YITP Workshop 19/08/20

Symmetry Breaking in Quantum Curves & Super Chern-Simons Matrix Models

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Main References:

S.M., S.Nakayama, T.Nosaka, JHEP, 2017;

S.M., T.Nosaka, T.Yano, JHEP, 2017;

N.Kubo, S.M., T.Nosaka, JHEP, 2018. ←

Matrix Model = Spectral Theory

- M-Theory, Mother, Membrane (M2), Mystery
- ABJM Theory for Multiple M2-branes

[Aharony-Bergman-Jafferis-Maldacena 2008]

- Partition Function is Localized to Matrix Model

[Kapustin-Willett-Yaakov 2009]

- Large N Expansion = $N^{3/2}$, Airy Function

[Drukker-Marino-Putrov 2010, Fuji-Hirano-M 2011]

- Matrix Model as Spectral Det, Det($1 + z\, H^{-1}$)

(Fermi Gas Formalism) [Marino-Putrov 2011]

Matrix Model = Topological String

- Matrix Model by Topological Strings

[Hatsuda-Marino-M-Okuyama 2013]

- Many Generalizations

[... ... ... 2013-2019]

But, Why Interesting? What is New?

No More Matrix Models

- ST / TS Correspondence

  (Spectral Theories / Topological Strings)

  [Grassi-Hatsuda-Marino 2014]

- On one hand,

  Matrix Model = Spectral Theory

- On the other hand,

  Matrix Model = Topological String

Advantages of ST / TS

- At Least Technically,

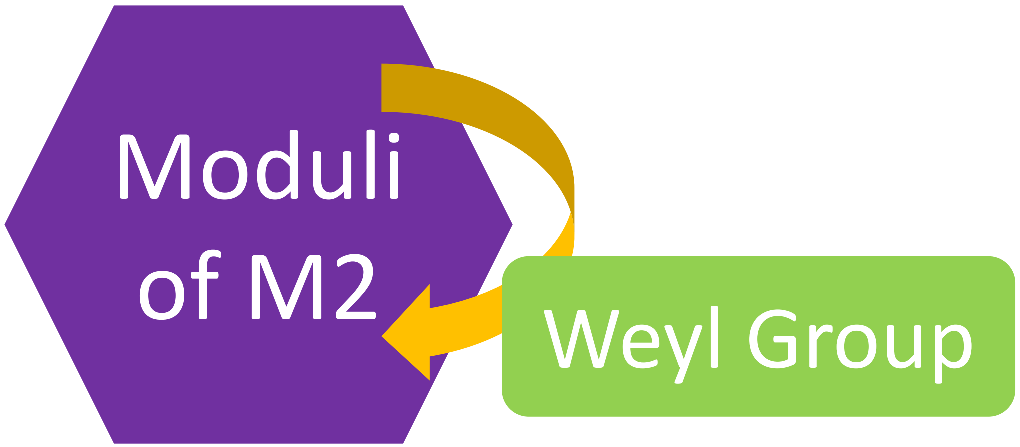
Group Theoretical Structure

- So Far, Free Energy of Topological Strings in Kahler Parameters

… Complicated & Ambigous …

- With Group Theoretical Structure, in Characters

- Conceptually, replace MM by ST / TS?



Especially, in "Strings & Fields 2017"

- Free Energy of Topological Strings

$$F = \Sigma\, N\, [(\text{characters})\, e^{-\mu/k}$$

$$+ \{\, (\text{characters})\, \mu + \partial(\text{characters})\, \}\, e^{-\mu}]$$

N: Multiplicities of Representations (BPS indices)

- (2,2) Model, so(10) → so(8)

- Rank Deformations, so(10) → [su(2)]^3

For D5$_{[=\text{so}(10)]}$ Del Pezzo Geometry

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- (2,2) Model, so(10) → so(8)

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Question: Explain the Symmetry Breaking!

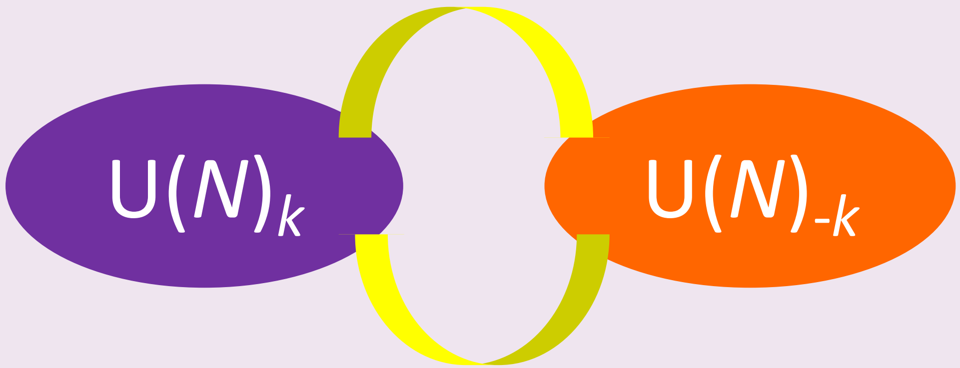
Contents

1. ABJM Theory

(Background)

