $A = 8 \qquad |\psi_0\rangle = \epsilon uu\underbrace{(Xu)\cdots(Xu)}_{\text{6 of } Xu}|0\rangle$

$X^I \quad I = 1, 2, 3$ ⇩ $2 \times 3 = 6$ of Xu

$A = 9 \qquad |\psi_0\rangle = \epsilon uu\underbrace{(Xu)\cdots(Xu)}_{\text{6 of } Xu}(Xu)|0\rangle = 0$

⇩

$A = 9 \qquad |\psi_0\rangle = \epsilon uu\underbrace{(Xu)\cdots(Xu)}_{\text{6 of } Xu}(XXu)|0\rangle$

Magic numbers

Magic number for either proton or neutron ($N_c = 3$)

$N_f = 2$ ⇨ Both quarks and nucleons have isospin $I = 1/2$

Example: ($N_p = 8$, $N_n = 2$)

$$|\psi_0\rangle = dd\,uu(Xu)(Xu)(Xu)(Xu)(Xu)(Xu)$$
$$\times dd\,uu(Xu)(Xu)(Xu)(Xu)(Xu)(Xu)$$
$$\times uu\,dd(Xd)(Xd)(Xd)(Xd)(Xd)(Xd)|0\rangle$$

2 neutrons   8 protons

3 sets of anti-sym. combinations
$\epsilon w \cdots (Xw)$

Proton and neutron configurations ⇨ ⇦? Quark configurations

Example: $N_u = 6$, $N_d = 3$; same to 3 protons (3 p's cannot be in ground state)

$$|\psi_0\rangle = (uu\,d)(uu\,d)(uu\,d)|0\rangle$$

State without X excitation is possible

⇨ corresponds to nucleus with Δ

will be heavier if 1st order perturbation V is taken into account

First order perturbation

Energy at first order = expectation value of H for $|\psi_0\rangle$

$$E = \langle\psi_0|H|\psi_0\rangle = E_0 + \langle\psi_0|V|\psi_0\rangle$$

Energy at 0-th order ⇒ $E_0 = N_c A M$

Linear order correction ⇒ $\langle V\rangle$ for $N_c A$ excitations of w

No X^I excitation for $A \leq 2N_f$ ⇒ $V = \lambda\left(\bar{w}^{\dot{\alpha}i}(\tau^I)_{\dot{\alpha}}{}^{\dot{\beta}} w_{\dot{\beta}i}\right)^2$

No excitations of X^I or $\bar{w}$ (only w excitations)

$$\langle V\rangle = \frac{4\lambda}{M^2} C_f + \frac{\lambda}{M^2}\frac{2A - N_f}{N_f A} N_w^2$$

C_f: quadratic Casimir of flavor $SU(N_f)$

N_w: Number of excitations of w

Smaller flavor charge ⇒ more stable

Allowed states ($N_f = 2$)

$w_{\dot{\alpha}i}$: raising operator of $w_{\dot{\alpha}i}$

$A = 1$ and $N_c = 3$

$$|\psi_0\rangle = w_{\dot{\alpha}_1 i_1} w_{\dot{\alpha}_2 i_2} w_{\dot{\alpha}_3 i_3} |0\rangle$$

(No baryon index)

w are bosonic ($=$ symmetric) $\Rightarrow$ Spin J and isospin I are in same rep.

$(J, I) = \left(\frac{1}{2}, \frac{1}{2}\right)$ or $(J, I) = \left(\frac{3}{2}, \frac{3}{2}\right)$

proton or neutron $\qquad$ Δ

Mass

$$E = 3M + \frac{4\lambda}{M^2} I(I+1)$$

proton and neutron have smaller mass than Δ

Allowed states ($N_f = 2$)

$A = 2$ and $N_c = 1$

$$|\psi_0\rangle = \epsilon_{a_1 a_2} w^{a_1}_{\dot{\alpha}_1 i_1} w^{a_2}_{\dot{\alpha}_2 i_2} |0\rangle$$

w are antisymmetric $\quad \begin{Bmatrix} \text{spin} \\ \text{isospin} \end{Bmatrix}$ is $\begin{Bmatrix} \text{symmetric} \\ \text{antisymmetric} \end{Bmatrix}$ or $\begin{Bmatrix} \text{antisymmetric} \\ \text{symmetric} \end{Bmatrix}$

$(J, I) = (1,0)$ stable $\qquad (J, I) = (0,1)$

$A = 2$ and $N_c = 3$

Symmetric combination of 3 sets of $A = 2$ and $N_c = 1$

$(J, I) = (1,0) \qquad (J, I) = (3,0) \qquad (J, I) = (1,2)$

$(J, I) = (0,1) \qquad (J, I) = (0,3) \qquad (J, I) = (2,1)$

Most stable states

$(J, I) = (1,0)$ Deuteron $\qquad (J, I) = (3,0)$ Dibaryon D_{03}

Hyperon

$N_f = 3$ u, d, s quarks

s quark has larger mass ⇨ Put larger mass for $w_{(s)} = w_{i=3}$ by hand

$$\sum_{i=1}^{3} M^2 \bar{w}_a^{\dot{\alpha} i} w_{\dot{\alpha} i}^a \Rightarrow \sum_{i=1}^{2} M^2 \bar{w}_a^{\dot{\alpha} i} w_{\dot{\alpha} i}^a + M_S^2 (\bar{w}_{(s)})_a^{\dot{\alpha}} (w_{(s)})_{\dot{\alpha}}^a$$

