# Peculiar Index Relations, 2D TQFT, and Universality of SUSY Enhancement

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w/ Matt Buican,
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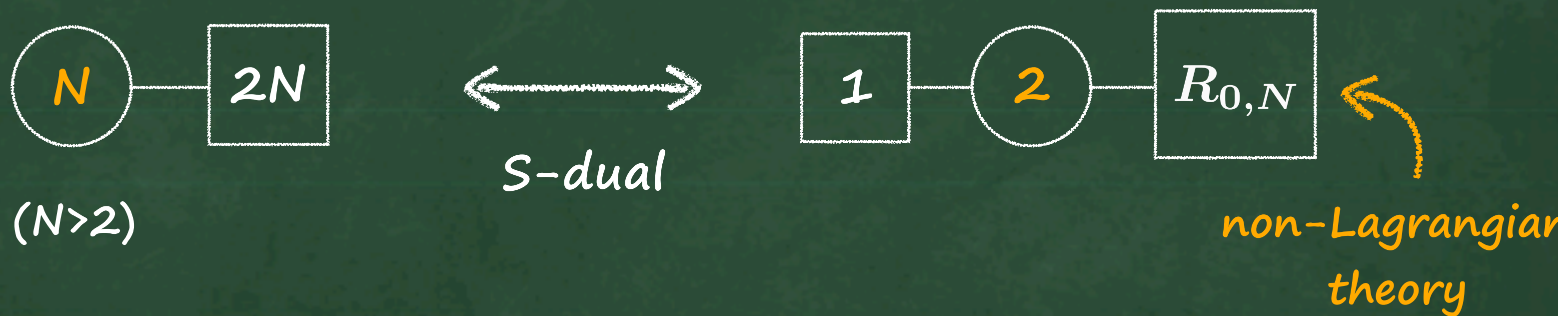
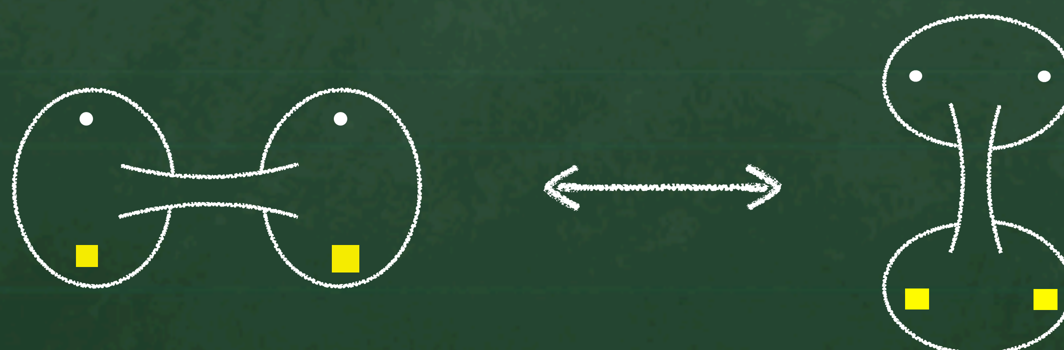
# 4d $\mathcal{N}=2$ S-dualities

[ Argyres-Seiberg ]
[ Gaiotto ]
[ Chacaltana-Distler ]

SU(N) gauge theory w/ 2N flavors




- This duality corresponds to N M5-branes wrapped on



- This interpretation led to the discovery of
  - class S dualities
  - AGT relation
  - etc.

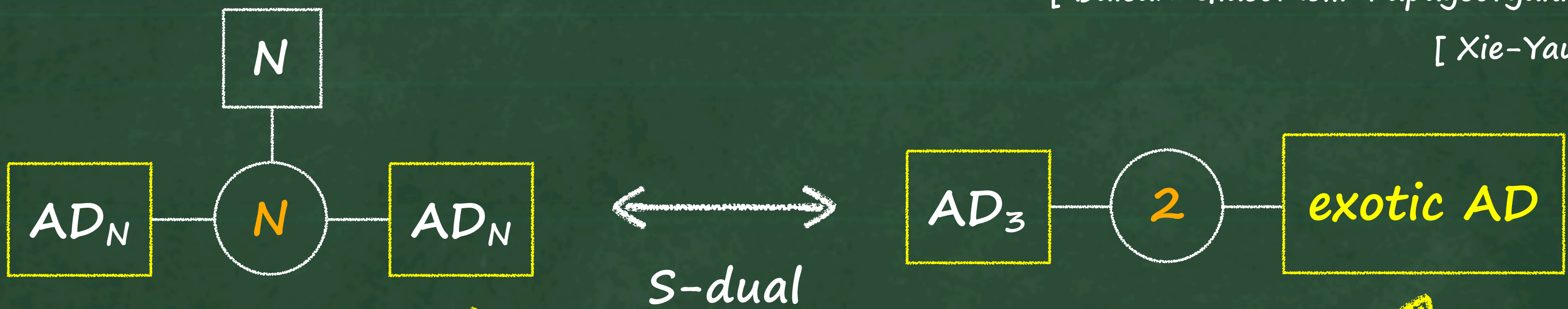
# $\mathcal{N}=2$ S-dualities w/ AD theories