ABJM Theory

$\mathcal{N}$=6 Chern-Simons Theory



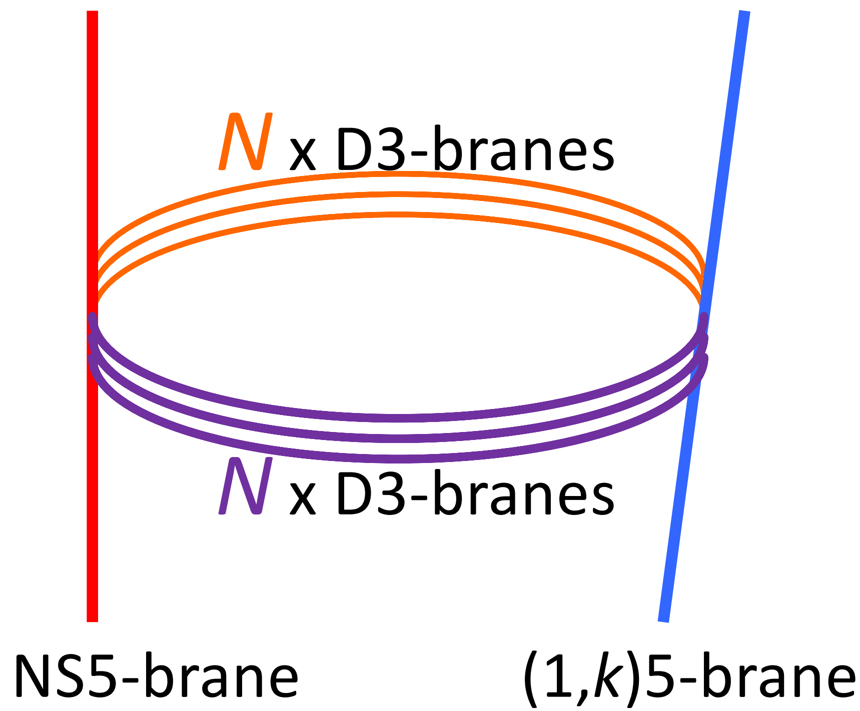
[Aharony-Bergman-Jafferis-Maldacena 2008]

N x M2 on C^4 / Z_k

Brane Configuration in IIB

From Large Supersymmetries

[Kitao-Ohta-Ohta 1998, ...]



(IIB String Theory)

→ T-duality to IIA

→ Lift to M-Theory

Grand Canonical Ensemble

- Partition Function $Z_k(N)$ & Grand Partition F

[Marino-Putrov 2011]

$$\Xi_k(z) = \Sigma_{N=0}^{\infty} z^N Z_k(N)$$

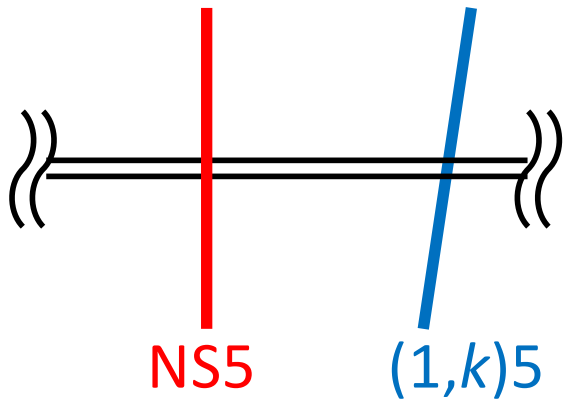
(N : Particle Number, z : Dual Fugacity)

- Spectral Determinant $\Xi_k(z) = \mathrm{Det}(1 + z H^{-1})$

$H^{-1} = (P^{1/2}+P^{-1/2})^{-1} (Q^{1/2}+Q^{-1/2})^{-1}$

or $H = (Q^{1/2}+Q^{-1/2}) (P^{1/2}+P^{-1/2})$

($Q = e^q$, $P = e^p$, $[q,p] = i\, 2\pi k$)



From Matrix Models To Curves

[Marino-Putrov 2011]

[..., Hatsuda-Marino-M-Okuyama 2013]

Grand Partition Function $\Xi_k(z)$

=

Spectral Det

Det (1 + $z\, H^{-1}$)

=

Free Energy of Top Strings

exp [$\Sigma\, N^d_{j\mathrm{L},j\mathrm{R}}\, F^d_{j\mathrm{L},j\mathrm{R}}\, (T)$]

$H = (Q^{1/2}+Q^{-1/2})\,(P^{1/2}+P^{-1/2})$

(Curve Eq of **Local P¹ x P¹**)

$N^d_{j\mathrm{L},j\mathrm{R}}$: BPS index on

Local P¹ x P¹

d: degree, $(j_\mathrm{L},j_\mathrm{R})$: spins

$T=T(z)$: Kahler Parameters

From Matrix Models To Curves

(Without Referring To Matrix Model)

[Grassi-Hatsuda-Marino 2014]

