Topological order in the color-flavor locked phase of (3+1)-dimensional $U(N)$ gauge-Higgs system

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Based on Y. Hidaka, Y Hirono, M. Nitta, Y. Tanizaki, RY,

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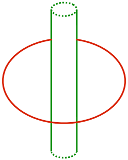
Overview of this talk



$$= \exp \frac{2\pi i k}{3k+1} \in \mathbb{Z}_{3k+1}$$

(for $N = 3$)

Color-flavor locked phase of a $U(N)$ gauge theory with N-Higgs fields is **topologically ordered** if the Higgs fields have non-trivial $U(1)$ charge k.

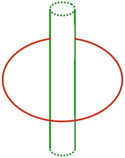
- Non-Abelian vortex and Wilson loop have a $\mathbb{Z}_{Nk+1}$ fractional linking phase.
- There are $\mathbb{Z}_{Nk+1}$ **1- and 2-form symmetries**, and both of them are spontaneously broken.

Higgs phase of gauge theories

- Massive gauge fields
- Some Nambu–Goldstone (NG) bosons are eaten
- Admitting extended objects e.g. vortex
- Vortex in many contexts:
  - magnetic vortex in superconductor (SC),
  - (local) cosmic strings in cosmology

Higgs phases can be further classified by “topological order”

Topological order [Wen '89, '91]



$$= \exp \frac{2\pi i}{k} \in \mathbb{Z}_k$$

Classification of phases by topology of non-local order parameters

- Order parameters: Wilson loop, vortex surface,...
- New classification e.g. SC $\neq$ charge 1 Abelian Higgs

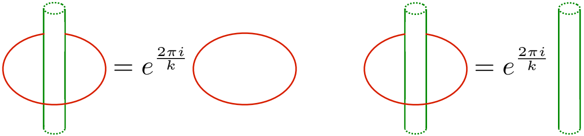
A characterization of the topologically ordered phase

1. Non-zero VEV of non-local order parameters
2. Fractional linking phases between non-local operators

Is topological ordered phase related to symmetry breaking?

p-form global symmetries & their breaking

[Banks & Seiberg '10; Kapustin & Seiberg '14; Gaiotto et al. 14]



Symmetry under transf. of p-dim. extended objects

- Charged objects: Wilson loop, vortex surface, ...

  $\rightarrow$ topological order parameters can be charged objects

- Example: phase rotation of a Wilson loop (1-form sym.)

- Symmetry breaking: non-zero VEV of charged objects

Topological order & p-form symmetry

[Banks & Seiberg '10; Kapustin & Seiberg '14; Gaiotto et al. 14]

Topologically ordered phase can be characterized by

1. p-form symmetry and their breaking
2. Fractional linking phases between non-local operators

In this talk, we consider the possibility of topologicaly ordered phase in

$3+1$ dim. non-Abelian gauge theories

in order to understand phases of non-Abelian gauge theories.

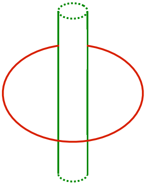
Topological order in Abelian Higgs model

Review based on

Hansson, et al. 04,; Banks & Seiberg '10;

Seiberg & Kapustin '14; Gaiotto, et al. '14

Message



$$= \exp \frac{2\pi i}{k} \in \mathbb{Z}_k$$

Abelian Higgs models can be classified by topological order.

- Order parameters are non-local: Wilson loop & Vortex surface.
- Fractional linking phase of extended objects
- Symmetry breaking: 1- & 2-form symmetries
- E.g. (s-wave) superconductor $\neq$ charge 1 Higgs model

In the following, we consider the low energy limit with no local excitations

BF-theory as a dual of Abelian Higgs model [Horowitz '89; Blau & Tompson '89]

Low energy ($\ll v$) limit of Abelian Higgs model is given by

Stückelberg action $\overset{\text{dual}}{\longleftrightarrow}$ BF-action

$$\frac{v^2}{2}\int |d\chi - kA|^2 \overset{\text{dual}}{\longleftrightarrow} \frac{ik}{2\pi}\int B \wedge dA \text{ or } \frac{ik}{2\pi}\int d^4x \epsilon^{mnpq} B_{mn}\partial_p A_q$$

- k: charge of Higgs field
- A: $U(1)$ 1-form gauge field
- NG boson $\chi \overset{\text{dual}}{\longleftrightarrow}$ 2-form gauge field B
- B can be coupled with a magnetic vortex $\int_{\mathcal{S}} B$ ($\mathcal{S}$: worldsheet of vortex)
- No local excitations in both theories

How about observables and correlation functions in BF-theory?

