Quantum chaos transition in a model dual to eternal traversable wormhole

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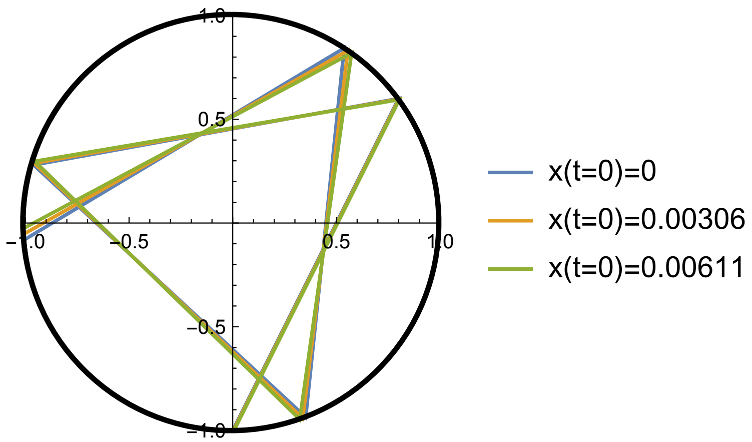
ref.: [Garcia-Garcia - TN - Rosa - Verbaarschot, 1901.06031]
(Phys. Rev. D 100, 026002)

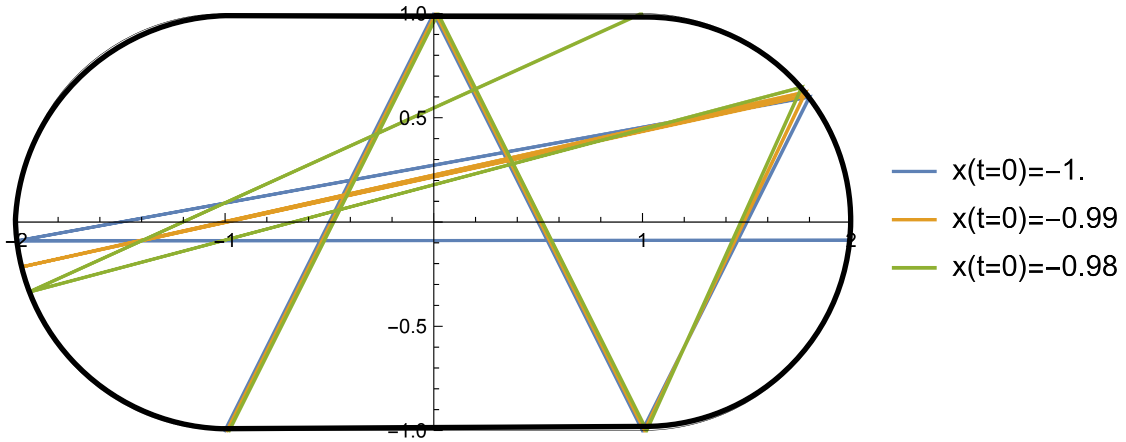
@Strings and Fields 2019 (2019/8/22)

Introduction

What is Chaos?

- In classical mechanics: initial value sensitivity of trajectory





Recently, **quantum chaos** is related to black holes

- There is a "bound on chaos", saturated by BH [Maldacena-Shenker-Stanford, '15]
- SYK model saturates bound and dual to AdS_2 BH [Sachdev-Ye, '92][Kitaev, '15][Maldacena-Stanford, '16]

This talk: to study relation between BHs and chaos more,

modify SYK s.t. gravity dual interpolates BH and non-BH (Hawking-Page transition)

Proposal: Hawking-Page transition ⇒ chaos/integrable transition

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How to define quantum chaos?

① out of time ordered correlators (early time chaos)

Quantum version of "initial value sensitivity" [Larkin-Ovchinnikov, '69]

$$\frac{\partial x(t)}{\partial x_0} \sim e^{\lambda_L t} \longleftrightarrow \{x(t), p(0)\}_{\mathrm{PB}} \sim e^{\lambda_L t} \Rightarrow \langle (i[W(t), V(0)])^2 \rangle \sim e^{\lambda_L t}$$

This talk

② energy spectrum (late time chaos)

Berry-Tabor/Bohigas-Giannoni-Schmit conjecture:

- *"Quantum spectrum of integrable system has same fluctuation properties as* ***independent random variables (Poisson)****."* [Berry-Tabor, '77]

- *"Quantum spectrum of chaotic system has same fluctuation properties as* ***eigenvalues of Random Matrix*** *in* GUE/GOE/GSE*, depending on* $\mathcal{T}$*-invariance property of Hamiltonian."* [Bohigas-Giannoni-Schmit, '84]