Spectral Det

Det (1 + $z\,H^{-1}$)

=

Free Energy of Top Strings

exp [$\Sigma\, N^{d}{}_{jL,jR}\, F^{d}{}_{jL,jR}\,(T)$]

H = (Curve Eq)

$Q = e^{q}$, $P = e^{p}$, $[q,p] = i\,2\pi k$

$N^{d}{}_{jL,jR}$: BPS index

on the Curve

2. Super Chern-Simons Theories

(Questions)

As a simple generalization

- (2,2) Model

[M-Nosaka 2014]

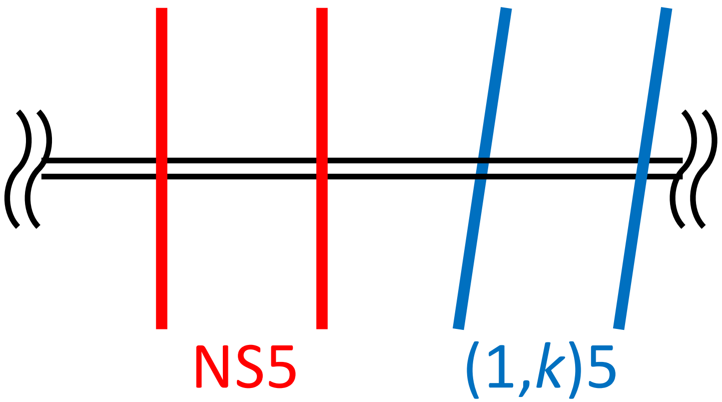
Spectral Det
Det (1 + z H^{-1})

=

Free Energy of Top Strings
exp [$\Sigma\, N^d_{jL,jR}\, F^d_{jL,jR}(T)$]

$H=(Q^{1/2}+Q^{-1/2})^2(P^{1/2}+P^{-1/2})^2$

$N^d_{jL,jR}$: BPS index on **Local D5 Del Pezzo**



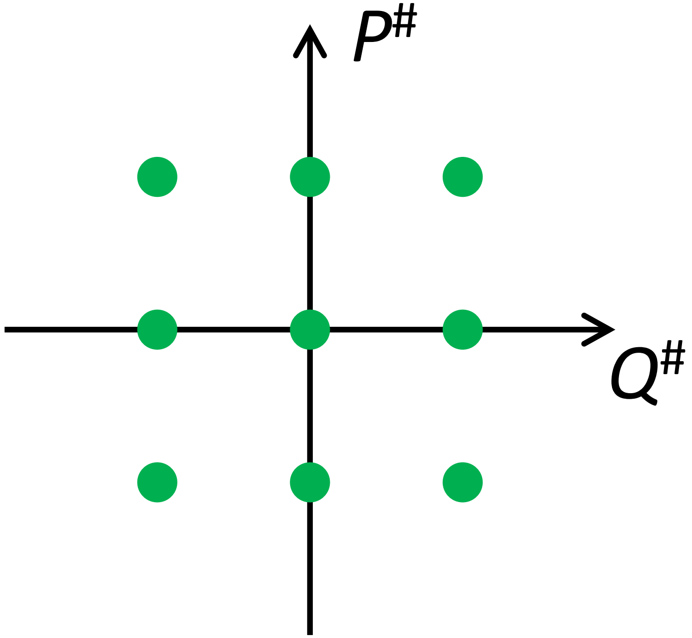
Natural Because

For (2,2) Model

$H = (Q^{1/2}+Q^{-1/2})^{2}\,(P^{1/2}+P^{-1/2})^{2}$

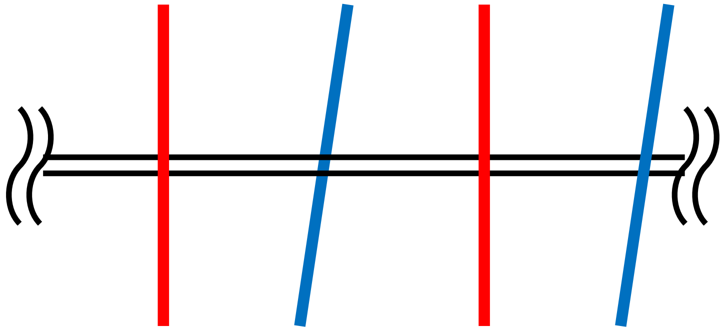
$= Q^{1}P^{1}+2P^{1}+Q^{-1}P^{1}+2Q^{1}+4+2Q^{-1}+Q^{1}P^{-1}+2P^{-1}+Q^{-1}P^{-1}$

Well-known Newton Polygon of D5[=so(10)] Curve



Also

- (1,1,1,1) Model



[Honda-M 2014]

$$H = (Q^{1/2}+Q^{-1/2})^{1}(P^{1/2}+P^{-1/2})^{1}(Q^{1/2}+Q^{-1/2})^{1}(P^{1/2}+P^{-1/2})^{1}$$

$$= Q^{1/2}\, P^{1/2}\, Q^{1/2}\, P^{1/2} + \dots$$

$$= q^{-1/4}\, Q\, P + \dots$$

(Since $P^{\alpha}Q^{\beta} = q^{-\alpha\beta}\, Q^{\beta}P^{\alpha}$, $q = e^{2\pi i k}$)

