Gravitational perturbations as $T\bar{T}$-deformations in 2D dilaton gravity systems

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Introduction

$T\bar{T}$-deformation : a irrelevant deformation of 2d QFT

[Yoshida-san's talk]

- As a deformation of the boundary CFT

cut-off AdS_3 / TT-deformed CFT_2

[Lauren McGough, Márk Mezei, Herman Verlinde, 1611.03470]
[Amit Giveon, Nissan Itzhaki, David Kutasov, 1701.05576]

- As a deformation of the bulk theory

The main issue of this talk

Introduction

flat-space JT gravity

$$S_{\text{matter}} + \frac{1}{16\pi G_N}\int d^2x\,\sqrt{-g}\,\left[\phi R - \Lambda\right]$$

$$\underset{G_N\sim 0}{\sim} S^{(0)} - 16\pi G_N \lambda \int d^2x\,\left[\,t_{\mu\nu}t^{\mu\nu} - (t^{\mu}_{\mu})^2\,\right]$$

The quadratic action can be regarded as a $T\overline{T}$ deformation of the original matter action

[Dubovsky, Gorbenko, Mirbabayi, 1706.06604]

Plan of this talk

0. Introduction

1. Gravitational perturbations around flat space

2. Gravitational perturbations around a curved b.g.

3. Summary and future direction

The JT gravity [Jackiw, 1985] [Teitelboin, 1983]

$$S_{\rm dg}[g_{\mu\nu},\phi] = \frac{1}{16\pi G_N}\int {\rm d}^2x\,\sqrt{-g}\,\left[\phi\,R - \Lambda + \frac{2}{L^2}\phi\right]$$

- A curvature : $R = -\dfrac{2}{L^2}$ Asymptotic AdS_2
- A candidate of the gravity dual of 1dim QM (SYK model)

[Almheili, Polchinski, 1402.6334]

[Maldacena, Stanford, Yang , 1606.01857]

[Engelsöy , Mertens, Verlinde, 1606.03438]

The flat-space limit of the JT gravity

[Dubovsky, Gorbenko, Mirbabayi, 1706.06604]

Let us consider $L \to \infty$ limit and add arbitrary matters ψ .

$$S[g_{\mu\nu},\, \phi,\, \psi] = S_{\text{matter}}[\psi,\, g_{\mu\nu}] + \frac{1}{16\pi G_N} \int \mathrm{d}^2 x \sqrt{-g}\, [\, \phi\, R - \Lambda\,]$$

Note : The integrability (or conformal symmetry) is not required.

eom

$$R = 0 \qquad\qquad \frac{1}{2} g_{\mu\nu}\Lambda - \left(\nabla_\mu \nabla_\nu \phi - g_{\mu\nu}\nabla^2\phi\right) = 8\pi G_N\, T_{\mu\nu}$$

vacuum solution ($T_{\mu\nu} = 0$ **)**

$$\mathrm{d}^2 s = -2\mathrm{d}x^+ \mathrm{d}x^- \qquad\qquad \overline{\phi} = \frac{\Lambda}{2} x^+ x^-$$

Gravitational perturbations around flat space

A gravitational perturbation is described as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \phi = \bar{\phi} + \sigma \qquad T_{\mu\nu} = 0 + t_{\mu\nu}$$

eom for the fluctuations

$$\partial^\nu(\partial^\mu h_{\mu\nu} - \partial_\nu h) = 0$$

$$\partial_\mu\partial_\nu\sigma = -8\pi G_N\,(t_{\mu\nu} - \eta_{\mu\nu}t^\rho_\rho) - \frac{1}{2}\Lambda\, h_{\mu\nu} + \frac{1}{2}\partial^\rho\bar{\phi}\,(\partial_\mu h_{\rho\nu} + \partial_\nu h_{\rho\mu} - \partial_\rho h_{\mu\nu})$$

solution of the fluctuation of the metric

$$h_{\mu\nu} = -16\pi G_N(t_{\mu\nu} - \bar{g}_{\mu\nu}t^\rho_\rho)\,k$$

solution of the fluctuation of the dilaton

$$\sigma_{\text{non-local}} = 4\pi G_N \Bigg[k\Lambda \int_0^{x^+} \mathrm{d}s\, s\, t_{++}(s, x^-) + k\Lambda \int_0^{x^-} \mathrm{d}s\, s\, t_{--}(x^+, s)$$

$$-2\,(k\Lambda - 1) \int_0^{x^+} \mathrm{d}s \int_0^{x^-} \mathrm{d}s'\, t_{+-}(s, s')$$

$$+ (k\Lambda - 2) \left(\int_{u_1^+}^{x^+} \mathrm{d}s \int_{u_2^+}^{s} \mathrm{d}s'\, t_{++}(s', 0) + \int_{u_1^-}^{x^-} \mathrm{d}s \int_{u_2^-}^{s} \mathrm{d}s'\, t_{--}(0, s') \right) \Bigg]$$