( poorly understood )

( N: odd )

[ Buican-Giacomelli-Papageorgakis-TN ]

[ Xie-Yau ] x 2

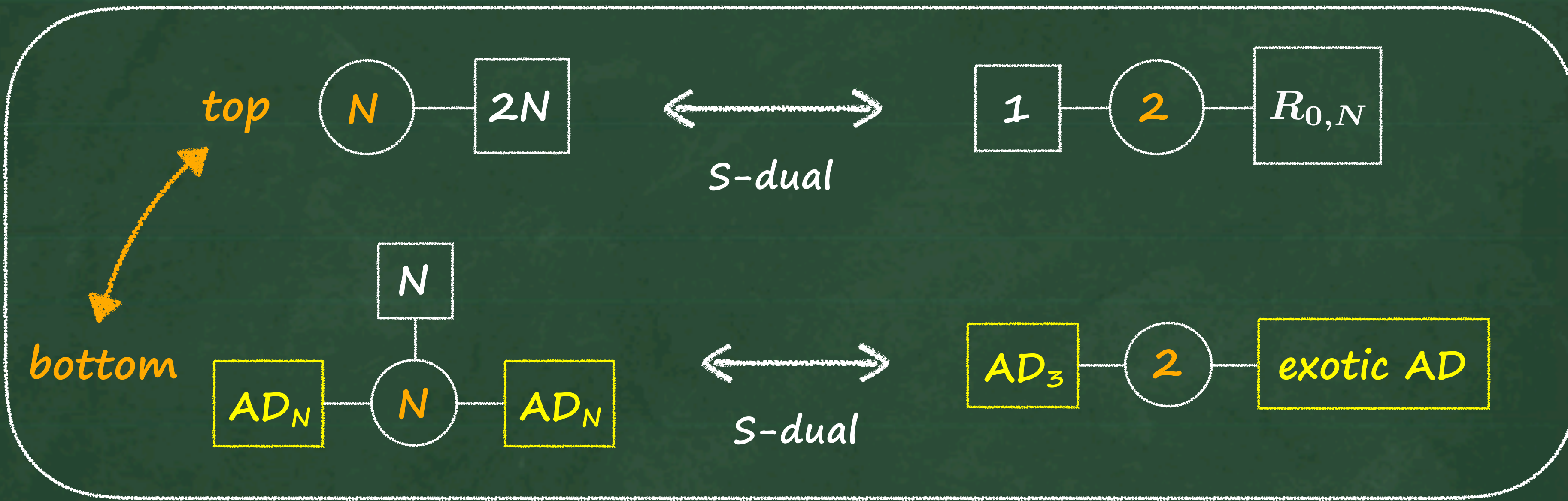



Argyres-Douglas (AD) theory w/ SU(N) flavor sym.

mysterious AD theory ( poorly understood )

No natural interpretation in terms of a Riemann surface!