The Same D5 Curve

Furthermore, Two Models are

- connected by Rank Deformations (M_1,M_2)

- described by Topological Strings
  in A Single Function:

Free Energy of Top Strings
$\exp [\, \sum N^{d}_{jL,jR}\, F^{d}_{jL,jR}\, (T)\,]$

- Prepare Six Kahler Parameters $T_i^{\pm}$ = ... (i = 1,2,3)
- Total BPS indices are distributed by Various Combinations

[M-Nakayama-Nosaka 2017]

Decomposition of BPS index

- Explicitly, 6 Degrees for 6 Kahler Parameters

$\Sigma\, N^{\boldsymbol{d}}_{(jL,jR)}\,(d_1^+,d_2^+,d_3^+;d_1^-,d_2^-,d_3^-)\cdot(T_1^+,T_2^+,T_3^+;T_1^-,T_2^-,T_3^-)$

- BPS Index

- d=1, (j_L,j_R)=(0,0)

$16 \rightarrow 2(1,0,0;0,0,0)+4(0,1,0;0,0,0)+2(0,0,1;0,0,0)$
$+2(0,0,0;1,0,0)+4(0,0,0;0,1,0)+2(0,0,0;0,0,1)$

From Tables in
[Huang-Klemm-Poretschkin 2013]

How About Higher Degrees?

Decomposition of BPS index

$\lvert \boldsymbol{d} \rvert$	$\{\boldsymbol{d} = (d_1^+, d_2^+, d_3^+; d_1^-, d_2^-, d_3^-)\}$		$\pm N^{\boldsymbol{d}}_{j_L, j_R}(j_L, j_R)$
1	$(1,0,0;0,0,0)$	$(0,0,0;1,0,0)$	$2(0,0)$
	$(0,1,0;0,0,0)$	$(0,0,0;0,1,0)$	$4(0,0)$
	$(0,0,1;0,0,0)$	$(0,0,0;0,0,1)$	$2(0,0)$
2	$(0,2,0;0,0,0), (1,0,1;0,0,0)$	$(0,0,0;0,2,0), (0,0,0;1,0,1)$	$(0,\frac{1}{2})$
	$(1,0,0;0,1,0), (0,1,0;0,0,1)$	$(0,1,0;1,0,0), (0,0,1;0,1,0)$	$2(0,\frac{1}{2})$
	$(1,0,0;1,0,0), (0,1,0;0,1,0), (0,0,1;0,0,1)$		$4(0,\frac{1}{2})$
3	$(2,0,0;1,0,0), (0,2,0;0,0,1$ $(1,1,0;0,1,0), (1,0,1;0,0,1$	$(1,0,0;2,0,0), (0,0,1;0,2,0),$ $0;1,1,0), (0,0,1;1,0,1)$	$2(0,1)$
	$(0,2,0;0,1,0), (1,0$ $(1,1,0;1,0,0), (0,1$	[illegible]	$4(0,1)$
	$(0,0,2;0,0,1), (0,2$ $(0,1,1;0,1,0), (1,0$	[illegible]	[illegible] $2(0,1)$

Decompositions Not Unique
Due to Relations among *T*'s
$2T_2^{\pm} = T_1^{\pm} + T_3^{\pm}$,
$T_1^{+}+T_1^{-} = T_2^{+}+T_2^{-} = T_3^{+}+T_3^{-}$, ...
Ambiguous, A Trouble ... ...

Organizing BPS Index Differently

[M-Nosaka-Yano 2018]

In $(M_1,M_2)=(M,0)$ Deformation,

d	$(j_{\rm L}, j_{\rm R})$	BPS	$(-1)^{d-1}\sum_{d_{\rm I}}\left(\sum_{d_{\rm II}} N^{(d,d_{\rm I},d_{\rm II})}_{j_{\rm L},j_{\rm R}}\right)_{d_{\rm I}}$
1	$(0,0)$	16	$8_{+1}+8_{-1}$
2	$(0,\frac{1}{2})$	10	$1_{+2}+8_0+1_{-2}$
3	$(0,1)$	16	$8_{+1}+8_{-1}$
4	$(0,\frac{1}{2})$	1	1_0
	$(0,\frac{3}{2})$	45	$8_{+2}+29_0+8_{-2}$
	$(\frac{1}{2},2)$	1	1[illegible]

