Quasilocal Smarr Relations

in collaboration with
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[1] arXiv:1809.07259 for AF (ver2 will be updated), arXiv:190X.XXXXX for AAdS

Content

Conserved charge in curved spacetime

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But some methods have been developed such as ADM method, Komar method, AD(T) method, and etc.

Smarr relation by Larry Smarr in 1973 [2]

$$ds^2_{RN} = \left(1 - \frac{2Mr - Q^2}{R^2}\right)du^2 + 2dudr + 2\frac{a\sin^2\theta}{R^2}(2Mr - Q^2)dud\phi - 2a\sin^2\theta drd\phi$$
$$- R^2 d\theta^2 + \frac{\sin^2\theta}{R^2}\left(\Delta a^2\sin^2\theta - (a^2 + r^2)^2\right)d\phi^2,$$
$$R^2 \equiv r^2 + a^2\cos^2\theta, \qquad \Delta \equiv r^2 + a^2 - 2Mr + Q^2, \qquad a \equiv \frac{J}{M}$$

The black hole's area is written as

$$S = 4\pi\left[2M^2 + 2(M^4 - J^2 - M^2Q^2)^{\frac{1}{2}} - Q^2\right],$$

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$$dM = TdS + \Omega dJ + \Phi dQ \implies M = 2TS + 2\Omega J + \Phi Q$$

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Euler theorem states that if a function $f(x, y, z)$ obeys the scaling relation

$$f(\alpha^p x, \alpha^q y, \alpha^k z) = \alpha^r f(x, y, z), \tag{1}$$

then it satisfies

$$rf(x, y, z) = p\left(\frac{\partial f}{\partial x}\right) x + q\left(\frac{\partial f}{\partial y}\right) y + k\left(\frac{\partial f}{\partial z}\right) z. \tag{2}$$

M in terms of S, L and Q satisfies this relation having a following scaling

$$M \propto [L], \quad S \propto [L]^2, \quad J \propto [L]^2, \quad Q \propto [L], \tag{3}$$

then the Euler's theorem yields

$$M = \left(\frac{\partial M}{\partial S}\right) S + \left(\frac{\partial M}{\partial J}\right) J + \left(\frac{\partial M}{\partial Q}\right) Q. \tag{4}$$

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Differential form is interpreted as the first law of black hole thermodynamics

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Phi_H \delta Q$$

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and also

- **Smarr relation**

Quasilocal Formalism

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1. Approaching infinity is not always an appropriate theoretical idealization, and is never satisfied in reality

2. black hole thermodynamics in a quasilocal frame can be applied to gravitational and matter fields within a bounded, finite spatial region

3. Intrinsically, thermodynamics for AdS black hole should be studied at a quasilocal frame, becuase

$$T_{\mathrm{Hawking}}(|-\xi^{\mu}\xi_{\mu}|^{1/2}=1) \tag{5}$$

$$|-\xi^{\mu}\xi_{\mu}|^{1/2}\neq 1 \quad r\to\infty \quad \text{for AAdS} \tag{6}$$

$$\text{because} \quad N(r)\sim r^2, \quad r\to\infty \tag{7}$$

.

Black Hole Thermodynamics

- For electrically charged and AF

$$dM = T_{\mathrm{Hawking}} dS + \Phi dQ,$$
$$M = 2TS + \Phi Q$$

- For electrically charged and AAdS

$$dM = T_{\mathrm{Hawking}} dS + \Phi dQ + VdP,$$
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How they will change in a quasilocal frame????

$$M?, \quad T_{\rm Hawking}?, \quad S?, \quad \Phi?, \quad Q? \tag{8}$$

Tolman Temperature

Hawking Temperature

$$T_{\rm Hawking} = \frac{1}{2\pi}\kappa \quad (9)$$

Tolman Temperature

$$T_{\rm Tolman} = \frac{1}{N(R)} T_{\rm Hawking} \quad (10)$$

Brown-York Quasilocal Formalism

- The renormalized gravity action

$$S_{\text{renormalized}} = S_{\text{EH}} + S_{\text{GH}} + S_{\text{MM-counterterm}}$$
$$= \frac{1}{16\pi G}\int_M d^4x\sqrt{-g}R + \frac{\epsilon}{8\pi G}\int_{\partial M} d^3x\sqrt{-h}\left(K - \hat{K}\right)$$

- The energy-momentum boundary stress tensor

$$\tau^{ab} = \frac{2}{\sqrt{-h}}\frac{\delta S_{\text{renormalized}}}{\delta h_{ab}} = -\frac{2}{\sqrt{-h}}\left(\pi^{ab} - \hat{\pi}^{ab}\right)$$

- The quasilocal quantities

$$\varepsilon = u_a u_b \tau^{ab},$$
$$j_i = -\sigma_{ia} u_b \tau^{ab},$$
$$s^{ij} = \sigma^i_a \sigma^j_b \tau^{ab}$$

where σ_{ij} is a two-dimensional induced metric.