GUE: $H \neq H^*$

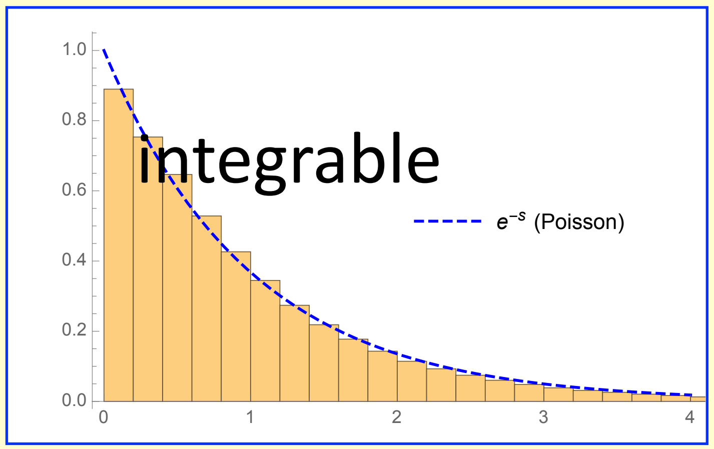
GOE: $H = H^*$

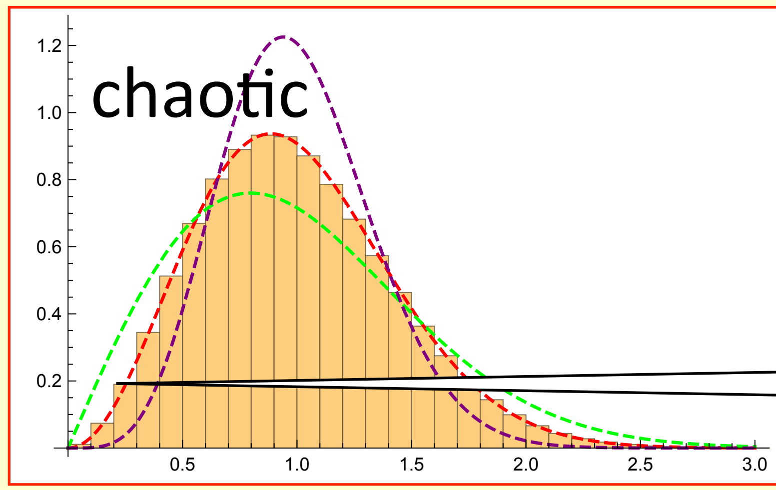
GSE: $H = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} H^* \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$

How to characterize RMT-likeness?

Nearest-neighbor spacing distribution (NNSD)

$$\left(-\frac{\hbar^2}{2m}\partial_x^2 + V(x)\right)\psi_i = E_i\psi_i$$ ➡ distribution of $\Delta E = E_i - E_{i-1}$





same level repulsion as RMT

$$\mathcal{Z}_{\mathrm{GUE}} = \int (dE_i)^L \prod_{i<j} (E_i - E_j)^2 e^{-\frac{L}{2}\sum_i E_i^2}$$

$$\mathcal{Z}_{\mathrm{GOE}} = \int (dE_i)^L \prod_{i<j} |E_i - E_j| e^{-\frac{L}{2}\sum_i E_i^2}$$

$$\mathcal{Z}_{\mathrm{GSE}} = \int (dE_i)^L \prod_{i<j} (E_i - E_j)^4 e^{-\frac{L}{2}\sum_i E_i^2}$$

Adjacent gap ratio

$$\langle \bar{r} \rangle = L^{-1} \sum_{i=1}^{L} \langle r_i \rangle$$

	Poisson	GOE	GUE	GSE
$\langle \bar{r} \rangle$	$2\log 2 - 1 = 0.39$	$4 - 2\sqrt{3} = 0.54$	$\frac{2\sqrt{3}}{\pi} - \frac{1}{2} = 0.6$	$\frac{32\sqrt{3}}{15\pi} - \frac{1}{2} = 0.68$

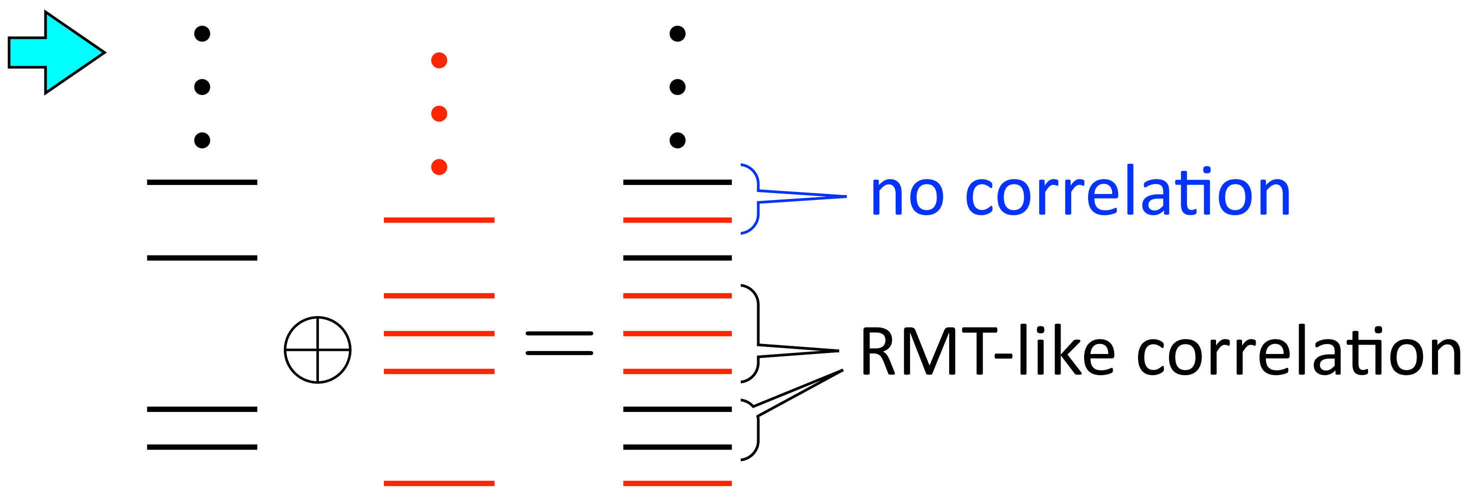
$$\langle r_i \rangle = \left\langle \frac{\min(E_i - E_{i-1}, E_{i+1} - E_i)}{\max(E_i - E_{i-1}, E_{i+1} - E_i)} \right\rangle$$