Reminiscent of $45 \to 28_0 + 8_{+2} + 8_{-2} + 1_0$
in $so(10) \to so(8)$

Organizing BPS Index Differently

In General (M_1,M_2) Deformation,

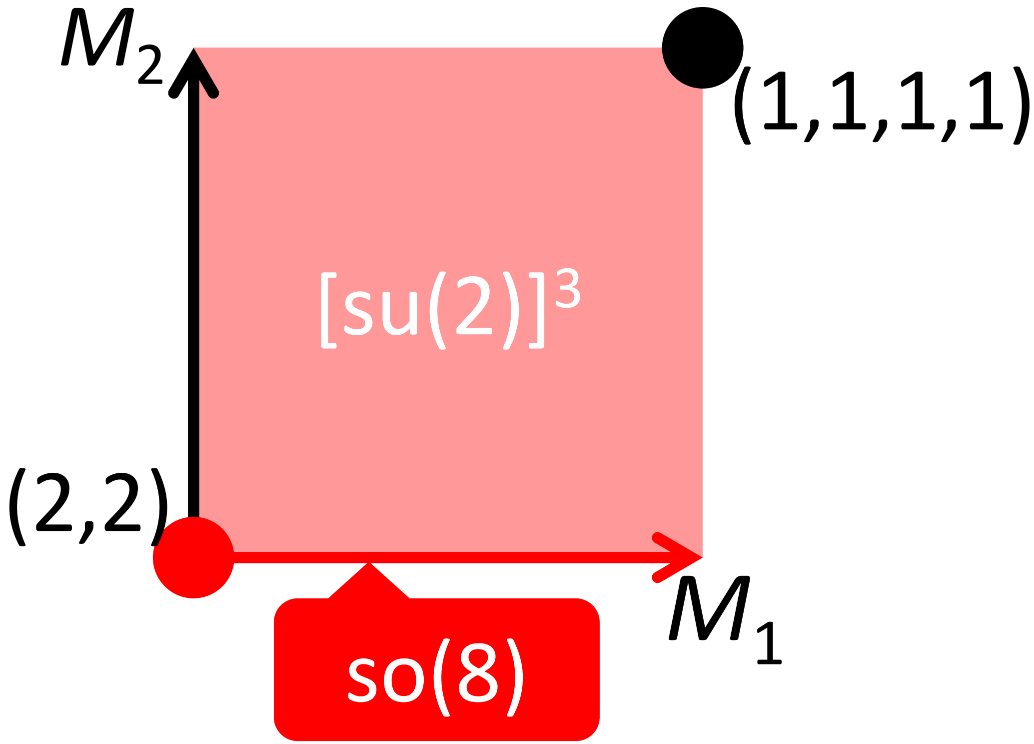
d	(j_L, j_R)	d_I	BPS	$(-1)^{d-1}\sum_{d_{II}}\left(N^{(d,d_I,d_{II})}_{j_L,j_R}\right)_{d_{II}}$
1	$(0,0)$	± 1	8	$2_{+1}+4_0+2_{-1}$
2	$(0,\frac{1}{2})$	0	8	$2_{+1}+4_0+2_{-1}$
		± 2	1	1_0
3	$(0,1)$	± 1	8	$2_{+1}+4_0+2_{-1}$
4	$(0,\frac{1}{2})$	0	1	1_0
	$(0,\frac{3}{2})$	0	29	$1_{+2}+8_{+1}+11_0+8_{-1}+1_{-2}$

Interpreted As Further Decomposition so(8)→[su(2)]^3

e.g. 28→(3,1,1,1)+(1,3,1,1)+(1,1,3,1)+(1,1,1,3)+(2,2,2,2)

in so(8) → [su(2)]^4

Finally,



A Natural Question

Nice to Summarize Numerical Results by

$so(10) \rightarrow so(8)$ & $so(8) \rightarrow [su(2)]^3$

- But Why ? Any Explanations ?

  [Also Raised by Y.Hikida & S.Sugimoto, "Strings & Fields 2017"]

- Now We Have Answer From Curve Viewpoint

3. Symmetry, Symmetry Breaking

(Answer)

Strategy

① D5 Weyl Action on D5 Curve

② (2,2) Model in D5 Curve

③ Unbroken Symmetry for Models

Quantum Curve

As Classical Curves are Defined by Zeros of Polynomial Rings

- Definition: Spectral Problem of

$$H = \Sigma\, c_{mn}\, Q^m\, P^n$$

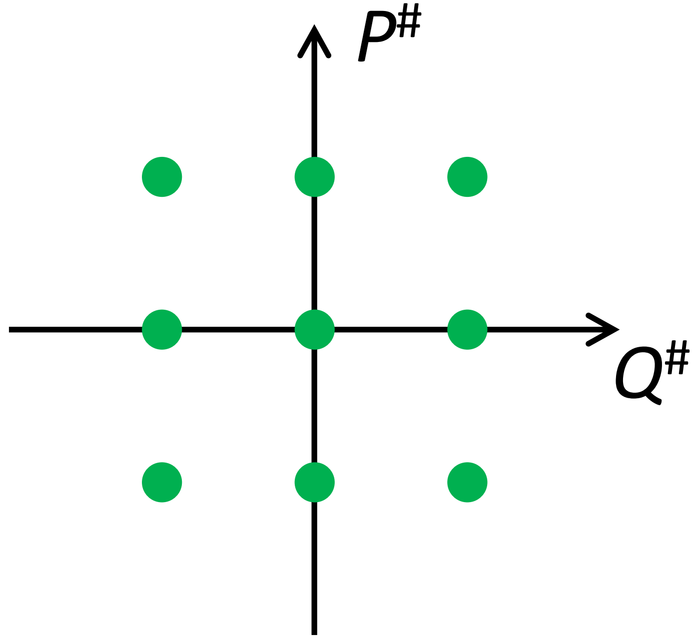
$$(\, P^\alpha Q^\beta = q^{-\alpha\beta}\, Q^\beta P^\alpha,\ q = e^{2\pi i k}\,)$$

