Nonperturbative definition of closed string theory with holes in the world-sheet via open string field theory

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"Strings and Fields 2019"
at Yukawa Institute for Theoretical Physics
August 20, 2019 in Kyoto

Introduction

One of the most important problems in theoretical physics is to formulate quantum gravity in a consistent manner.

While the quantization of general relativity in the framework of quantum field theory turned out to be difficult, it was found that string theory consistently describes on-shell scattering amplitudes involving gravitons.

However, string theory only provides a perturbative definition of such on-shell scattering amplitudes with respect to the string coupling constant.

One possible approach to a nonperturbarive formulation of string theory would be to introduce a spacetime field for each oscillation mode of the string and construct an action of those spacetime fields. The resulting theory in this approach is called string field theory.

Since gravitons are described as states of the closed string, a natural approach to quantum gravity would be to construct closed string field theory.

Closed string field theory is useful when we handle phenomena such as vacuum shift and mass renormalization in the perturbative string theory.

arXiv:1703.06410, de Lacroix, Erbin, Kashyap, Sen and Verma

However, formulating closed string field theory at the quantum level nonperturbatively by the path integral does not seem to be promising because we need quantum corrections to the action at each loop order to compensate anomaly.

Then how can we formulate string theory nonperturbatively?

The typical origin of the string perturbation theory is the $1/N$ expansion of gauge theories with $N \times N$ matrix degrees of freedom.

Nucl. Phys. B72 (1974) 461, 't Hooft

The long history of research on string theory indicates that string theory can be defined nonperturbatively in terms of such gauge theory.

The AdS/CFT correspondence can be regarded as providing a nonperturbative definition of closed string theory in terms of a quantum field theory without containing gravity.

Type IIB superstring theory on $AdS_5 \times S^5$, for example, is conjectured to be defined nonperturbatively by $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions, and the string coupling constant of type IIB superstring theory on $AdS_5 \times S^5$ is given by $1/N$.

Question 1

What kind of closed string theory should we consider?

Let us recall the explanation of the AdS/CFT correspondence.

Consider type IIB superstring theory on a flat spacetime in ten dimensions with N coincident D3-branes.

In the low-energy region where the energy of the system is much lower than the string scale $1/\sqrt{\alpha'}$, closed strings and open strings are decoupled.

Then closed string theory becomes a free theory in ten dimensions and open string theory becomes $\mathcal{N} = 4$ $U(N)$ super Yang-Mills theory in four dimensions.

Next consider type IIB superstring theory on the three-brane solution of supergravity.

Because of the redshift factor, an object brought closer and closer to the three-brane appears to have lower and lower energy for the observer at infinity.

In the same low-energy limit, excitations propagating in ten dimensions and excitations in the near horizon region are decoupled, and we have a free theory in ten dimensions and type IIB superstring theory on $AdS_5 \times S^5$, which is the near horizon geometry of the three-brane solution.