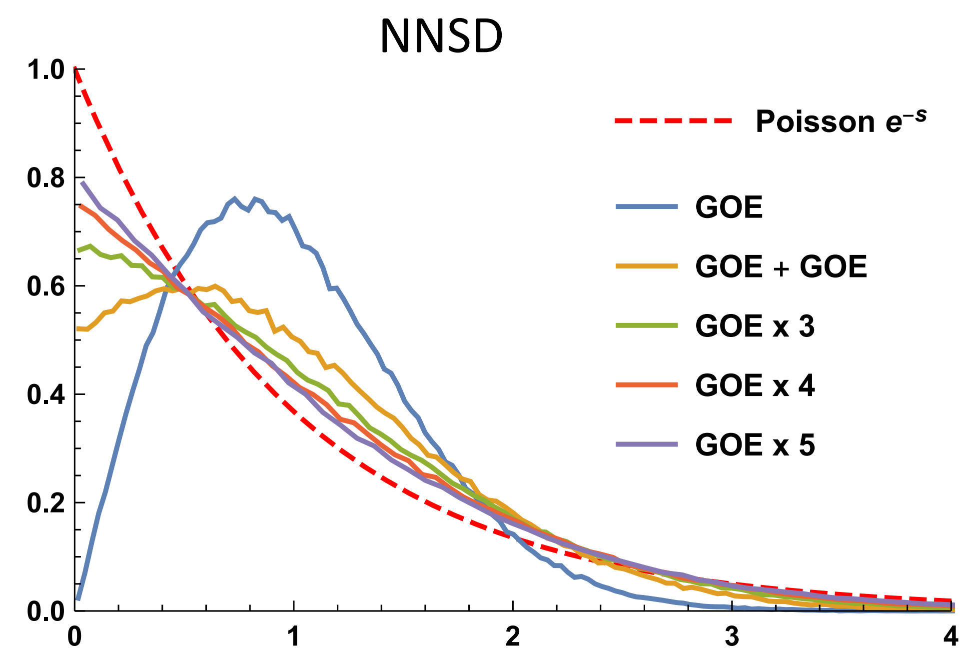
○ $\langle r_i \rangle$: RMT-likeness of each part of spectrum around E_i

Note

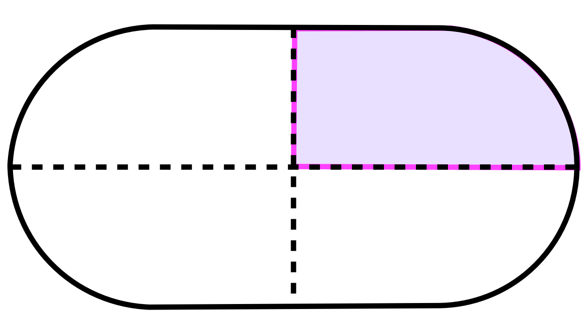
To diagnose quantum chaos correctly, we have to extract only one symmetry sector

○ suppose $H = H^{(1)} \oplus H^{(2)}$





ex.: stadium billiard



$$H^{(1)} : \ \psi(x,y) = \psi(-x,y) = \psi(x,-y)$$
$$H^{(2)} : \ \psi(x,y) = \psi(-x,y) = -\psi(x,-y)$$
$$H^{(3)} : \ \psi(x,y) = -\psi(-x,y) = \psi(x,-y)$$
$$H^{(4)} : \ \psi(x,y) = -\psi(-x,y) = -\psi(x,-y)$$