[Ishii, S.O. , Sakamoto, Yoshida, 1906.03865]

Comment : originally Dubovsky etc. didn't give the explicit solution.

$$S[g_{\mu\nu}, \phi, \psi] = S^{(0)} + S^{(1)} + S^{(2)} + \cdots$$

eom

$$S^{(2)} = -16\pi G_N \left(\frac{1}{2k} - \frac{\Lambda}{8} \right) k^2 \int \mathrm{d}^2 x \left[t_{\mu\nu} t^{\mu\nu} - (t^\mu_\mu)^2 \right]$$

[Dubovsky, Gorbenko, Mirbabayi, 1706.06604] for $k = \frac{2}{\Lambda}$

[Ishii, S.O. , Sakamoto, Yoshida, 1906.03865] for arbitrary k

The quadratic action can be regarded

as a TT deformation of the original matter action

The dressing factor of the S-matrix

[Dubovsky, Gorbenko, Mirbabayi, 1706.06604]

In $k = 0$ case,

$$h_{\mu\nu} = 0 \qquad \partial_\mu \partial_\nu \sigma = -8\pi G_N \left(t_{\mu\nu} - \eta_{\mu\nu}\, t^\sigma_\sigma \right)$$

Dynamical coordinates are defined as

$$X^\mu \equiv -\frac{2}{\Lambda}\partial^\mu \phi = x^\mu + Y^\mu$$

↓

$$Y^\mu = -\frac{2}{\Lambda}\partial^\mu \sigma \; = \frac{16\pi G_N}{\Lambda}\epsilon^{\mu\nu} \left(2 \int_{-\infty}^{x} \mathrm{d}x \, t_{t\mu}(t,x) - P_\mu \right)$$

The dressing factor of the S-matrix

Before introducing the gravity,
any matter field can be decomposed in the asymptotic region.

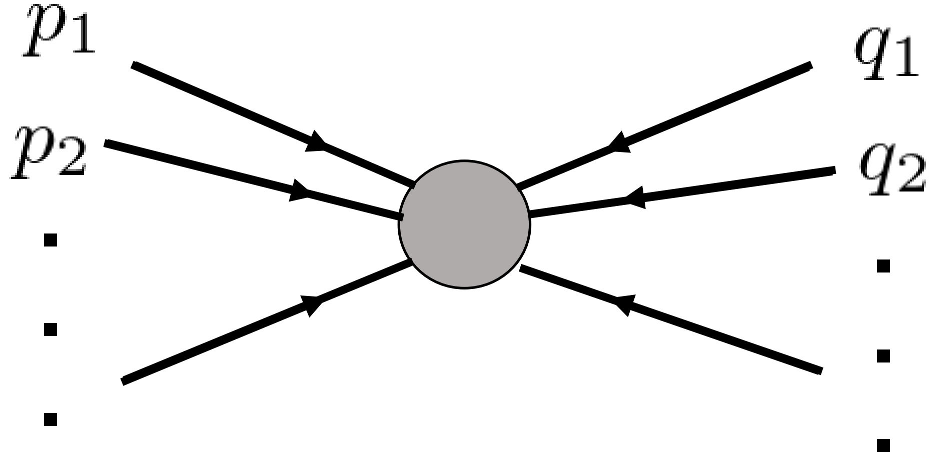
$$\psi = \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{\sqrt{2\pi}} \frac{1}{\sqrt{2E}} \left(a_{in}^{\dagger}(p) \mathrm{e}^{-ip_\alpha x^\alpha} + h.c. \right)$$

To describe the effect of the gravity,
it is needed to define $A_{in}^{\dagger}(p_i)$ by using the dynamical coordinates.

$$A_{in}^{\dagger}(p) = a_{in}^{\dagger}(p) \mathrm{e}^{ip_\alpha Y^\alpha}$$

$$[A_{in}^{\dagger}(p),\, A_{in}^{\dagger}(p')] = 0$$

The dressing factor of the S-matrix



The dressed asymptotic states are evaluated as

$$|\{p_i\},\, in\rangle_{dressed} = \Pi\, A^\dagger_{in}(p_i)|0\rangle = \exp\left(-i\,\frac{16\pi G_N}{\Lambda}\sum_{i<j} p_i * p_j\right)|\{p_i\},\, in\rangle$$

$$|\{q_i\},\, out\rangle_{dressed} = \Pi\, A^\dagger_{out}(q_i)|0\rangle = \exp\left(i\,\frac{16\pi G_N}{\Lambda}\sum_{i<j} q_i * q_j\right)|\{q_i\},\, out\rangle$$

The dressing factor of the S-matrix

$$\hat{S}_{dressed} = {}_{dressed}\langle \{q_i\},\, out | \{p_i\},\, in \rangle_{dressed}$$

$$= \hat{S} \exp\left(-i\, \frac{16\pi G_N}{\Lambda} \sum_{i<j} p_i * p_j - i\, \frac{16\pi G_N}{\Lambda} \sum_{i<j} q_i * q_j \right)$$

The gravitational dressing factor

the modification of the CDD factor in the case of IQFT

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Gravitational perturbations around a curved b.g.

