On perturbation theory of supersymmetric gradient flow in $\mathcal{N}=1$ SQCD

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1. Motivation

- Studies on SUSY are diverse and attractive.
  - superstring, GUT, mathematical structure of field theory, duality, ...
  - SYM⇒black hole, origin of gravitational wave, ...
- We want to solve the dynamics of SUSY with lattice gauge theory.

However,

1. CANNOT define energy momentum tensor $T_{\mu\nu}$ on lattice
   - EMT⇒SUSY breaking, thermodynamics of dual black holes
2. Difficulty constructing the SUSY continuum limit

⇒ Approaching with supersymmetric gradient flow

2. Properties of gradient flow

flow equation and solution of Yang-Mills theory

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + \alpha_0 D_\mu \partial_\nu B_\nu$$

grad $S_{YM}[B_\mu]$ — "gauge fixing"

$$B_\mu(t,x) = \int \mathrm{d}^D y \left\{ K_t(x-y)_{\mu\nu} A_\nu(y) + \int_o^t \mathrm{d}s K_{t-s}(x-y)_{\mu\nu} R_\nu(s,y) \right\}$$

non linear

$$\left(\begin{array}{l} B_\mu|_{t=0} = A_\mu, \quad K_t(z)_{\mu\nu} = \int_p \frac{e^{ipz}}{p^2}\{(\delta_{\mu\nu}p^2 - p_\mu p_\nu)e^{-tp^2} + p_\mu p_\nu e^{-\alpha_0 t p^2}\} \\ R_\mu = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] + (\alpha_0 - 1)[B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]] \end{array} \right)$$

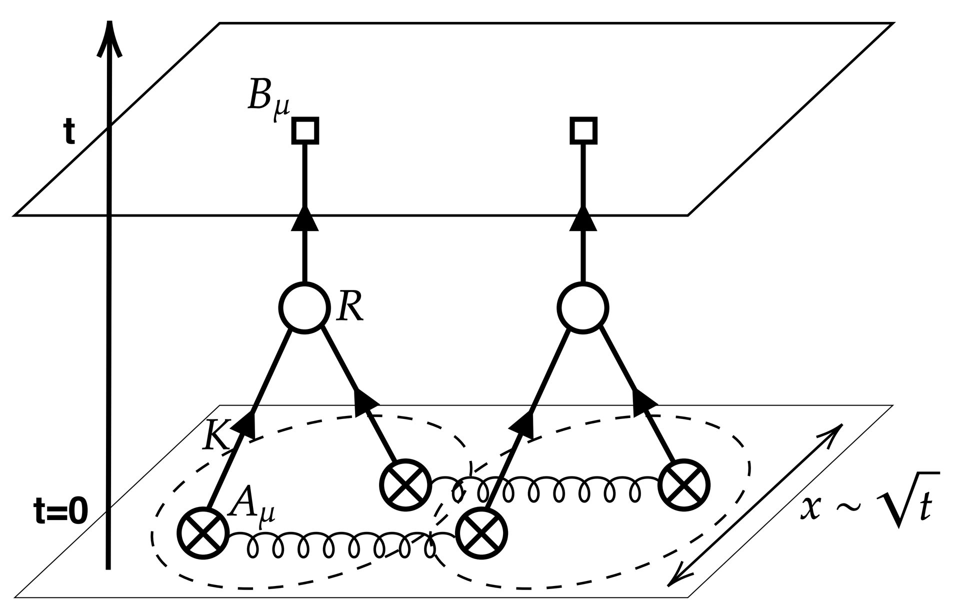

Figure: Graphical representation of flowed fields

- No extra UV divergence in flowed-correlation functions
  [Luscher (2010); Luscher-Weisz (2011)]
- Formulation and numerical calculation in QCD are in progress.
  [H. Suzuki (2017); Hatsuda, et al. (2015); Kanaya, et al.(2017)]
- Other applications of the gradient flow approach
  - The nonlinear sigma model
    [Makino-H. Suzuki (2015), Aoki-Kikuchi-Onogi (2015), Bietenholz, et al. (2018)]
  - Nonperturbative renormalization group
    [Yamamura (2016), Abe-Fukuma (2018), Sonoda-H. Suzuki(2019)]
  - A theory with anti-de Sitter geometries
    [Aoki-Kikuchi-Onogi (2015), Aoki-Yokoyama (2018)]

  e.t.c.