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SYK model

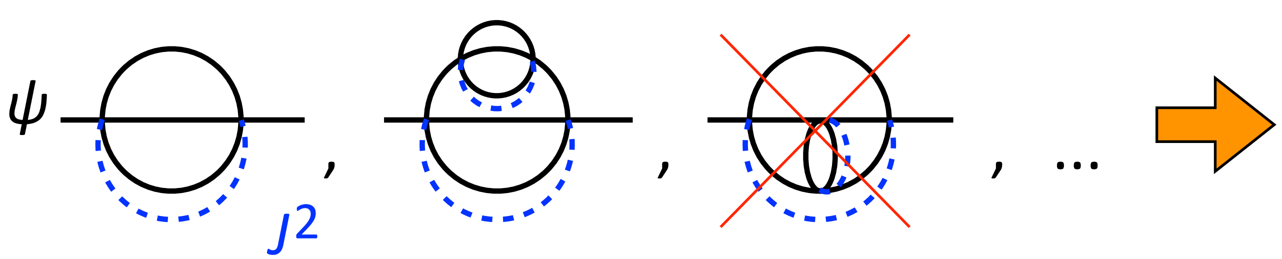
$$S = \int d\tau \Big(\frac{1}{2} \sum_i \psi_i \partial_\tau \psi_i - H \Big)$$

$$H = \sum_{i<j<k<\ell} J_{ijk\ell} \psi_i \psi_j \psi_k \psi_\ell \qquad \{\psi_i, \psi_j\} = \delta_{ij}$$

$J_{ijk\ell}$: Gaussian-distributed random couplings with $\langle J_{ijk\ell}^2 \rangle = N^{-3}$

○ 1d QM with disorder: $\langle \mathcal{O} \rangle = \Big\langle \int \mathcal{D}\psi_i \mathcal{O} e^{-S} \Big\rangle_J$

○ large *N* perturbation is dominated by melonic diagrams



$$\widetilde{G}(\omega) \cdot (-i\omega - \widetilde{\Sigma}(\omega)) = 1, \quad \Sigma(\tau, \tau') = G(\tau, \tau')^3$$

○ low energy emergent reparametrization symmetry (spontaneously broken to SL(2,R))

$$\begin{cases} G(\tau, \tau') & \to (\dot{f} \dot{f}')^{\frac{1}{4}} G(f(\tau), f(\tau')) \\ \Sigma(\tau, \tau') & \to (\dot{f} \dot{f}')^{\frac{3}{4}} \Sigma(f(\tau), f(\tau')) \end{cases}$$

"Nearly CFT$_1$"

[Maldacena-Stanford,'16]

gravity dual: AdS_2 with UV cutoff (Nearly AdS_2)

Jackiw-Teitelboim gravity (2d gravity + dilaton) [Teitelboim,'83][Jackiw,'85]

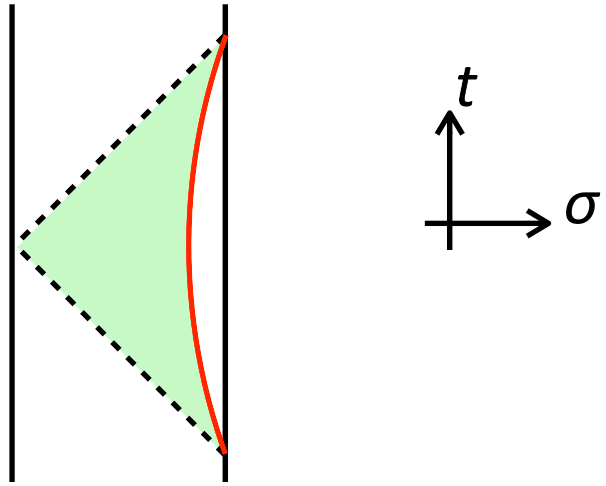
$$S = \frac{\phi_0}{2}\Big[\int_M R + 2\int_{\partial M} K\Big] + \frac{1}{2}\int_M \phi(R+2) + \phi_b \int_{\partial M} K$$

○ $\int \mathcal{D}\phi$ ➡ $R+2=0$: AdS_2 $ds^2 = \frac{-dt^2 + d\sigma^2}{\sin\sigma^2}$

○ No bulk graviton ➡ dynamical d.o.f. = shape of boundary $(t(u), \sigma(u))$ $\sin\sigma \sim 0$

○ low energy reparametrization symmetry (u) : spontaneously broken to SL(2,R)