Number of w_s $(= w_{i=3})$ in $|\psi_0\rangle$ ⇨ Number of s quarks in nucleus

Assumption: no $SU(3)$ breaking effect in $V = \lambda \left(\bar{w}_a^{\dot{\alpha} i} (\tau^I)_{\dot{\alpha}}{}^{\dot{\beta}} w_{\dot{\beta} i}^a \right)^2$

Effect of mass in raising and lowering operators

$$w = \frac{1}{\sqrt{M}} \left(a^\dagger + \bar{a} \right)$$

$a^\dagger$: raising (creation) operator of w

$\bar{a}$: lowering (annihilation) operator of $\bar{w}$

Hyperon mass

A: Number of baryons

Y: Hypercharge

Mass formula

$$M_{\mathrm{hyperon}} = \widetilde{M}_{\mathrm{D4}} + 4\tilde{\lambda}(1-\delta)C_f - \left(M_S\delta - 2\tilde{\lambda}\delta(1-\delta)\right)Y + 4\tilde{\lambda}\delta\left(I(I+1) - \frac{1}{4}Y^2\right) + \tilde{\lambda}\delta^2 Y^2$$

$\widetilde{M}_{\mathrm{D4}}$: D-brane tension $\quad \tilde{\lambda} = \frac{\lambda}{M^2} \quad \delta = 1 - \frac{M}{M_S}$

Global fit with hyperon mass

$\widetilde{M}_{\mathrm{D4}} = 933$ [MeV] $\quad M_S = 603$ [MeV] $\quad \tilde{\lambda} = 24.9$ [MeV] $\quad \delta = 0.339$

	N	**Λ**	**Σ**	**Ξ**	**Δ**	**Σ***	**Ξ***	**Ω**
I	1/2	0	1	1/2	3/2	1	1/2	0
Y	1	0	0	−1	1	0	−1	−2
Exp.	939	1116	1193	1318	1232	1385	1533	1672
GMO	939	1117	1183	1328	1238	1383	1528	1673
Our	941	1115	1182	1327	1240	1380	1525	1676

GMO: Gell-Mann-Okubo formula $\quad$ Our: Our mass formula

Dibaryon

2nd order corrections are partially calculated

Octet	$\mathbf{N(939)}$	$\mathbf{\Lambda(1116)}$	$\mathbf{\Sigma(1193)}$	$\mathbf{\Xi(1318)}$
GMO	939	1117	1183	1328
Our	975	1126	1237	1347

Decuplet	$\mathbf{\Delta(1232)}$	$\mathbf{\Sigma^*(1385)}$	$\mathbf{\Xi^*(1533)}$	$\mathbf{\Omega(1672)}$
GMO	1238	1383	1528	1673
Our	1311	1407	1516	1639

Dibaryon	$D(1876)$	$D_{03}(2370)$	H	$\mathbf{\Omega\Omega}$
Our	1876	2285	2084	3007

Dibaryon?	D_{10}	D_{30}	$D_{12}(2160?$	D_{21}
Our	1855	2157	2100	2058

Threshold	$\mathbf{N+N}$	$\mathbf{\Delta+\Delta}$	$\mathbf{N+\Delta}$	$\mathbf{\Lambda+\Lambda}$	$\mathbf{\Omega+\Omega}$
Experiment	1878	2464	2171	2232	3344
Our	1950	2622	2286	2252	3278

Baryon resonance

Internal excitations in baryons

$A = 1$ ⇨ X^I is $U(1)$ ⇨ No internal excitation from X^I

Only possible source of internal excitation is w

Constraint: $(\text{Number of } w) - (\text{Number of } \bar{w}) = N_c A$

⇩

$\bar{w}w$ pair excitation(s)

Wave function

$$|\psi_0\rangle = w_{\dot{\alpha}_1 i_1} w_{\dot{\alpha}_2 i_2} w_{\dot{\alpha}_3 i_3} w_{\dot{\alpha}_4 i_4} \bar{w}^{\dot{\alpha}_5 i_5} |0\rangle$$

$w_{\dot{\alpha}i}$: raising operator of $w_{\dot{\alpha}i}$

$\bar{w}^{\dot{\alpha}i}$: raising operator of $\bar{w}^{\dot{\alpha}i}$

Baryon resonance

Input $N(938), \Delta(1232)$ and excited state of N at 1440 MeV

$N(I=1/2)$	PDG	Our
$J=1/2$	938(****)	(938)
	1440(****)	(1440)
	1720(***)	
	1880(***)	1859
	2100(*)	
$J=3/2$	1720(****)	1821
	1900(***)	
	2040(*)	

$\Delta(I=3/2)$	PDG	Our
$J=1/2$	1750(*)	
	1910(****)	1821
$J=3/2$	1232(****)	(1232)
	1600(***)	1478
	1920(***)	2409
$J=5/2$	1905(****)	
	2000(**)	2213

$I=5/2$	Our
$J=3/2$	2213
$J=5/2$	1723

All parity +

Summary

Nuclear matrix model from D-branes in holographic QCD

X^I: position of baryons (nucleons)

$w^{\dot{\alpha} i}$: carries charges of quarks (spin, flavor)

EOM of gauge field ⇨ constraints

U(1) constraint ⇨ appropriate quark number as nuclei

SU(A) constraint ⇨ anti-symmetric (fermionic) wave function

Interaction terms

$[X^I, X^J]^2$ ⇨ Effective trapping potential (?)

$\epsilon^{IJK}[X^I, X^J]\bar{w}\tau^K w$ ⇨ Spin-orbit interaction?

$(\bar{w}\tau^I w)^2$ ⇨ Flavor dependence of mass

Matrix model gives good results for small baryon number

We calculated mass of hyperons, dibaryons and baryon resonance

For larger baryons number, more studies on X^I are necessary

Thank you