the D-brane description

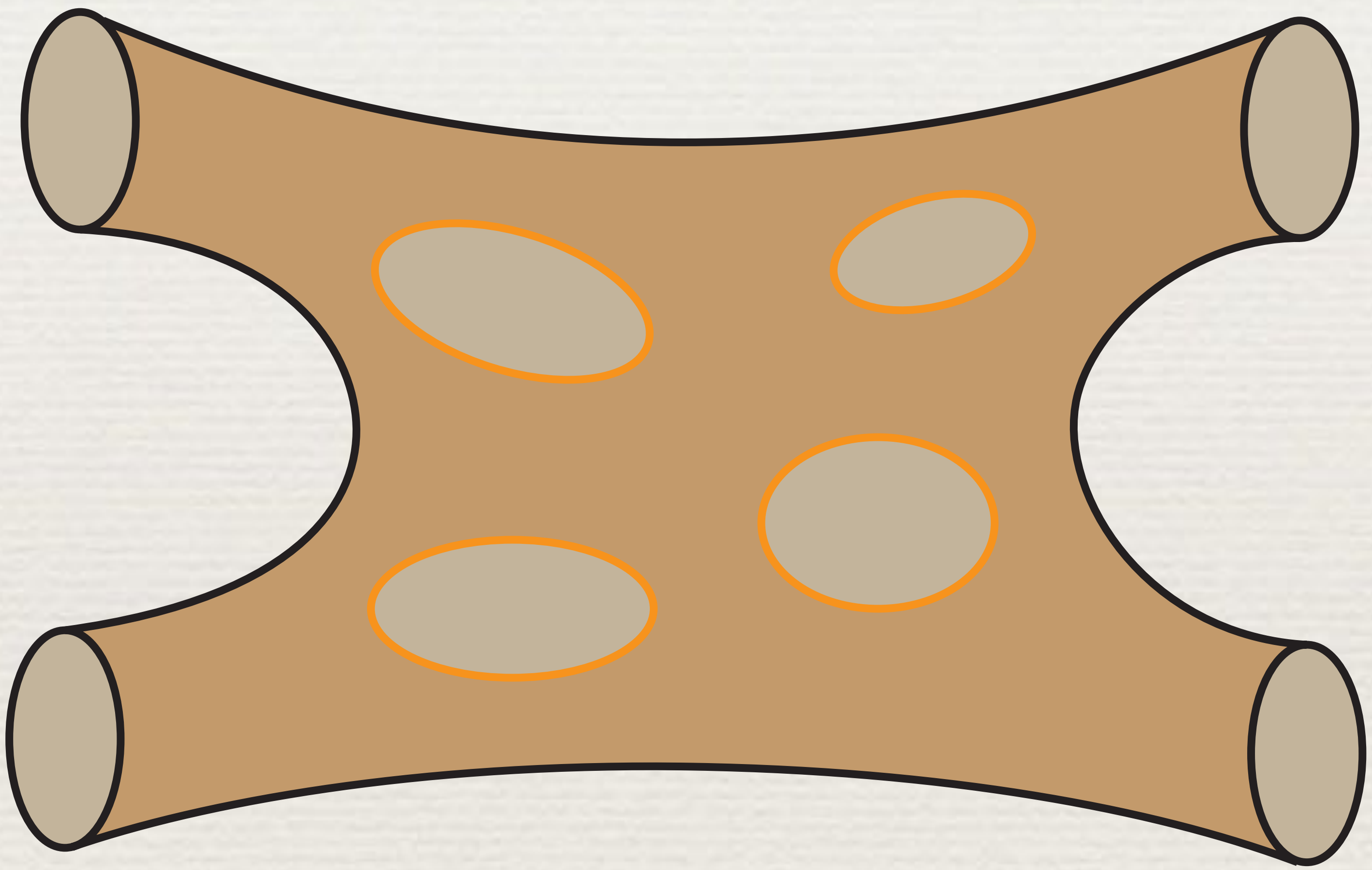
open strings + closed strings
$\rightarrow \mathcal{N} = 4$ super Yang-Mills theory + free closed theory in ten dimensions

the three-brane description

closed strings near the horizon + closed strings away from the horizon
$\rightarrow$ superstring theory on $AdS_5 \times S^5$ + free closed theory in ten dimensions