Invariant under Similarity Transf.

$$H \sim G\, H\, G^{-1}$$

- For D5 Quantum Curve

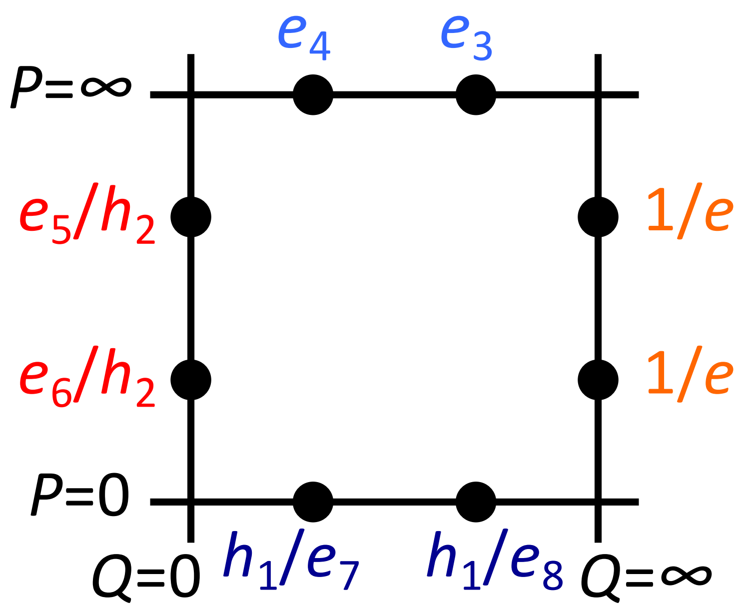
$$H = \Sigma_{(m,n)\in\{-1,0,1\}\times\{-1,0,1\}}\, c_{mn}\, Q^m\, P^n$$



Parameterization

Parameterize D5 Curve by "Asymptotic Values"

$$H/\alpha = Q\,P \quad - (e_3+e_4)\,P \quad + e_3 e_4\,Q^{-1}\,P$$
$$- (e_1^{-1}+e_2^{-1})\,Q \quad + E/\alpha \quad - \ldots\,Q^{-1}$$
$$+ (e_1 e_2)^{-1}\,Q\,P^{-1} \quad - \ldots\,P^{-1} \quad + \ldots\,Q^{-1}\,P^{-1}$$



Subject to Vieta's Formula

解と係数の関係 $(h_1 h_2)^2 = e_1 \ldots e_8$

① D5 Weyl Transformation

Trivial Transformations

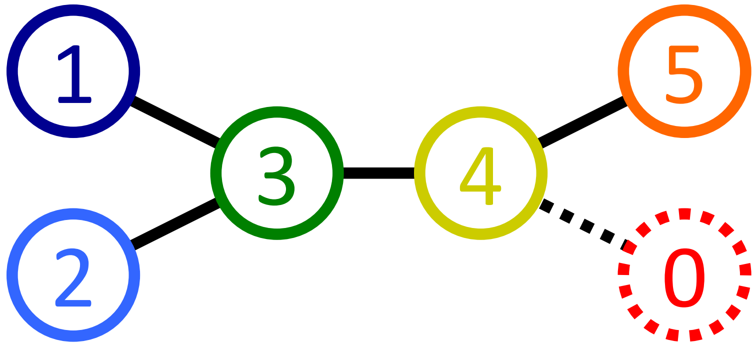
(Switching Asymptotic Values)

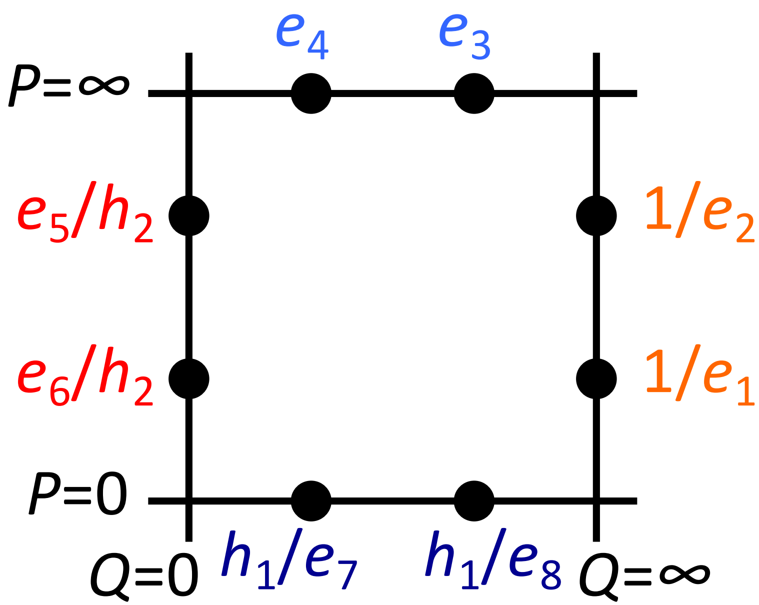
$\boldsymbol{s_1}$: $h_1/e_7 \Leftrightarrow h_1/e_8$