## Q: Any quantitative relation between these two S-dualities?

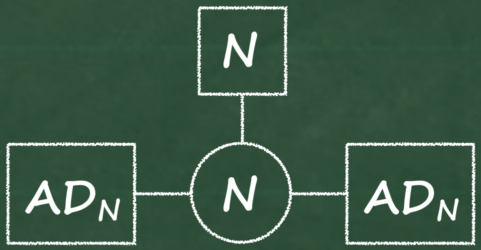



## Our Answer : YES!!!

- The Schur index of the top and bottom are identical up to a simple change of variables.
- This relation enables us to read off the Schur index of exotic AD

# Outline

1. What is $AD_N$ ?

2. Schur index of 


3. Index relation in the S-duality

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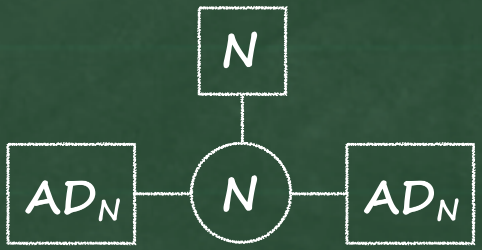
2. Schur index of 

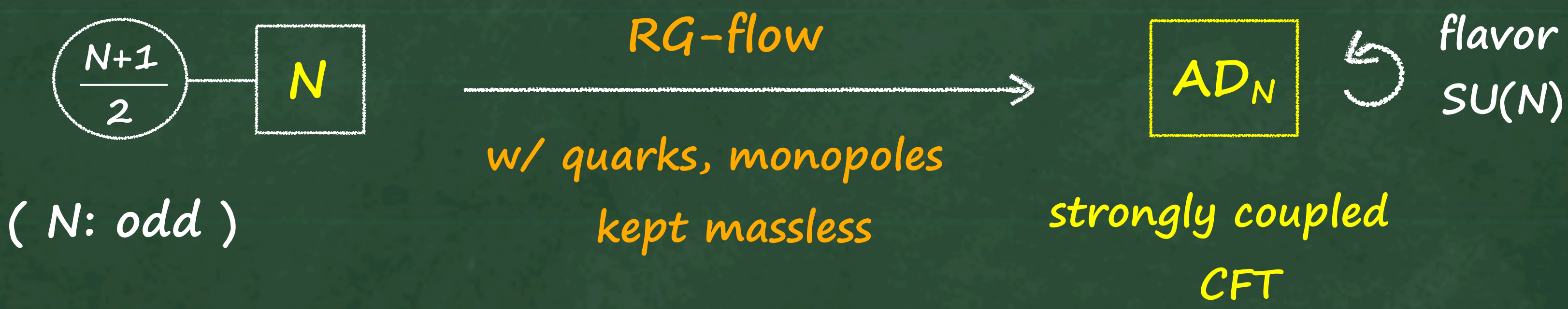

3. Index relation in the S-duality

# What is $AD_N$ ?

[ usually refereed to as $D_2(SU(N))$ ]

[ Cecotti – Del Zotto ]

$SU(\frac{N+1}{2})$ w/ $N$ flavors




- Flavor $SU(N)$ symmetry
- Coulomb branch operators of dimension $\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \cdots, \frac{N}{2}$

# What is $AD_N$ ?

$AD_N$ ↺ flavor SU(N)

## Schur index

space of local operators; scaling dim.; $SU(2)_R$ charge; flavor sym.; flavor charge

$$\mathcal{I}(q;\vec{z}) \equiv \mathrm{Tr}_{\mathcal{H}}\, (-1)^F q^{E-R} \prod_{k=1}^{\mathrm{rank}G_F} (z_k)^{A_k}$$

flavor fugacity

This index counts 1/4 BPS ops.

## Conjecture for $AD_N$

[ Xie-Yan-Yau ]

character of adj. rep. of SU(N)

$$\mathcal{I}_{\mathrm{AD}_N}(q;\vec{z}) = \exp\left[\sum_{n=1}^{\infty} \frac{1}{n}\frac{q^n}{1-q^{2n}}\chi_{\mathrm{adj}}^{SU(N)}({z_k}^n)\right]$$

$$= \prod_{k=0}^{\infty}\left[\frac{1}{(1-q^{2k+1})^{N-1}} \prod_{1\le i\neq j\le N} \frac{1}{1-q^{2k+1}z_i z_j^{-1}}\right]$$

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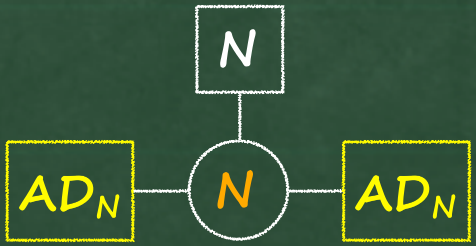
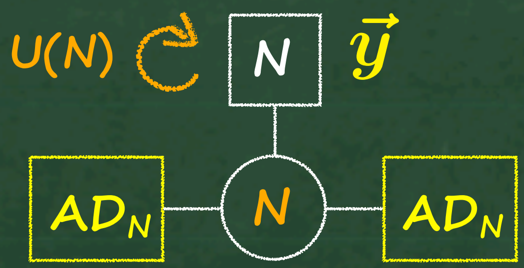
2. Schur index of 