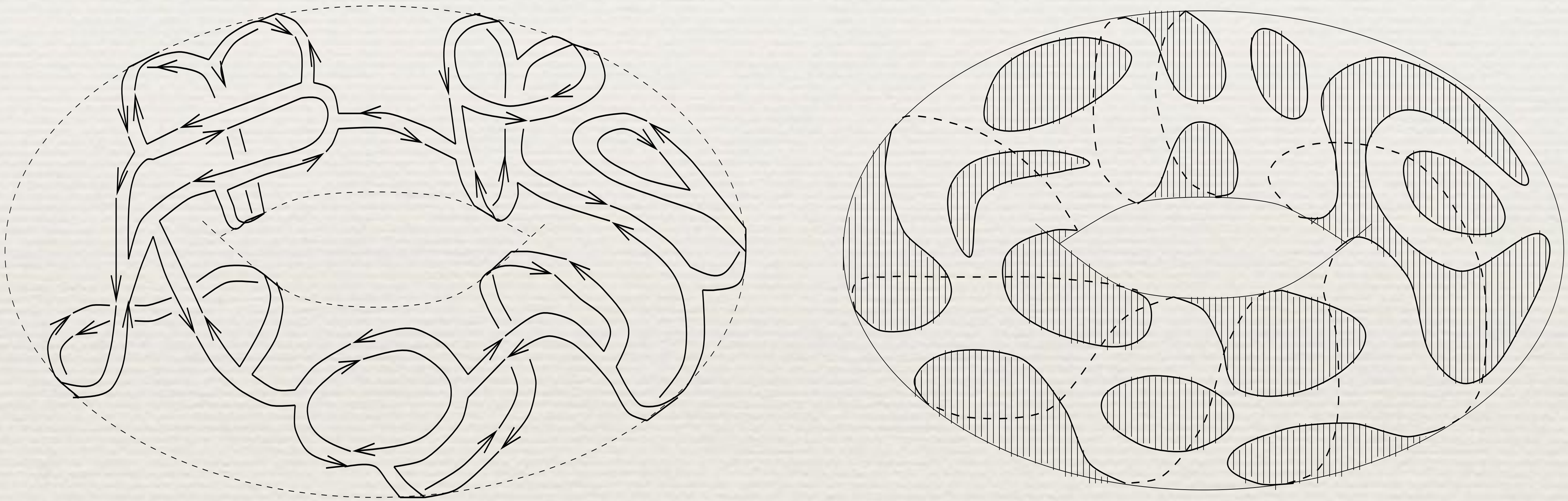
We are then led to the conjecture that $\mathcal{N} = 4$ $U(N)$ super Yang-Mills theory in four dimensions is the same as type IIB superstring theory on $AdS_5 \times S^5$.

In the D-brane description, the world-sheet of closed strings contains holes.



The AdS/CFT correspondence tells us that this closed string theory with holes in the world-sheet is equivalent to a closed string theory without holes on a curved background in the low-energy limit.

This was discussed in the context of the large N duality of the topological string. hep-th/0205297, Ooguri and Vafa



Figures taken from hep-th/0205297 by Ooguri and Vafa

This was also discussed recently using the pure-spinor formalism.