$\boldsymbol{s_2}$: $e_3 \Leftrightarrow e_4$

$\boldsymbol{s_5}$: $1/e_1 \Leftrightarrow 1/e_2$

$\boldsymbol{s_0}$: $e_5/h_2 \Leftrightarrow e_6/h_2$





① D5 Weyl Transformation

Nontrivial s_3 and s_4

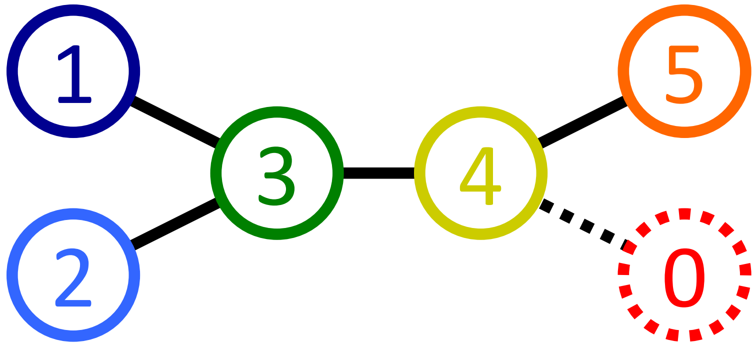
by Suitable Similarity Transf.

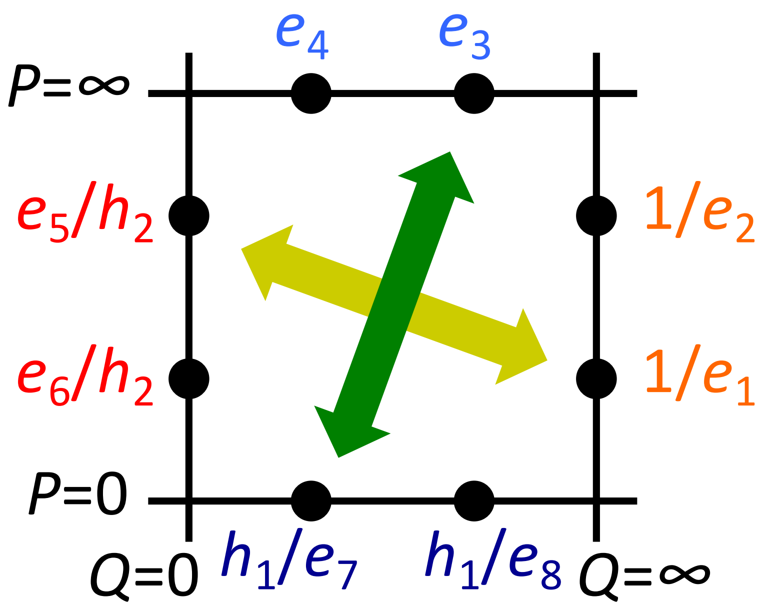
$$Q' = GQG^{-1}, \quad P' = GPG^{-1}$$

s_3: $e_3 \Leftrightarrow h_1/e_7$

s_4: $1/e_1 \Leftrightarrow e_5/h_2$

Totally, D5 Weyl Transf.





Gauge Fixing

- Redundancies in Parametrization

- $(h_1,h_2,e_1,\ldots,e_8)$: 10 Parameters for 8 Asymptotic Values

- Similarity Transformation to Rescale Q & P

$$(Q,P)\sim(AQ,P),\ (Q,P)\sim(Q,BP)$$

- Totally, **4** Gauge Fixing Conditions

$$e_2 = e_4 = e_6 = e_8 = 1$$

- 6 Parameters Subject to 1 Vieta's Constraint

$$\rightarrow 5 \text{ DOF } (h_1,h_2,e_1,e_3,e_5)$$

$$\boldsymbol{s_1}, \boldsymbol{s_2}, \boldsymbol{s_3}, \boldsymbol{s_4}, \boldsymbol{s_5} : (h_1,h_2,e_1,e_3,e_5) \rightarrow (*,*,*,*,*)$$

② Matrix Models

- (2,2), $H = (Q^{1/2}+Q^{-1/2})^2\,(P^{1/2}+P^{-1/2})^2$

$(h_1,h_2,e_1,e_3,e_5)=(1,1,1,1,1)$

- (1,1,1,1), $H = (Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})(Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})$

$(h_1,h_2,e_1,e_3,e_5)=(1,1,q^{-1/2},q^{+1/2},q^{-1/2})$ $\qquad q = e^{2\pi ik}$