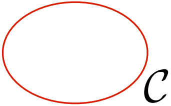
Observables and correlation functions in BF-theory

[Horowitz & Srednicki '89; Oda & Yahikozawa '89]

Observables: non-local, gauge invariant

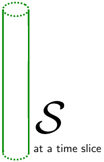
Wilson loop

$$W(\mathcal{C}) = e^{i\int_{\mathcal{C}} A}$$



Surface of vortex

$$V(\mathcal{S}) = e^{i\int_{\mathcal{S}} B}$$

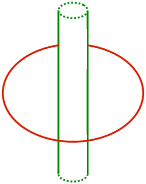


Correlation functions are topological: linking number $\mathrm{link}\,(\mathcal{C},\mathcal{S})$ & $\mathbb{Z}_k$

- $\langle W(\mathcal{C})V(\mathcal{S})\rangle$

$$= e^{\frac{2\pi i}{k}\,\mathrm{link}\,(\mathcal{C},\mathcal{S})}\,\langle W(\mathcal{C})\rangle = e^{\frac{2\pi i}{k}\,\mathrm{link}\,(\mathcal{C},\mathcal{S})}\,\langle V(\mathcal{S})\rangle$$

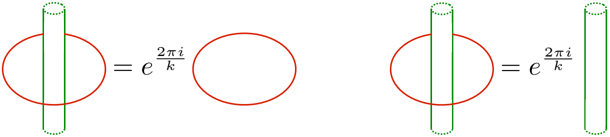
$$= \exp(\frac{2\pi i}{k}\,\mathrm{link}\,(\mathcal{C},\mathcal{S}))$$



- $\langle W(\mathcal{C})\rangle = \langle V(\mathcal{S})\rangle = 1$

1- and 2-form symmetries exist, and are broken spontaneously.

1- & 2-form symmetries [Kapustin & Seiberg '14; Gaiotto, et al. '14]



Symmetry under transf. of 1- & 2-dim. objects

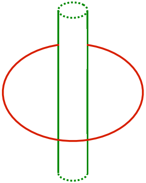
	Group	Object	Generator	Transf.
1-form	$\mathbb{Z}_k$	$W(\mathcal{C})$	$V(\mathcal{S})$	$\langle V(\mathcal{S})W(\mathcal{C})\rangle = e^{\frac{2\pi i}{k}} \langle W(\mathcal{C})\rangle$
2-form	$\mathbb{Z}_k$	$V(\mathcal{S})$	$W(\mathcal{C})$	$\langle W(\mathcal{C})V(\mathcal{S})\rangle = e^{\frac{2\pi i}{k}} \langle V(\mathcal{S})\rangle$

- Symmetry breaking: $\langle W(\mathcal{C})\rangle \neq 0$, $\langle V(\mathcal{S})\rangle \neq 0$ (at large distance limit)

Notes

- ordinary symmetry is 0-form symmetry for particles
- BF-action is invariant under 1- & 2-form transf. (up to $2\pi\mathbb{Z}$).

Topological order in Abelian Higgs model [Hansson et al. '04]



$$= \exp \frac{2\pi i}{k} \in \mathbb{Z}_k$$

1. Existence of fractional linking phase

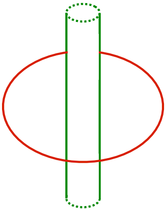
   $$\langle W(\mathcal{C}) V(\mathcal{S}) \rangle = \exp\left(\frac{2\pi i}{k} \operatorname{link}(\mathcal{C}, \mathcal{S})\right)$$

2. 1- & 2-form symmetries are broken spontaneously.

   - Order parameters = charged objects $W(\mathcal{C})$ & $V(\mathcal{S})$
   - Non-zero VEVs of order parameters: $\langle W(\mathcal{C}) \rangle = \langle V(\mathcal{S}) \rangle = 1$

E.g. superconductor ($\mathbb{Z}_2$) $\neq$ charge 1 Higgs system ($\mathbb{Z}_1 = 1$)

Summary of Abelian Higgs model



$$= \exp \frac{2\pi i}{k} \in \mathbb{Z}_k$$

Abelian Higgs models can be classified by topological order.

- Order parameters are Wilson loop & Vortex surface.
- Fractional linking phase of extended objects
- Symmetry breaking: 1- & 2-form symmetries

Topological order in CFL phase of $U(N)$ gauge-Higgs system

Y. Hidaka, Y Hirono, M. Nitta, Y. Tanizaki, RY,

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Topological order in non-Abelian Higgs models?