arXiv:1903.08264, Berkovits

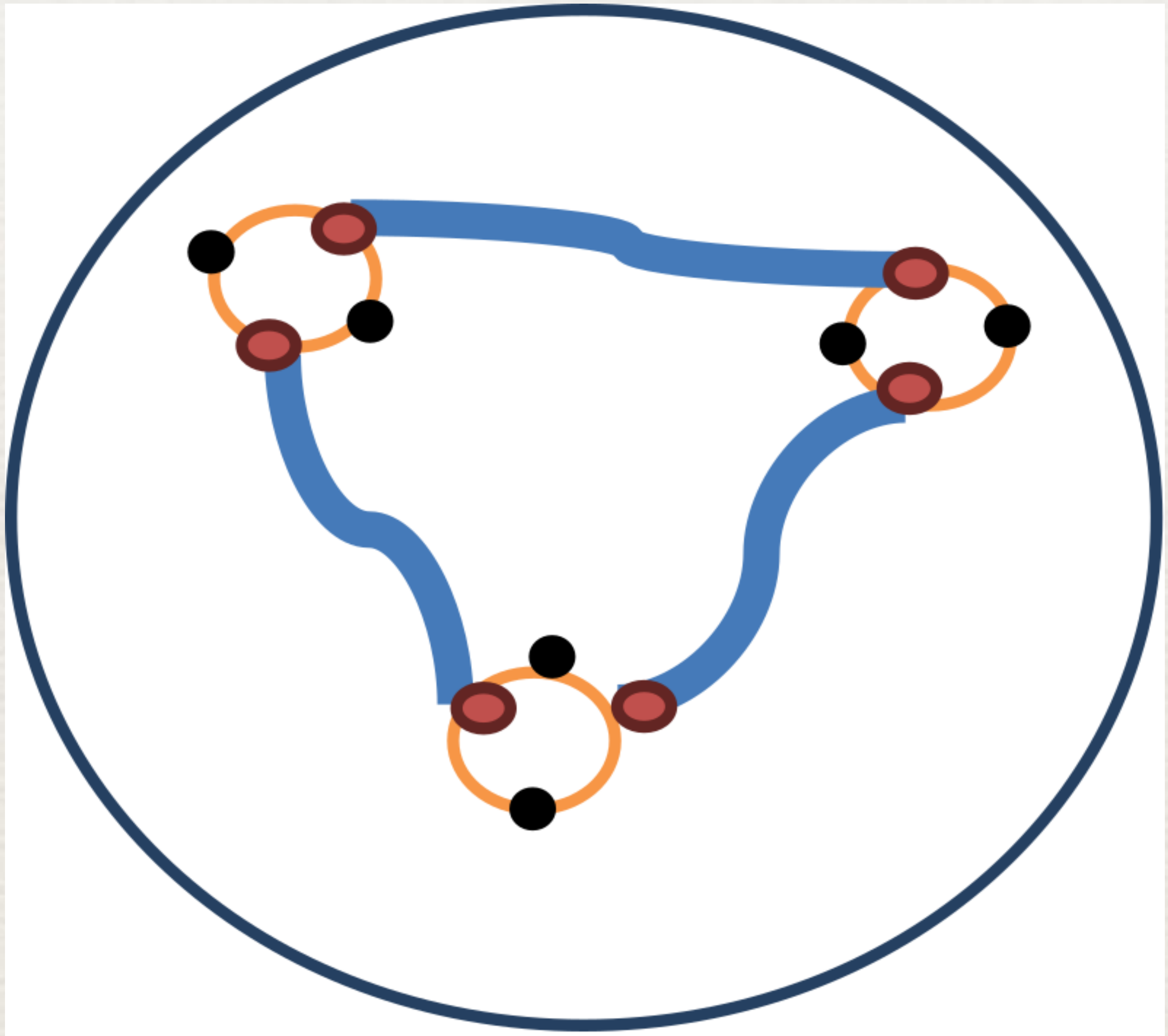


Figure taken from arXiv:1903.08264 by Berkovits

In this talk we do not discuss this problem, and we assume that closed string theory with holes in the world-sheet is a consistent perturbation theory and it contains gravity.

When there are N coincident D-branes, we describe the interactions in terms of the 't Hooft coupling constant as usual. Then the coupling constant of closed string theory with holes in the world-sheet is given by $1/N$.