3. Index relation in the S-duality

# Schur index of quiver

$AD_N$ ↺ flavor $SU(N)$ ( N: odd )

- The following gauging is **exactly marginal:**




$$\beta = 2N + N + N - 4N = 0$$

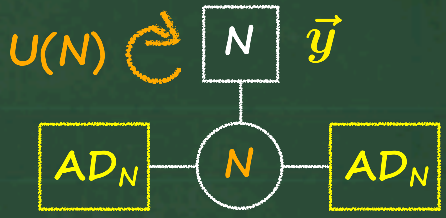
N $AD_N$ N

- This quiver theory has U(N) flavor symmetry.

$\Longrightarrow$ Schur index $\mathcal{I}(q, \vec{y})$

↑ fugacity for flavor U(N)

# Schur index of quiver




## Schur index

extracting gauge invariant ops.

N    AD$_N$ × 2    N $\vec{y}$

$$\mathcal{I}(q,\vec{y}) = \oint_{|z_k|=1}\left(\prod_{k=1}^{N-1}\frac{dz_k}{2\pi i z_k}\right)\mathcal{I}_{\text{vec}}(q,\vec{z})\times\left[\mathcal{I}_{\text{AD}_N}(q,\vec{z})\right]^2\times\mathcal{I}_{\text{hyp}}(q,\vec{z},\vec{y})$$

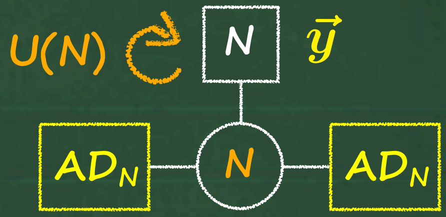
flavor fugacity

NEW identity

$$= \mathcal{I}_{\text{vec}}(q^2,\vec{z})$$

N

# Schur index of quiver




## Schur index

extracting gauge invariant ops.

$$\mathcal{I}(q,\vec{y}) = \oint_{|z_k|=1} \left( \prod_{k=1}^{N-1} \frac{dz_k}{2\pi i z_k} \right) \mathcal{I}_{\text{vec}}(q,\vec{z}) \times \left[ \mathcal{I}_{\text{AD}_N}(q,\vec{z}) \right]^2 \times \mathcal{I}_{\text{hyp}}(q,\vec{z},\vec{y})$$

flavor fugacity

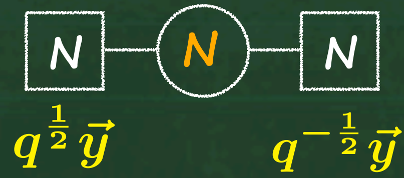
N → $\mathcal{I}_{\text{vec}}$; $AD_N$ x 2 → $[\mathcal{I}_{\text{AD}_N}]^2$; N $\vec{y}$ → $\mathcal{I}_{\text{hyp}}$

NEW identities

$$= \mathcal{I}_{\text{vec}}(q^2,\vec{z}) \quad \mathcal{I}_{\text{hyp}}(q^2,\vec{z},q^{\frac{1}{2}}\vec{y})\, \mathcal{I}_{\text{hyp}}(q^2,\vec{z},q^{-\frac{1}{2}}\vec{y})$$

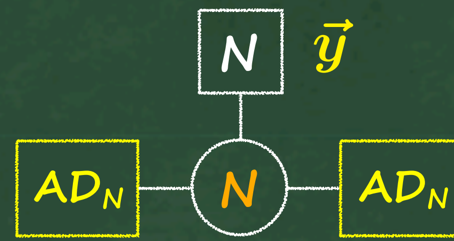
N    N $q^{\frac{1}{2}}\vec{y}$    N $q^{-\frac{1}{2}}\vec{y}$

= Schur index of




with $q \to q^2$

Schur index of




$=$ WOW!

Schur index of


w/ $q \to q^2$

This side is a Lagrangian theory!