- Many symmetry breaking patterns
  - Some of gauge fields are massive, others are not.
  - Some NG boson may not be eaten.

    (cf. no topological order in CFL of QCD [Hirono & Tanizaki '18])
- Hints from Abelian case:
  - No local excitations in low-energy limit
  - Existence of extended objects

The hints suggest:

there may be a topologically ordered phase

in a $U(N)$ gauge theory with N-flavor Higgs fields

$U(N)$ gauge theory with N-flavor Higgs

[Hanany & Tong '03; Auzzi et al. '03; Gorsky et al. 04]

Action

$$\int \frac{1}{2g_1^2} \operatorname{tr} |F|^2 + \frac{1}{2g_2^2} |\operatorname{tr} F|^2 + |d\Phi - iA\Phi - ik \operatorname{tr}(A)\Phi|^2 + V(\Phi)$$

- Higgs fields: N(color) $\times$ N(flavor) matrix $\Phi = (\Phi_{cf})$
- Transf. law $\Phi \to (\det U_{\mathrm{col}})^k \, U_{\mathrm{col}} \, \Phi \, U_{\mathrm{flav}}^T$

  $U_{\mathrm{col}} \in U(N)_{\mathrm{col}}$, $U_{\mathrm{flav}} \in SU(N)_{\mathrm{flav}}/(\mathbb{Z}_N)_{\mathrm{flav}}$
- A: $U(N)$ 1-form gauge field

How about Higgs phase?

Higgs phase with non-Abelian vortex

[Hanany & Tong '03; Auzzi et al. '03; Gorsky et al. 04]

1. There is a Higgs phase with no massless excitation

   (# of gauge fields = # of Higgs fields)

   - VEV of Higgs can be diagonized $\langle\Phi\rangle = v\mathbf{1}_{N\times N}$
   - Color-flavor locked (CFL) phase:

     simultaneous color-flavor transf. remains

     (cf. QCD case [Alford et al. '98])

2. CFL phase Admits non-Abelian vortices $\Phi \sim \mathrm{diag}\,(e^{i\theta}, 1, ..., 1)$

   - Fractional $(1/N)$ magnetic flux

We will show that this CFL phase can be a topologically ordered phase.

$$\mathrm{diag}\,(e^{i\theta}, 1, ..., 1) = e^{i\theta/N}\,\mathrm{diag}\,(e^{i(N-1)\theta/N}, e^{-i\theta/N}, ..., e^{-i\theta/N}) \sim e^{i\theta/N}\mathbf{1}_{N\times N}$$

How to see topological order?

Conditions for the topological ordered phase

1. Existence of fractional linking phase,
2. Spontaneously broken 1- and 2-form symmetries

Procedure: similar to the case of Abelian Higgs model

1. Dual theory: $\int |d\Phi - iA\Phi - ik\,\mathrm{tr}\,(A)\Phi|^2 \quad \overset{\text{dual}}{\longleftrightarrow} \quad \frac{i}{2\pi} K_{iA} \int b_i \wedge da_A$
2. Fractional linking phase
3. 1- & 2-form symmetry breaking

In this talk, we consider $N = 3$ case for simplicity.

Low energy limit in CFL phase

Action is simplified in Abelian gauge: (cf. ['t Hooft '81])

Stückelberg action

$$\frac{v^2}{2}\int |d\phi_i - K_{iA}a_A|^2, \quad K_{iA} = \begin{pmatrix} k+1 & k & k \\ k & k+1 & k \\ k & k & k+1 \end{pmatrix}$$

- Abelian gauge: $\Phi = \frac{1}{\sqrt{2}} v \,\mathrm{diag}\left(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}\right) \in U(3)$
- a_A: Cartan of $U(3)$ gauge field
- K_{iA}: matrix of charges $(\det(K_{iA}) \neq 0)$

$$|d\phi_i - K_{iA}a_A|^2 = |d\phi_1 - (k+1)a_1 - ka_2 - ka_3|^2 + |d\phi_2 - ka_1 - (k+1)a_2 - ka_3|^2 + |d\phi_3 - ka_1 - ka_2 - (k+1)a_3|^2$$