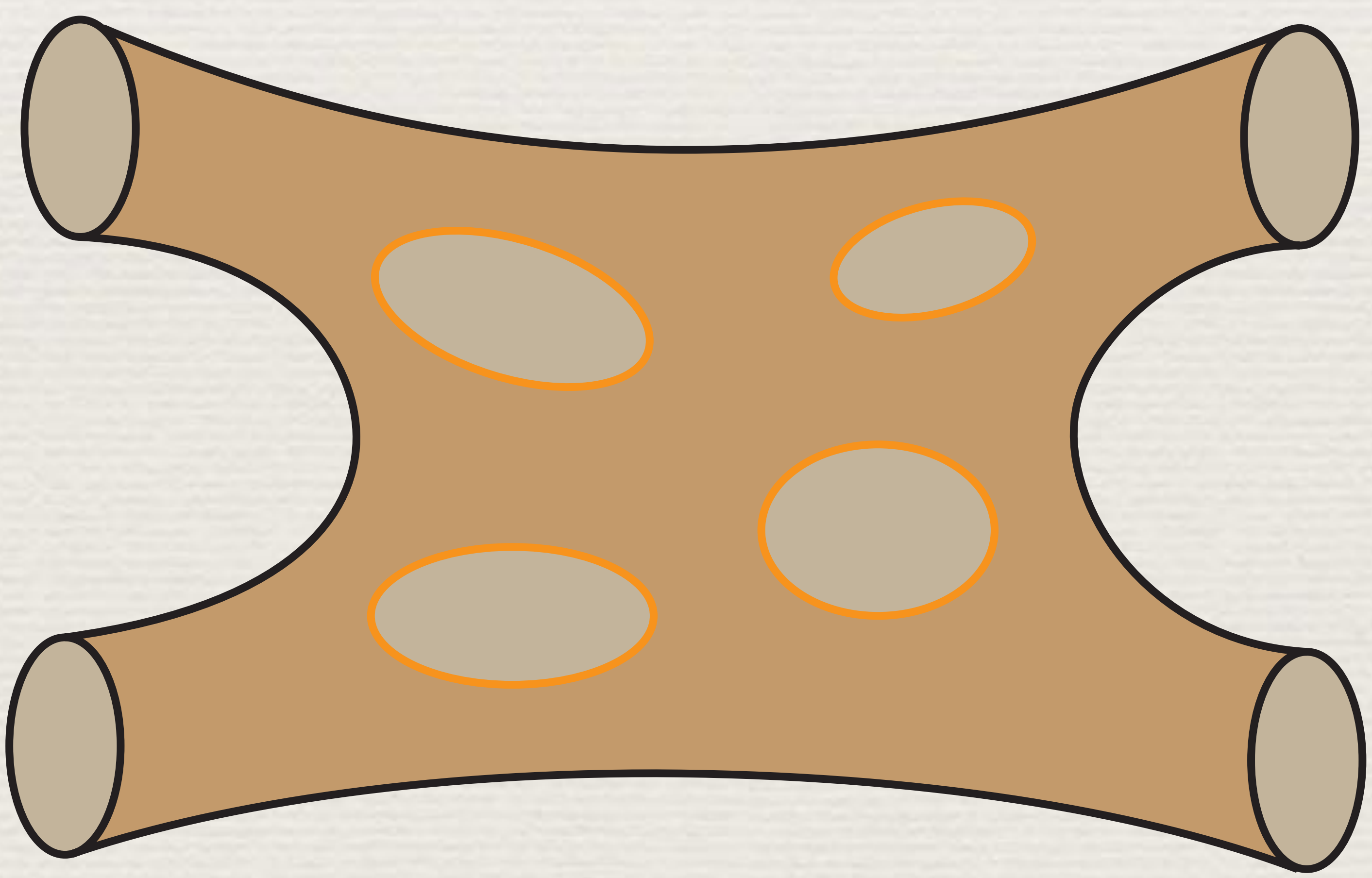
This is the perturbation theory including gravity that we want to reproduce by a theory without gravity.

Question 1

What kind of closed string theory should we consider?

Answer

We should consider closed string theory with holes in the world-sheet.



It would be difficult to see this world-sheet picture in $\mathcal{N} = 4$ super Yang-Mills theory because it is the theory after taking the low-energy limit.

Before taking the low-energy limit, the dynamics on the D-branes is described by open string field theory. In open string field theory, it would be more promising to see the world-sheet picture.

In string field theory, as we mentioned before, we introduce a spacetime field for each oscillation mode of the string. These component fields are incorporated into a single object called string field.

In open bosonic string field theory, the string field Ψ is a state of the boundary conformal field theory which describes the D-brane we are considering. For a D25-brane on a flat spacetime, a tachyonic scalar field $T(k)$, a massless vector field $A_\mu(k)$, and other component fields are incorporated into Ψ as

$$\Psi = \int \frac{d^{26}k}{(2\pi)^{26}} \Big[\frac{1}{\sqrt{\alpha'}} T(k)\, c_1 |\, 0; k\,\rangle + \frac{1}{\sqrt{\alpha'}} A_\mu(k)\, \alpha^\mu_{-1} c_1 |\, 0; k\,\rangle + \frac{i}{\sqrt{2}} B(k)\, c_0 |\, 0; k\,\rangle + \ldots \Big] .$$

The action of open bosonic string field theory is given by

Witten, Nucl. Phys. B268 (1986) 253

$$S = -\frac{1}{g^2} \left[\frac{1}{2} \langle \Psi, Q\Psi \rangle + \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle \right],$$

where g is the open string coupling constant, Q is the BRST operator, $\langle A, B \rangle$ and $A * B$ are the BPZ inner product and the star product, respectively, defined for a pair of states A and B.