But, not supersymmetric

	Yang-Mills	matter
Boundary $\to$ Bulk	$A_\mu(x) \to B_\mu(t,x)$	$\psi(x) \to \chi(t,x)$
flow equation	$\partial_t B_\mu = D_\nu G_{\nu\mu}$	$\partial_t \chi = D_\mu D_\mu \chi$
renomalization	$B_\mu = B_\mu^R$	$\chi = Z_\chi^{-1/2}\chi^R$
flowed-Correlation function	UV-finite	UV-finite

3. UV-finiteness of SUSY flow

Strategy of proof [Luscher-Weisz (2011)]

1. Constructing appropriate flow eq. (gauge covaiant, SUSY, ...)
   [Kikuchi-Onogi (2014); Kadoh-Ukita (2018); Kadoh-Kikuchi-Ukita(2019)]
2. Formulating a perturbation theory (Boundary & Bulk)
3. (4+1)D theory which reproduces perturbation theory of 2nd step
4. Ward-identities of (4+1)D theory forbid extra counterterms.

4. Constructing SUSY flow eq.

$S_{SQCD} = S_{SYM} + S_{matter}(m)$ (Euclid, four components)

$$S_{SQCD} = \int \mathrm{d}^4x \left\{ \frac{1}{g^2}\mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}^2 + \bar\lambda \not{D}\lambda + D^2\right) + |D_\mu\varphi_+|^2 + |D_\mu\varphi_-|^2 + \bar\psi\not{D}\psi + |G_+|^2 + |G_-|^2 \right.$$
$$- i(\varphi_+^\dagger D\varphi_+ - \varphi_-^\dagger D\varphi_-) + \sqrt{2}i(\bar\psi P_-\lambda\varphi_+ + \bar\psi P_+\lambda\varphi_- \varphi_+^\dagger\bar\lambda P_+\psi - \varphi_-^\dagger\bar\lambda P_-\psi)$$
$$\left. + m(\bar\psi\psi - i\varphi_-^\dagger G_+ - iG_-^\dagger\varphi_+ - i\varphi_+^\dagger G_- - iG_+^\dagger\varphi_-) \right\}$$

(gauge multiplet: A_μ, λ, D, matter multiplet: $\varphi_\pm, \psi_\pm, G_\pm$)

∗ SUSY transformations

$$\delta_\xi A_\mu = \bar\xi\gamma_\mu\lambda, \quad \delta_\xi\lambda = \frac{1}{2}F_{\mu\nu}\gamma_\mu\gamma_\nu\xi - D\gamma_5\xi, \quad \delta_\xi D = \bar\xi\gamma_5\not{D}\lambda, \quad \delta_\xi\varphi_\pm = \sqrt{2}\bar\xi P_\pm\psi, \quad \cdots\cdots$$

SQCD-flow equation in superfields

$$\partial_t V^a = -\frac{1}{2}g^{ab}\frac{\delta S_{SYM}}{\delta V^b} + \delta_\Phi V^a$$
$$\partial_t \Phi_+ = -\frac{1}{4}\bar D\bar D\left(e^{-2gV}\frac{\delta S_{matter}(m=0)}{\delta\Phi_+^\dagger}\right) + \delta_\Phi\Phi_+$$
$$\cdots\cdots$$

- Simplified by using S_{SYM} & massless matter
- (grad term) + (for preserving Wess-Zumino gauge fixing)
- $\bar D\bar D$ for preseving chiral condition $\bar D\Phi_\pm = 0$

SQCD-flow equation in component fields

$$\partial_t A_\mu = D_\rho F_{\rho\mu} + i\bar\lambda\gamma_\mu\lambda, \quad \partial_t\lambda = \not{D}^2\lambda - i[\gamma_5\lambda, D]$$
$$\partial_t\varphi_\pm = D_\mu D_\mu\varphi_\pm + \sqrt{2}i\bar\lambda P_\pm\psi, \quad \cdots\cdots$$

(Bulk fields are expressed by the same characters as the boundary fields.)