➡ effective theory for boundary (= Schwarzian) ⟷ effective theory of single SYK



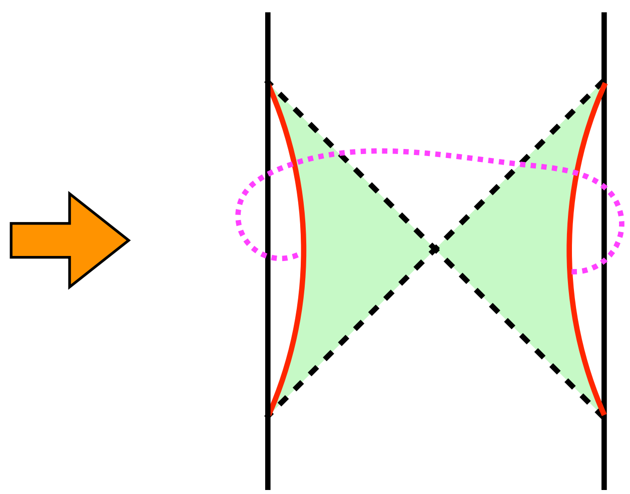
Two coupled SYK model

$$H = \sum_{i<j<k<\ell}^{N/2} J_{ijk\ell}\psi_i^L\psi_j^L\psi_k^L\psi_\ell^L + \sum_{i<j<k<\ell}^{N/2} J_{ijk\ell}\psi_i^R\psi_j^R\psi_k^R\psi_\ell^R + i\mu\sum_i^{N/2}\psi_i^L\psi_i^R$$

$$\{\psi_i^L,\psi_j^L\} = \delta_{ij}, \quad \{\psi_i^L,\psi_j^R\} = 0 \qquad \langle J_{ijk\ell}^2\rangle = N^{-3}$$

[Maldacena-Qi, 18]

- two SYK ➡ two boundaries
- $|\text{ground state}\rangle \approx |\text{TFD}\rangle_\beta \equiv \frac{1}{\sqrt{Z(\beta)}}\sum_{n=1}^{2^{N/4}} e^{-\frac{\beta}{2}E_n^{\text{SYK}}}|n\rangle_L \otimes |\widetilde{n}\rangle_R$ with some $\beta(\mu)$



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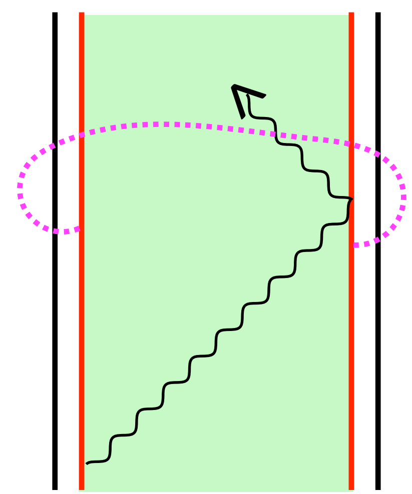
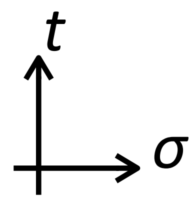




: traversable wormhole

Prohibited by ANEC

But if we couple L and R directly, $T_{--} < 0$ is possible by Casimir effect

[Gao-Jafferis-Wall,'16]

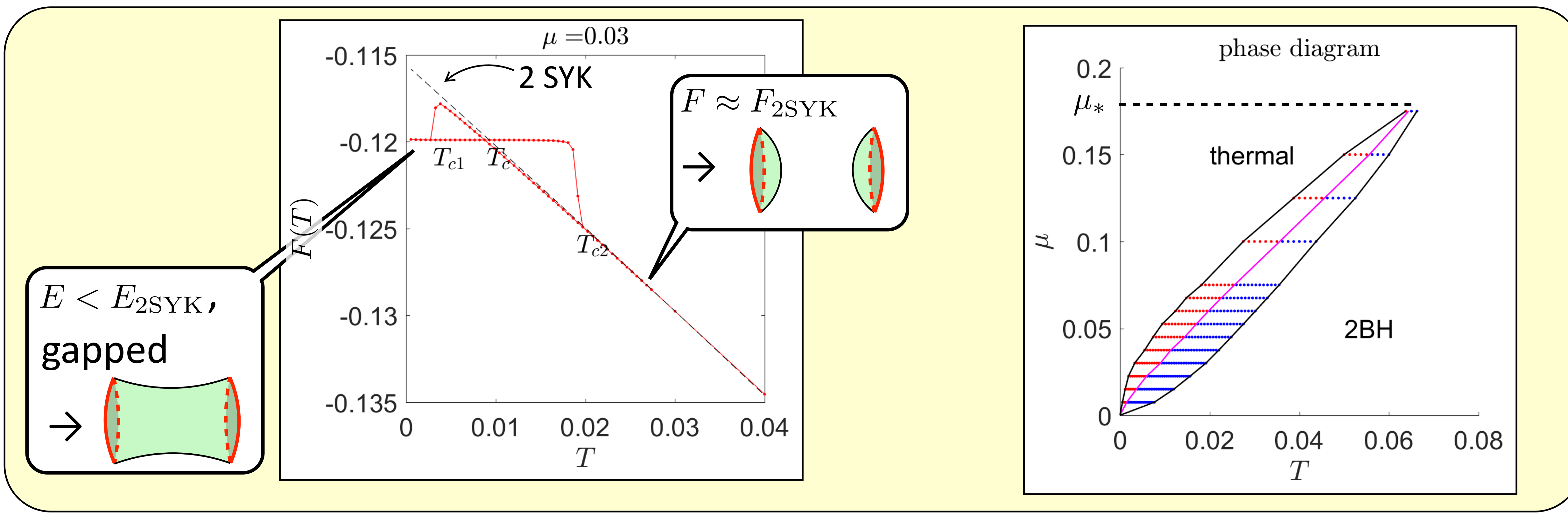
Hawking-Page transition

$$Z = \left\langle \int \mathcal{D}\psi_i^a e^{-\int d\tau \left(\frac{1}{2}\psi_i^a \partial_\tau \psi_i^a - H\right)} \right\rangle_J = \int \mathcal{D}G_{ab}\mathcal{D}\Sigma_{ab} e^{-{\color{red}N} S'}$$