Entropy in QLF from Euclidean Formulation

Define quasilocal free energy

$$F_R \equiv T_{\mathrm{R}} I_{E,(r_h,R)} \tag{11}$$

for example,

$$I_E = -\int_{\mathcal{M}} \sqrt{g}\left(\frac{1}{16\pi G}R - \frac{1}{4}F^2\right) - \int_{\partial\mathcal{M}} \sqrt{h}F^{\mu\nu}n_\mu\mathcal{A}_\nu - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} \sqrt{h}(K - \hat{K})$$

Entropy in QLF from Euclidean Formulation

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quasilocal entropy

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Electric charge and potential

$$\Phi_{\rm R}(r) = \mathcal{A}_\mu(r) u^\mu(r)\Big|_R^{r_h}, \qquad Q_{\rm M} = \int F. \tag{13}$$

Construction Smarr Relation at a quasilocal frame

How can we find Smarr Relation at a quasilocal frame???

QL Smarr Relation for AF by Euler Theorem

Euler theorem says

$$E \propto [L]^{D-3}, \quad S \propto [L]^{D-2}, \quad A \propto [L]^{D-2}, \quad Q \propto [L]^{D-3},$$

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$$(D-3)E = (D-2)\left(\frac{\partial M}{\partial S}\right) S - (D-2)\left(\frac{\partial M}{\partial A}\right) A + (D-3)\left(\frac{\partial M}{\partial Q}\right) Q. \tag{14}$$

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The first law of thermodynamics for charged black holes is

$$dE = TdS - PdA + \Phi Q$$

Example : Charged BH in *n*-dimensions

$$ds^2 = -N(r)^2 dt^2 + h(r)^2 dr^2 + r^2 d\Omega_{n-2}, \quad N(r) = \frac{1}{h(r)} = \sqrt{1 - \frac{\mu}{r^{n-3}} + \frac{q^2}{r^{2(n-3)}}}$$

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- Tolman temperature : $T_{\rm Tolman} = \frac{1}{N(R)} T_{\rm Hawking}$
- Entropy : $S = \frac{\omega_{n-2} r_h^{n-2}}{4G}$
- quasilocal area : $A = \omega_{n-2} R^{n-2}$
- electric charge : $Q = \pm\sqrt{\frac{(n-2)(n-3)}{8\pi}}\, q\,\omega_{n-2}$
- electric potential : $\Phi(R) = \frac{1}{N(R)}\sqrt{\frac{1}{8\pi}\frac{n-2}{n-3}}\left(\frac{q}{r_h^{n-3}} - \frac{q}{R^{n-3}}\right)$

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From the Brown-York formalism with the background subtraction method,

- Quasilocal energy : $E = \frac{(n-2)}{8\pi} S_{n-2} R^{n-3}\left[1 - \sqrt{1 - \frac{\mu}{r^{n-2}} + \frac{q^2}{r^{2(n-3)}}}\right]$
- surface pressure : $p = \frac{(n-3)}{8\pi r}\left[\frac{1}{\sqrt{1 - \frac{\mu}{r^{n-2}} + \frac{q^2}{r^{2(n-3)}}}}\left(1 - \frac{\mu}{2r^{n-3}}\right) - 1\right]\Bigg|_{r=R}$

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Example : Charged BH in *n*-dimensions

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The Smarr relation is satisfied

$$(n-3)E = (n-2)TS + (n-3)\Phi_R Q + (n-2)PA$$

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The cosmological constant is identified as the pressure

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- Tolman temperature : $T_{\text{Tolman}} = \frac{1}{N(R)} T_{\text{Hawking}}$
- Entropy : $S = \frac{\omega_{n-2} r_h^{n-2}}{4G}$
- quasilocal area : $A = \omega_{n-2} R^{n-2}$
- electric charge : $Q = \pm\sqrt{\frac{(n-2)(n-3)}{8\pi}} q\omega_{n-2}$
- The electric potential : $\Phi_R = \frac{1}{N(R)}\sqrt{\frac{32\pi(n-2)}{(n-3)}}\left(\frac{q}{R^{n-3}} - \frac{q}{r_h^{n-3}}\right)$
- pressure : $\mathcal{P} = \frac{(n-1)(n-2)}{16\pi l^2}$

Example : Charged BH in n-dimensions

From the Brown-York formalism with the background subtraction method,

- Quasilocal energy : $E = \frac{2(n-2)}{\kappa}\omega_{n-2}R^{n-3}\Big(N(R) - N_0(R)\Big)$
- surface pressure :
  $p = \frac{-l^2(n-3)\big(kr_h^n\big(R^3 r_h^{n-2} r_h^3 R^n\big)+q^2 R^3 r_h^6\big)-R^2 r_h^{n+2}\big((n-3)Rr_h^{n-2}(n-2)r_h R^n\big)}{\kappa l^2 r_h^{n-3} R^{n-1} N(R)}$
- The thermodynamic volume : $V = \frac{\partial E}{\partial \mathcal{P}} = \frac{16\pi\omega_{n-2}}{(n-1)\kappa}\left(\frac{(R^{n-1}-r_h^{n-1})}{N(R)} - \frac{R^{n-1}}{N_0(R)}\right)$

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Conclusion and Future works

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- Interpretation of thermodynamic volume for AdS Black holes