③ Matrix Models

- (2,2), $H = (Q^{1/2}+Q^{-1/2})^2\,(P^{1/2}+P^{-1/2})^2$

$(h_1,h_2,e_1,e_3,e_5)=(1,1,1,1,1)$

Unbroken Symmetry

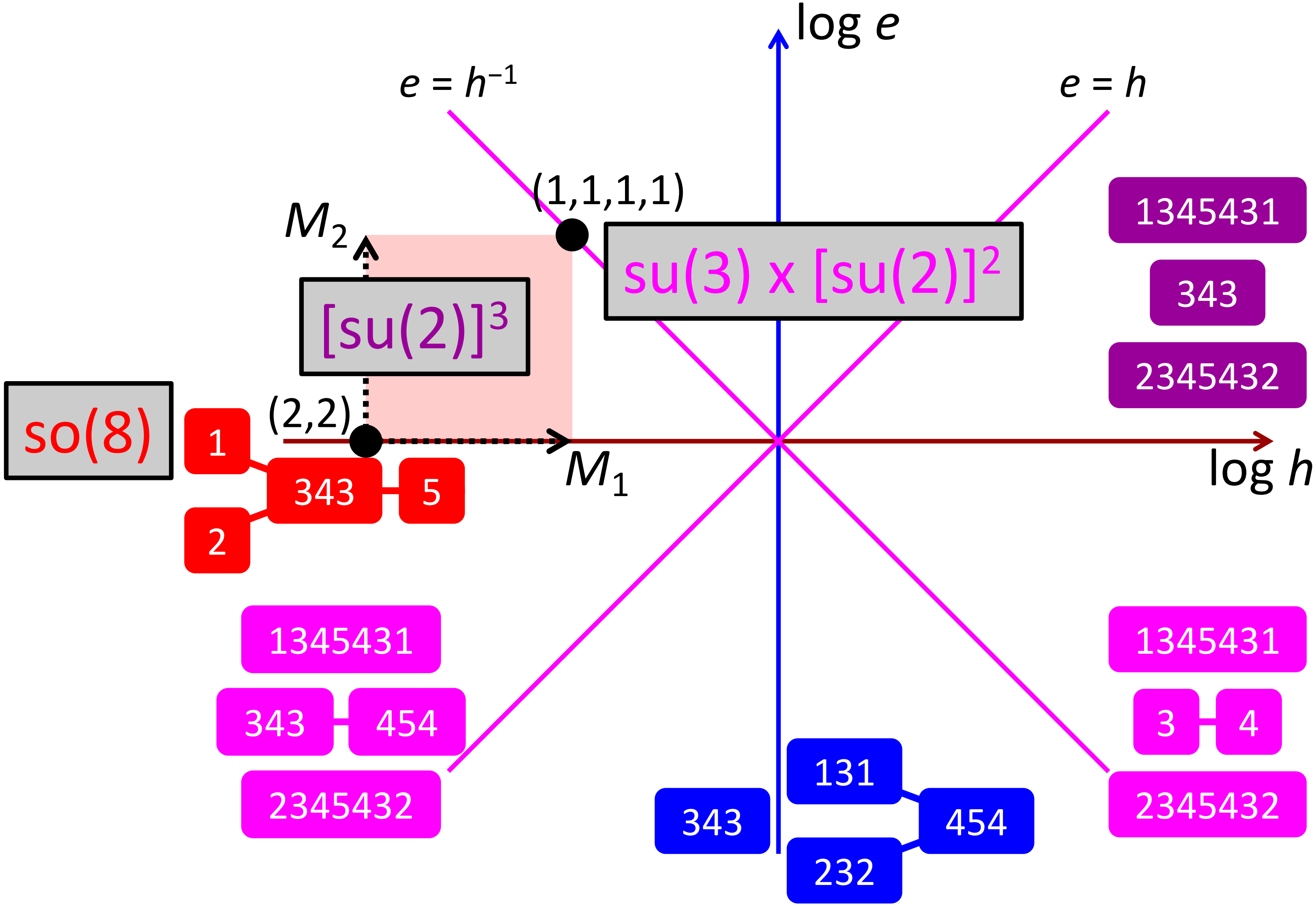
→ so(8)

- (1,1,1,1), $H = (Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})(Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})$

$(h_1,h_2,e_1,e_3,e_5)=(1,1,q^{-1/2},q^{+1/2},q^{-1/2})$ $\quad q = e^{2\pi i k}$

→ su(3) x $[\mathrm{su}(2)]^2 \neq [\mathrm{su}(2)]^3$

"Moduli Space of M2?"



$$(h_1,h_2,e_1,e_3,e_5)=(e/(qh),qh/e,1/e,e,1/e)$$

Conclusion & Further Progress

- Group-Theoretical Structure is useful
- Weyl Group acts on M2-brane configurations
- From Matrix Models To Quantum Curves

- 3D Relative Ranks vs 5D Parameter Space

- Group-Theoretical Structure works also for Mirror Map

- Integrability Hierarchy also acts on M2-brane configurations

→ N. Kubo's, Y. Sugimoto's, T. Furukawa's talks and posters