$$\left(\begin{aligned} S' = &-\log \mathrm{Pf}\begin{pmatrix} -\frac{1}{2}\partial + \frac{\Sigma_{LL}}{2} & \frac{\Sigma_{LR}}{2} \\ \frac{\Sigma_{RL}}{2} & -\frac{1}{2}\partial + \frac{\Sigma_{RR}}{2} \end{pmatrix} + \frac{1}{2}\int\int \Sigma_{ab}(\tau,\tau')G_{ab}(\tau,\tau') \\ &- \frac{1}{8}\int\int G_{ab}(\tau,\tau')^4 - \frac{i\mu}{2}\int (G_{LR}(\tau,\tau) - G_{RL}(\tau,\tau)) \end{aligned} \right)$$

large *N* limit = classical limit ➡

$$\frac{F}{N} = -\frac{1}{N\beta}\log Z = -\frac{1}{N\beta}\log \sum_{\text{saddles}} e^{-S_{saddle}}$$



(figures from [Maldacena-Qi,'18])

○ phase transition disappears for $\mu > \mu_* \sim 0.177$

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*S*ymmetry and *T*-reversal property of *H*

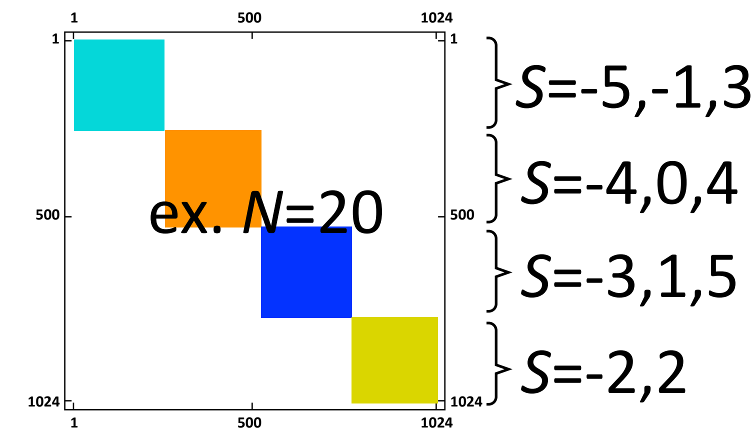
To evaluate quantum chaos, we have to extract single symmetry-protected sector and identify *T*-reversal property (GUE/GOE/GSE)

○ $H = \sum_{i<j<k<\ell}^{N/2} J_{ijk\ell}\psi_i^L\psi_j^L\psi_k^L\psi_\ell^L + \sum_{i<j<k<\ell}^{N/2} J_{ijk\ell}\psi_i^R\psi_j^R\psi_k^R\psi_\ell^R + i\mu\sum_i^{N/2}\psi_i^L\psi_i^R$

$S = i\sum_{i=1}^{N/2}\psi_i^L\psi_i^R = \sum_i s_i$: "total spin"

$s_i = \psi_i^{(+)}\psi_i^{(-)} - \frac{1}{2}$ $\quad \psi_i^{(\pm)} = \psi_i^R \pm i\psi_i^L$

$= \frac{1}{8}\sum_{i<j<k<\ell}^{N/2} J_{ijk\ell}(\psi_i^{(+)}\psi_j^{(+)}\psi_k^{(+)}\psi_\ell^{(+)} + \psi_i^{(-)}\psi_j^{(-)}\psi_k^{(-)}\psi_\ell^{(-)} + \psi_i^{(+)}\psi_j^{(+)}\psi_k^{(-)}\psi_\ell^{(-)} + \cdots) + \mu S$



➡ *H* preserves ***S* mod 4** for any $J_{ijk\ell}$ (c.f. *S* mod 2 = parity)

○ In spin basis $\{|s_1 s_2 \cdots {}_{N/2}\rangle\}_{s_i=\pm 1}$, $\psi_i^{(\pm)} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}_i, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}_i$ are real