Dual BF-type theory

The action

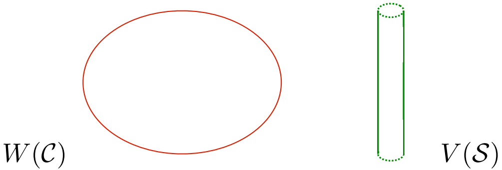
$$\frac{i}{2\pi} K_{iA} \int b_i \wedge da_A$$

NG boson $\phi_i \overset{\text{dual}}{\longleftrightarrow}$ 2-form b_i

Deriavtion

1. Original action $\frac{v^2}{2} \int |d\phi_i - K_{iA} a_A|^2$
2. First order action by adding 3-form H_{3i}
   $\frac{1}{8\pi^2 v^2} \int |H_{3i}|^2 + \frac{i}{2\pi} \int H_{3i} \wedge (d\phi_i - K_{iA} a_A)$
3. Eliminating ϕ_i by EOM: $dH_{3i} = 0 \to H_{3i} = db_i$
4. Take low-energy limit ($v \to \infty$)

Observables in $\frac{i}{2\pi} K_{iA} \int b_i \wedge da_A$

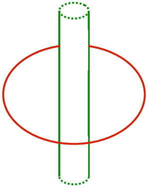


- Wilson loop: $W(\mathcal{C}) = \frac{1}{N} \operatorname{tr} \mathcal{P} e^{i \int_{\mathcal{C}} A} = \frac{1}{N} \sum_{A=1}^{N} e^{i \int_{\mathcal{C}} a_A}$
- Vortex surface operator: $V_i(\mathcal{S}) = e^{i \int_{\mathcal{S}} b_i}$

Observables depends on only topology

(No local DOF: $da_A = 0, db_i = 0$ by EOM)

CFL phase is topologically ordered!



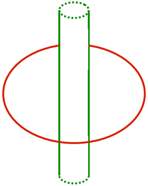
$= \exp \frac{2\pi i k}{3k+1} \in \mathbb{Z}_{3k+1}$

1. $\mathbb{Z}_{3k+1}$ fractional phase in correlation function

$$\langle W(\mathcal{C}) V_i(\mathcal{S}) \rangle = \exp\left(\frac{2\pi i k}{3k+1} \operatorname{link}(\mathcal{C}, \mathcal{S}) \right)$$

2. $\mathbb{Z}_{3k+1}$ 1- & 2-form symmetries & breaking
   - $\langle W(\mathcal{C}) V_i(\mathcal{S}) \rangle = \exp\left(\frac{2\pi i k}{3k+1} \operatorname{link}(\mathcal{C}, \mathcal{S}) \right) \langle W(\mathcal{C}) \rangle$
   - $\langle W(\mathcal{C}) V_i(\mathcal{S}) \rangle = \exp\left(\frac{2\pi i k}{3k+1} \operatorname{link}(\mathcal{C}, \mathcal{S}) \right) \langle V(\mathcal{S}) \rangle$
   - Non-zero VEVs $\langle W(\mathcal{C}) \rangle = \langle V_i(\mathcal{S}) \rangle = 1$

Some comments



$$= \exp \frac{2\pi i k}{3k+1} \in \mathbb{Z}_{3k+1}$$

- $U(1)$ charge k in $\Phi \to (\det U_{\rm col})^k \, U_{\rm col} \, \Phi \, U_{\rm flav}^T$ is important for the existence of topological order.
- Non-Abelian vortices in the previous research [Gorsky, et al. 04]: $k = 0$ case ($\mathbb{Z}_1$). No nontrivial topological order.
- Possible fractional phase is restricted by color N:
  e.g. $\mathbb{Z}_4, \mathbb{Z}_7, \mathbb{Z}_{11}, \ldots$ are allowed for $N = 3$ case.

Summary

CFL phase of a $U(N)$ gauge theory with N-Higgs fields is topologically ordered if the Higgs fields have non-trivial $U(1)$ charge k.

- Non-Abelian vortex and Wilson loop has $\mathbb{Z}_{Nk+1}$ fractional linking phase.
- There are $\mathbb{Z}_{Nk+1}$ 1- and 2-form symmetries, and both of them are spontaneously broken.

Future work: more general gauge group, numerical solutions of charge k non-Abelian vortex, adding θ-term,...