- gauge & SUSY covariant equation
  $\partial_t\delta_\xi - \delta_\xi\partial_t = \delta_\omega^g \quad (\omega \equiv -(D_\nu - \alpha_0\partial_\nu)(\bar\xi\gamma_\nu\lambda))$

5. Constructing (4+1)D theory

$$S_{total} = S_{SQCD} + S_{GF+FP} + S_{fl} + S_{d\bar d}$$

Boundary — Bulk

$$S_{GF+FP} = s\int \mathrm{d}^4x\mathrm{tr}\{\bar c(2\partial_\mu A_\mu - i\xi B)\} \quad (s \equiv \delta_\omega^{BRS} + \delta_\xi^{BRS})$$

$$S_{fl} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^4x\left[\mathrm{tr}\{L_\mu(t,x) \times E_\mu(t,x) + \cdots\cdots\}\right]$$

Lagrange multiplier — flow eq.

$$S_{d\bar d} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^4x\mathrm{tr}\{\bar d(\partial_t d - \alpha_0 D_\mu\partial_\mu d + \bar\xi\not{D}\lambda - \alpha_0\bar\xi\not{\partial}\lambda)$$

∗ flow equation (+"gauge fixing")

$$E_\mu = \partial_t A_\mu - D_\rho F_{\rho\mu} - i\bar\lambda\gamma_\mu\lambda - \alpha_0 D_\mu\partial_\nu A_\nu, \quad \cdots\cdots$$

"gauge fixing"

- $LA^2, LA^3, \ldots$ correspond to flow vertices.

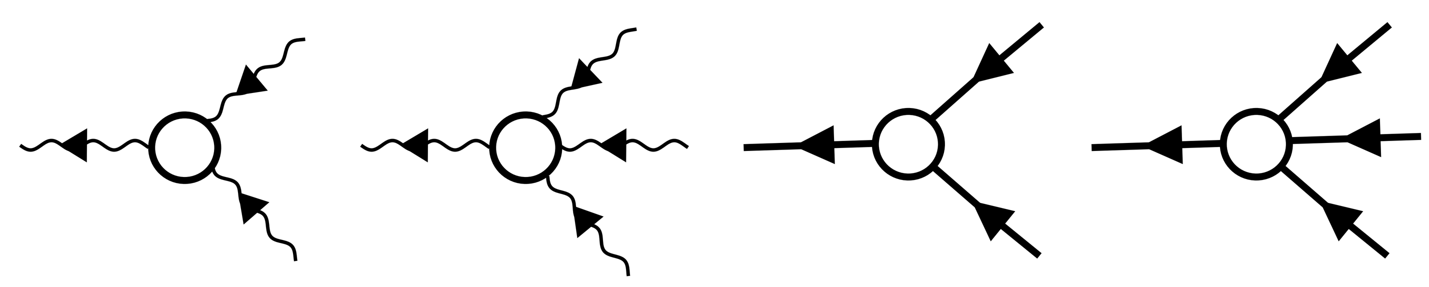

Figure: flow vertices

- Using $s(S_{tot}) = 0$, we try to prove UV finiteness of flowed-correlation functions at all order.

6. Future work

- Research UV-finiteness at all order
- Define physical quantities (EMT, supercurrent, ...)
- Arbitrary number of chiral/anti-chiral matter fields, $\mathcal{N} > 1$