# Outline

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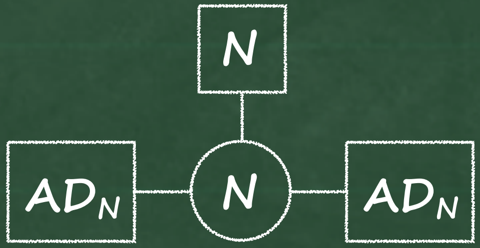
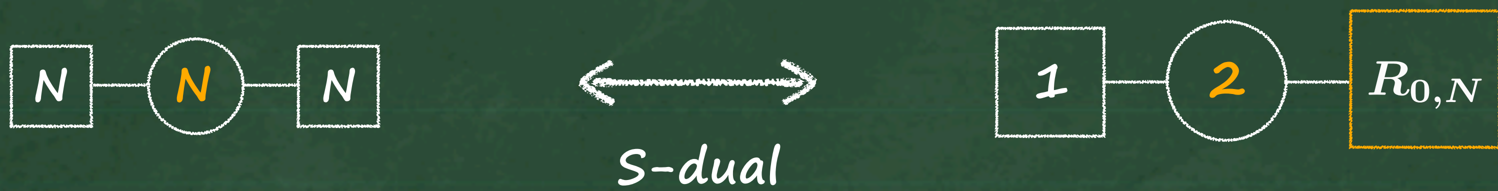
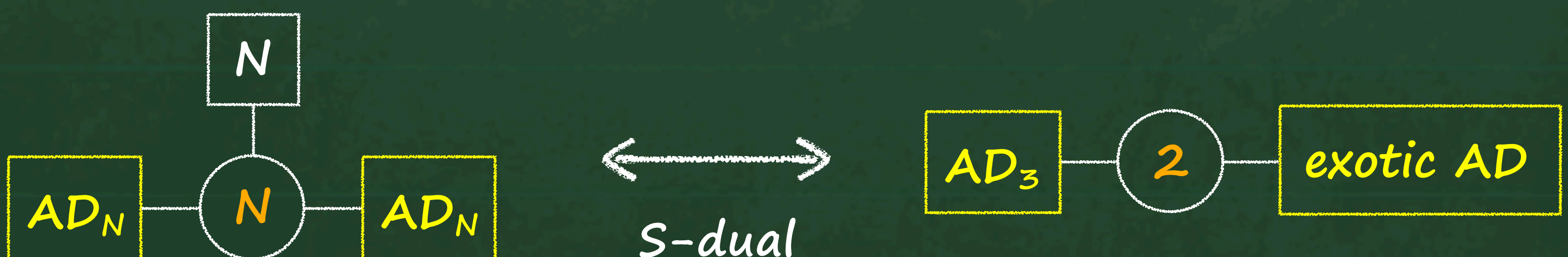
2. Schur index of 