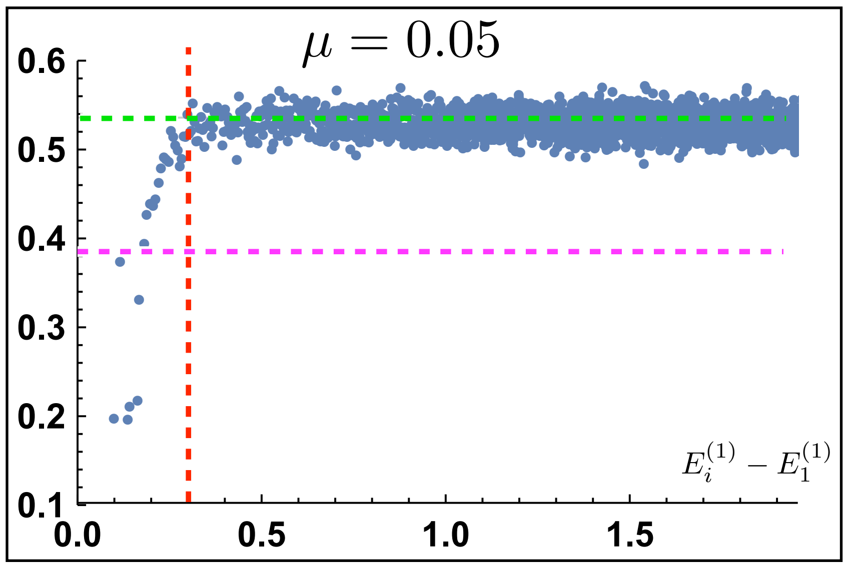
➡ $(HP_{S\mathrm{mod}4})^* = HP_{S\mathrm{mod}4}$ **: GOE-type**

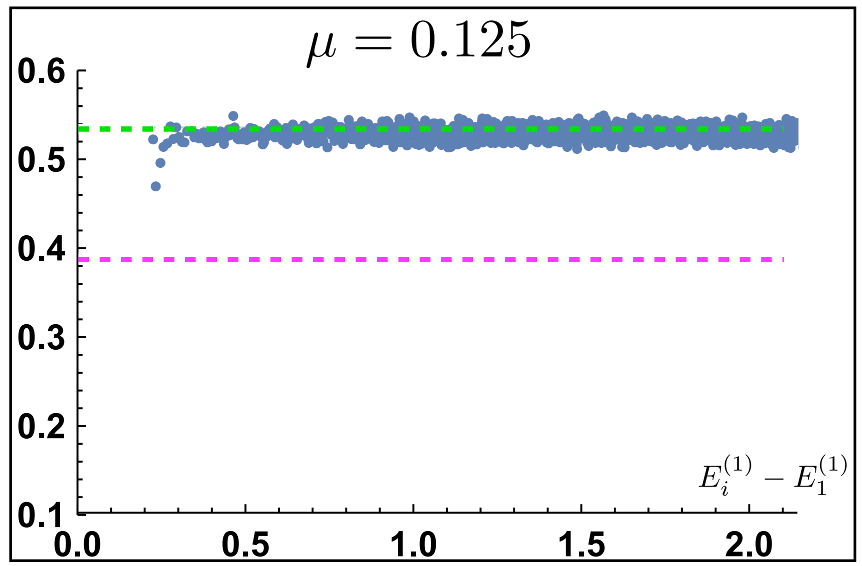
Result (N=28)

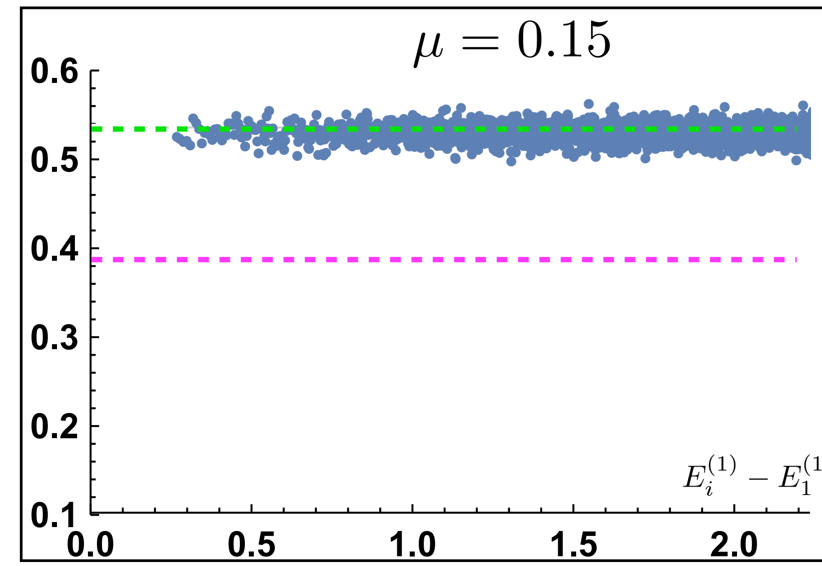
We use adjacent gap ratio

$$r_i = \frac{\min(E_i^{(1)} - E_{i-1}^{(1)}, E_{i+1}^{(1)} - E_i^{(1)})}{\max(E_i^{(1)} - E_{i-1}^{(1)}, E_{i+1}^{(1)} - E_i^{(1)})}$$