Appendix

Derivation of correlation function in BF-theory I

We begin with the following correlation function [Chen, et al. '15]

$$\langle W(\mathcal{C})V(\mathcal{S})\rangle = \mathcal{N}\int \mathcal{D}A\mathcal{D}B e^{\frac{ik}{2\pi}\int B\wedge dA + i\int A\wedge J_3(\mathcal{C}) + i\int B\wedge J_2(\mathcal{S})}$$

where $J_3(\mathcal{C})$ and $J_2(\mathcal{S})$ are delta function forms. They can be rewritten as

$$J_3(\mathcal{C}) = J_3(\partial\mathcal{S}(\mathcal{C})) = dJ_2(\mathcal{S}(\mathcal{C})), \quad J_2(\mathcal{S}) = J_2(\partial\mathcal{V}(\mathcal{S})) = -dJ_1(\mathcal{V}(\mathcal{S})),$$

where $\mathcal{S}(\mathcal{C})$ and $\mathcal{V}(\mathcal{S})$ are boundaries of $\mathcal{C}$ and $\mathcal{S}$:

$$\partial\mathcal{S}(\mathcal{C}) = \mathcal{C}, \qquad \partial\mathcal{V}(\mathcal{S}) = \mathcal{S}$$

Derivation of correlation function in BF-theory II

By the redefinition $A \to A + \frac{2\pi}{k} J_1(\mathcal{V}(\mathcal{S}))$, we obtain

$$\langle V(\mathcal{S}) W(\mathcal{C}) \rangle = \mathcal{N} \int \mathcal{D}A \mathcal{D}B \, e^{\frac{ik}{2\pi} \int B \wedge d(A - \frac{2\pi}{k} J_1(\mathcal{V}(\mathcal{S}))) + i \int A \wedge J_3(\mathcal{C})}$$

$$= \mathcal{N} \int \mathcal{D}A \mathcal{D}B \, e^{\frac{ik}{2\pi} \int B \wedge dA + i \int A \wedge J_3(\mathcal{C}) + \frac{2\pi i}{k} \int J_1(\mathcal{V}(\mathcal{S})) \wedge J_3(\mathcal{C})}$$

$$= e^{-\frac{2\pi i}{k} \mathrm{link}\,(\mathcal{C}, \mathcal{S})} \langle W(\mathcal{C}) \rangle \, .$$

By the redefinition $B \to B - \frac{2\pi}{k} J_2(\mathcal{S}(\mathcal{C}))$, we also obtain

$$\langle V(\mathcal{S}) W(\mathcal{C}) \rangle = \mathcal{N} \int \mathcal{D}A \mathcal{D}B \, e^{\frac{ik}{2\pi} \int (B + \frac{2\pi}{k} J_2(\mathcal{S}(\mathcal{C}))) \wedge dA + i \int B \wedge J_2(\mathcal{S})}$$

$$= \mathcal{N} \int \mathcal{D}A \mathcal{D}B \, e^{\frac{ik}{2\pi} \int B \wedge dA + i \int B \wedge J_2(\mathcal{S}) - \frac{2\pi i}{k} \int J_2(\mathcal{S}(\mathcal{C})) \wedge J_2(\mathcal{S})}$$

$$= e^{-\frac{2\pi i}{k} \mathrm{link}\,(\mathcal{C}, \mathcal{S})} \langle V(\mathcal{S}) \rangle$$

Derivation of correlation function in BF-theory III

We can similarly show $\langle V(\mathcal{S})\rangle = 1$ and $\langle W(\mathcal{C})\rangle = 1$ as follows:

$$\begin{aligned}\langle V(\mathcal{S})\rangle =& \mathcal{N}\int \mathcal{D}A\mathcal{D}B\, e^{\frac{ik}{2\pi}\int B\wedge d(A-\frac{2\pi}{k}J_1(\mathcal{V}(\mathcal{S})))} \\ =& \mathcal{N}\int \mathcal{D}A\mathcal{D}B\, e^{\frac{ik}{2\pi}\int B\wedge dA} = 1,\end{aligned}$$

and

$$\begin{aligned}\langle W(\mathcal{C})\rangle =& \mathcal{N}\int \mathcal{D}A\mathcal{D}B e^{\frac{ik}{2\pi}\int (B+\frac{2\pi}{k}J_2(\mathcal{S}(\mathcal{C})))\wedge dA} \\ =& \mathcal{N}\int \mathcal{D}A\mathcal{D}B e^{\frac{ik}{2\pi}\int B\wedge dA} = 1.\end{aligned}$$

By using there relations, we also obtain

$$\langle W(\mathcal{C})V(\mathcal{S})\rangle = e^{-\frac{2\pi i}{k}\,\mathrm{link}\,(\mathcal{C},\mathcal{S})}.$$