[Ishii, S.O. , Sakamoto, Yoshida, 1906.03865]

Starting from a general dilaton gravity coupled with arbitrary matters

$$S[g_{\mu\nu},\, \phi,\, \psi] = S_{\text{matter}}[\psi,\, g_{\mu\nu}] + \frac{1}{16\pi G_N}\int \mathrm{d}^2x\,\sqrt{-g}\,\left[\,\phi\, R - U(\phi)\right]$$

eom $$R - U'(\phi) = 0$$

$$\frac{1}{2} g_{\mu\nu} U(\phi) - \left(\nabla_\mu \nabla_\nu \phi - g_{\mu\nu}\nabla^2\phi\right) = 8\pi G_N\, T_{\mu\nu}$$

A gravitational perturbation is described as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \phi = \bar{\phi} + \sigma \qquad T_{\mu\nu} = 0 + t_{\mu\nu}$$

eom for the fluctuations

$$\bar{\nabla}^{\mu}\bar{\nabla}^{\nu}h_{\mu\nu} - \bar{\nabla}^2 h - \frac{1}{2}U'(\bar{\phi})\,h - \underline{U''(\bar{\phi})\sigma} = 0$$

$$\bar{\nabla}_{\mu}\bar{\nabla}_{\nu}\sigma + \frac{1}{2}\bar{g}_{\mu\nu}\,U'(\bar{\phi})\,\sigma$$
$$= -8\pi G_N\left(t_{\mu\nu} - \bar{g}_{\mu\nu}t^{\rho}_{\rho}\right) - \frac{1}{2}U(\bar{\phi})\,h_{\mu\nu} + \frac{1}{2}\bar{\nabla}^{\rho}\bar{\phi}\left(\bar{\nabla}_{\mu}h_{\rho\nu} + \bar{\nabla}_{\nu}h_{\rho\mu} - \bar{\nabla}_{\rho}h_{\mu\nu}\right)$$

In the general potential case, we cannot solve the equations…

Assuming $\begin{cases} U''(\phi) = 0 \\ t^{\mu}_{\mu} = 0 \end{cases}$ e.g. $U(\phi) = \Lambda - \frac{2}{L^2}\phi$

JT gravity (AdS_2)

A solution to the eom is $h_{\mu\nu} = -k \cdot 16\pi G_N\, t_{\mu\nu}$

$$S^{(2)} = (16\pi G_N)\,\frac{k^2}{8}\int \mathrm{d}^2x\,\sqrt{-\bar{g}}\left(U(\bar{\phi}) - \frac{4}{k}\right)\,t_{\mu\nu}t^{\mu\nu}$$

Comments on the solution of the JT gravity

$$\sigma_{\mathrm{non-local}} = \frac{1}{x^+ - x^-}\left[I^+(x^+, x^-) - I^-(x^+, x^-)\right]$$

$$I^\pm(x^+, x^-) \equiv 8\pi G_N \int_{u^\pm}^{x^\pm} \mathrm{d}s\,(s - x^+)(s - x^-)\,\mathcal{T}_\pm(s)$$

$$\mathcal{T}_\pm(x^\pm) \equiv (1 \mp b\,k \mp c\,k\,x^\pm)\,t_{\pm\pm} \mp \frac{k}{4}\left(a + 2b\,x^\pm + c\,(x^\pm)^2\right)\partial_\pm t_{\pm\pm}$$

Comment :

If $t^\mu_\mu = 0$, trace of eom is equivalent to the massive Klein-Gordon eq.

$$\left(\bar{\nabla}^2 - \frac{2}{L^2}\right)\sigma = 0$$

Local solutions (hypergeometric functions or Gegenbauer polynomials) don’t satisfy another constraints.

Summary

Under some conditions (including AdS_2 and dS_2 cases),

gravity perturbations in 2d dilaton gravity system can be regarded

as a $T\overline{T}$-deformation of the original matter action.

Future direction

- Evaluating the S-matrix and the partition function

  The application of AdS_2 holography

- Relaxing the conditions

  Non-conformal matter, more general types of dilaton potential