The equation of motion derived from the action is

$$Q\Psi + \Psi * \Psi = 0 \, .$$

The action is invariant under the gauge transformation given by

$$\delta\Psi = Q\Lambda + \Psi * \Lambda - \Lambda * \Psi \, .$$

Question 2

What quantities should we consider in open string field theory?

In the context of the AdS/CFT correspondence, we consider correlation functions of gauge-invariant operators in $\mathcal{N} = 4$ super Yang-Mills theory.

In string field theory, it is in general difficult to construct gauge-invariant operators. It is in fact an important feature of string field theory and it is part of the reason that the interacting string field theory is believed to be unique up to field redefinition given a free theory.

In open bosonic string field theory there are a class of gauge-invariant operators and we can define a gauge-invariant operator for each on-shell closed string vertex operator.

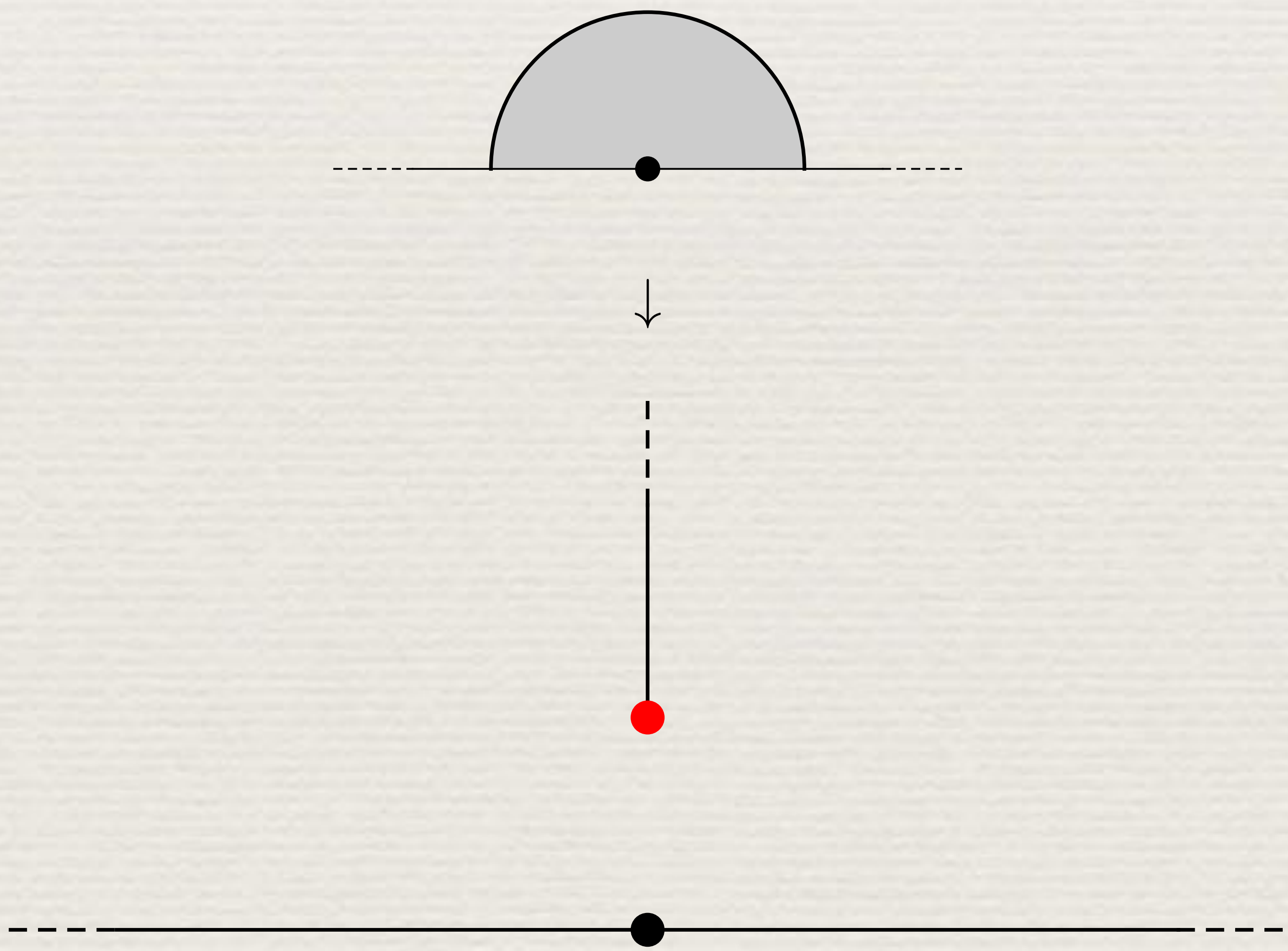
hep-th/0111092, Hashimoto and Itzhaki
hep-th/0111129, Gaiotto, Rastelli, Sen and Zwiebach