$\{E_i^{(1)}\}$: spectrum of $H\Big|_{S\mathrm{mod}\ 4=1(S=-7,-3,1,5)}$



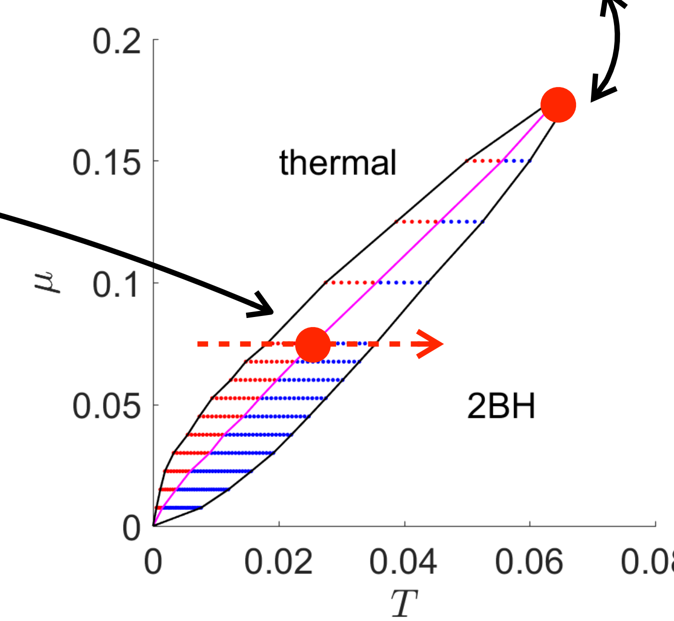




$r \sim r_{\mathrm{Poisson}}$ $r \sim r_{\mathrm{GOE}}$

$r \sim r_{\mathrm{GOE}}$ everywhere: C/I transition disappears

chaos/integrable transition in energy (= T)



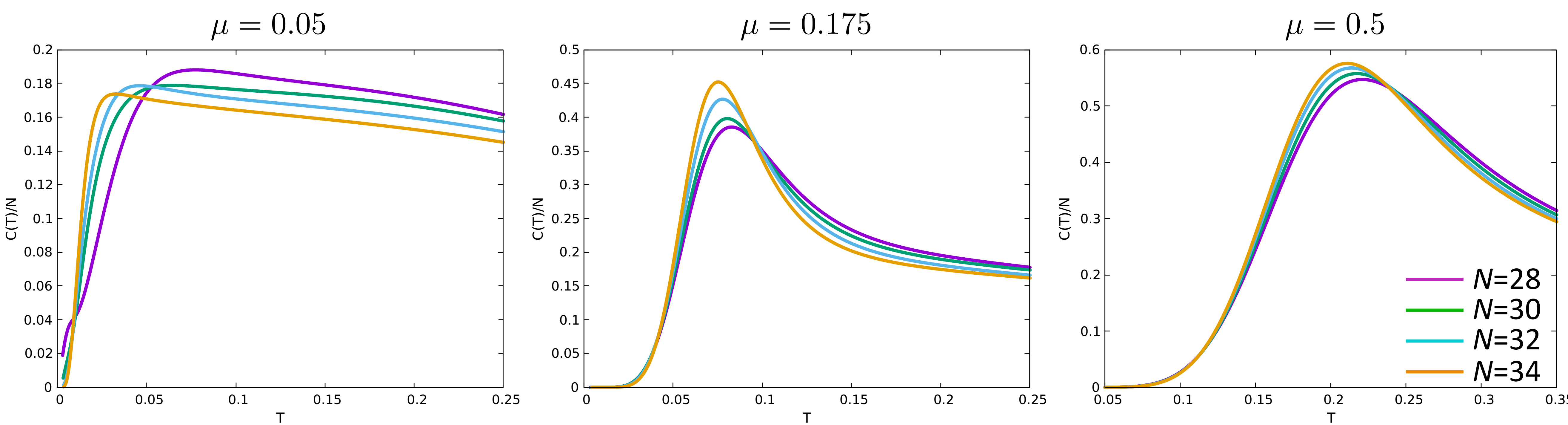
Quantitative mismatch?

SD eq. approach ➔ phase transition disappears at $\mu = 0.177$

adjacent gap ratio ➔ non-RMT region disappears at $\mu = 0.125 \sim 0.15$

Finite *N* effect?

Computing specific heat $c = -T\dfrac{d^2F}{dT^2}$ from spectrum, we obtained



... difficult to distinguish phase transition $c = \infty$ from smooth (though sharp) peak

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Summary and Future works

Summary:

- Studied late time chaos of two coupled SYK model (dual to traversable wormhole)
- Propose Hawking-Page transition ⇒ chaos/integrable transition

Future works:

- Does C/I transition remains and becomes sharp transition in large N limit?
- Check C/I transition in Lyapunov exponent [TN-Rosa, work in progress]
- Gravity interpretation for $\mu > \mu_*$?
- Generalization to non-equal (independent) random coupling for $\mathrm{SYK_L}$ and $\mathrm{SYK_R}$ [TN-Numasawa, in preparation]