3. Index relation in the S-duality

# Index relation in the S-duality



# Index relation in the S-duality

Schur index of


integral formula

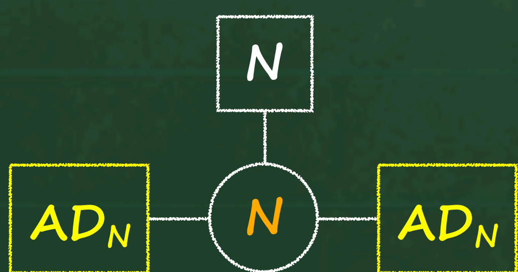
→

order by order

Schur index of

$R_{0,N}$

Schur index of




integral formula

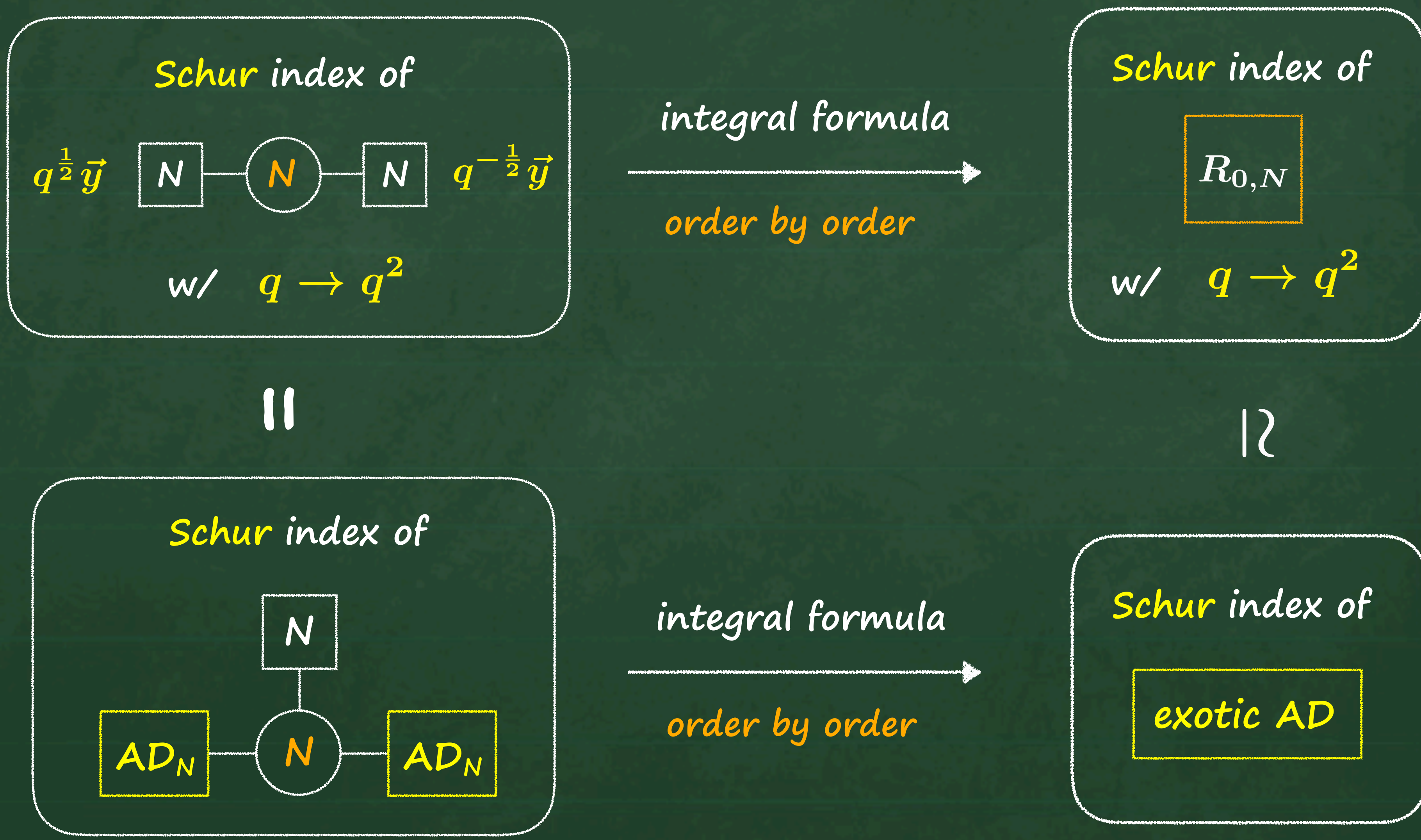
→

order by order

Schur index of

exotic AD

# Index relation in the S-duality



$\exists$ closed form expression in terms of 2D TQFT

Schur index of

$R_{0,N}$

w/ $q \to q^2$

Riemann surface interpretation

Schur index of

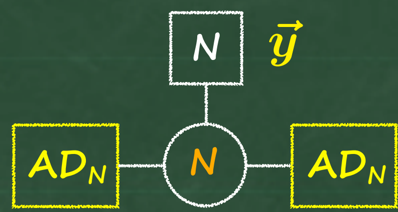
exotic AD

We obtain a closed-form formula for the Schur index of exotic AD !!

# Summary

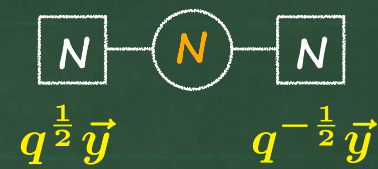
- We found

Schur index of




=

Schur index of




w/ $q \to q^2$

from which you can read off the index of exotic AD.

- We also have infinite generalizations of this relation.

$\Longrightarrow$ predictions for many NEW S-dualities

- Open problems
  - String theory interpretation?
  - Simple relation between the operator spectra?
  - Only for the Schur index?