The gauge-invariant operator $\mathcal{A}_{\mathcal{V}}[\,\Psi\,]$ for an on-shell closed string vertex operator $\mathcal{V}$ is defined by

$$\mathcal{A}_{\mathcal{V}}[\,\Psi\,] = \langle\, \mathcal{V}(i)\, f_I \circ \Psi(0)\,\rangle_{\mathrm{UHP}}$$

with

$$f_I(z) = \tan\Big(\,2\,\arctan z\,\Big) = \frac{2\,z}{1-z^2}\,.$$



These gauge-invariant operators have an interesting origin in open-closed string field theory.

A one-parameter family of formulations for open-closed bosonic string field theory were constructed, and it was observed that in a singular limit the action reduces to that of the cubic open bosonic string theory with an additional vertex which couples one off-shell open string field and one on-shell closed string field.

hep-th/9202015, Zwiebach

$$S = -\frac{1}{2}\langle\,\Psi, Q\Psi\,\rangle - \frac{1}{3}\langle\,\Psi, \Psi * \Psi\,\rangle + \langle\,V(\Phi), \Psi\,\rangle\,,$$

where Φ is the on-shell closed string field and $V(\Phi)$ is a map from a closed string field to an open string field.

The kinetic term of the closed string field is absent so that the resulting theory is no longer open-closed string field theory.

It is open string field theory and the on-shell closed string field can be regarded as a source for a set of gauge-invariant operators.

An important consequence from this relation of the gauge-invariant operators and open-closed string field theory is that Feynman diagrams for correlation functions of the gauge-invariant operators are given by Riemann surfaces containing holes with bulk punctures and the moduli space of such Riemann surfaces is covered.

Let us now consider the theory on N coincident D-branes. If we evaluate correlation functions of the gauge-invariant operators in the $1/N$ expansion, by construction it reproduces the closed-string perturbation theory with holes in the world-sheet we mentioned before.

Question 2

What quantities should we consider in open string field theory?

Answer

We should consider correlation functions of the gauge-invariant operators. They can be evaluated in the $1/N$ expansion in terms of Feynman diagrams of closed string theory with holes in the world-sheet.

However, Riemann surfaces associated with such Feynman diagrams contain at least one hole, and contributions from Riemann surfaces without any holes are missing.

In the context of on-shell scattering amplitudes, they are necessary for factorization.

Question 3

What do we lose in the missing Feynman diagrams?

Let us again recall the explanation of the AdS/CFT correspondence.

the D-brane description

open strings + closed strings
$\to \mathcal{N} = 4$ super Yang-Mills theory + free closed theory in ten dimensions

the three-brane description

closed strings near the horizon + closed strings away from the horizon
$\to$ superstring theory on $AdS_5 \times S^5$ + free closed theory in ten dimensions

For each of the two descriptions, there are two decoupled sectors in the low-energy limit.

One of them is a free theory in ten dimensions from the closed string sector, and we identified the two descriptions of the other sector.

In the low-energy limit, the missing contributions correspond to those of the free theory. In the low-energy limit, the contributions from open string field theory are exactly what we want.

<u>Question 3</u>

What do we lose in the missing Feynman diagrams?

<u>Answer</u>

Nothing in the low-energy limit!

To summarize, we claim that the evaluation of correlation functions of the gauge-invariant operators in the $1/N$ expansion can be interpreted as a closed string perturbation theory in the low-energy limit.

Therefore, if open string field theory for finite N is a consistent quantum theory, it provides a nonperturbative definition of closed string theory.

Question 4

Is open string field theory
a consistent quantum theory?

We do not expect open bosonic string field theory to be a consistent quantum theory in general.

It is interesting to consider topological strings. For example, three-dimensional Chern-Simons gauge theory can be formulated as open string field theory. hep-th/9207094, Witten

The duality in the B-model topological string theory is also discussed recently. arXiv:1812.09257, Costello and Gaiotto

It is also interesting to consider the Kontsevich model in this context.
hep-th/0312196, Gaiotto and Rastelli

On the other hand, open superstring field theory can be a consistent quantum theory.

While the action of open superstring field theory involving the Ramond sector had not been constructed for many years, this problem was recently overcome and we now have several formulations of open superstring field theory which are complete at the classical level.

arXiv:1508.00366, Kunitomo and Okawa
arXiv:1508.05387, Sen
arXiv:1602.02582, Erler, Okawa and Takezaki
arXiv:1602.02583, Konopka and Sachs

We consider that the formulations of open superstring field theory need to be developed further and it is an important question to address whether or not open superstring field theory is consistent as a quantum theory.

At the same time, however, we consider that we are in a position to discuss how we use open superstring field theory to understand the mechanism which realizes the AdS/CFT correspondence. This is one of the main messages of the talk.

Question 4

Is open string field theory a consistent quantum theory?

Answer

Open string field theory for the topological string or the non-critical string can be a consistent quantum theory. Open superstring field theory can also be a consistent quantum theory, which motivates us to extend our discussion to the superstring.

Question 5

Can we make sense of the path integral of open string fields?

As we mentioned before, we do not consider the path integral of closed string field theory to be promising for a nonperturbative definition of closed string theory.

On the other hand, there will be a better chance of making sense of the path integral of open string field theory. However, it may still be difficult because the open string field contains infinite component fields.

Actually, we define closed string theory by taking the low-energy limit of open string field theory so that we can in principle integrate out the massive fields of open string field theory.

In general, the resulting theory in terms of the massless fields will be very complicated, but if we can identify a theory in the same universality class, we can use it to define closed string theory nonperturbatively.

In the case of D3-branes, for example, the resulting theory will be equivalent to $\mathcal{N} = 4$ super Yang-Mills theory in the low-energy limit.

As we mentioned before, it is difficult to see the world-sheet picture in $\mathcal{N} = 4$ super Yang-Mills theory, but the theory after integrating out the massive fields by construction reproduces the world-sheet structure because we keep track of the relation to closed string theory with holes in the world-sheet.

This can be a promising way for proving the AdS/CFT correspondence.

Puzzling?

The gauge-invariant operator is a linear functional of the open string field and apparently it does not look like operators which couple to closed strings such as the energy-momentum tensor.

In the process of integrating out the massive fields, couplings of the closed string and multiple open string fields are generated, and the gauge-invariant operators in terms of the massless fields will resemble single-trace operators of $U(N)$ gauge theories in the low-energy limit.

Kouyama, Okawa and Suzuki, *work in progress*

Question 5

Can we make sense of the path integral of open string fields?

Answer

We can integrate out the massive fields to obtain a theory in terms of the massless fields. If we can identify a theory in the same universality class, we can use it to define closed string theory nonperturbatively.

Summary

- We want to have a consistent formulation of quantum gravity.
- For this purpose we want to define closed string theory nonperturbatively.
- Instead of closed string theory without holes in the world-sheet, we can consider closed string theory with holes in the world-sheet as a consistent perturbation theory including gravity.
- Instead of scattering amplitudes of open strings, we consider gauge-invariant operators in open string field theory.
- The $1/N$ expansion of correlation functions of gauge-invariant operators in open bosonic string field thoery on N D-branes reproduces this perturbation theory.
- We hope that this can be extended to open superstring field theory and it provides a nonperturbative formulation of quantum gravity.