Kaluza-Klein Graviton from Primordial Non-Gaussianities

based on arXiv:1906.11840 w/Suro Kim, Toshifumi Noumi and Siyi Zhou.

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1.Introduction

- Typical inflation scale: $H \lesssim 10^{14}$GeV (cf. GUT scale $\sim 10^{15}$GeV, string scale $\sim 10^{16}$ GeV)
- Primordial non-Gaussianities can be used to probe the particle spectrum at the inflation scale. (cf. Pimentel's talk, Suro Kim's talk, Toshiaki Takeuchi' talk)
- The coefficients of the effective couplings of inflaton depend on intermediate particles, and tell us the spin of particles at the inflation scale. (cf. Suro Kim's talk)
- The method discussed by Suro Kim cannot be applied to the Kaluza-Klein graviton case directly because the Froissart-Martin bound was assumed there.

In this poster

we discuss the four-point amplitude mediated by Kaluza-Klein graviton, and show its signs are the same as the massive spin-2 exchange although it violate the bound $< s^2$.

2.Inflation

Inflation is the era when the universe expanded exponentially, and its simplest model is slow-roll inflation;

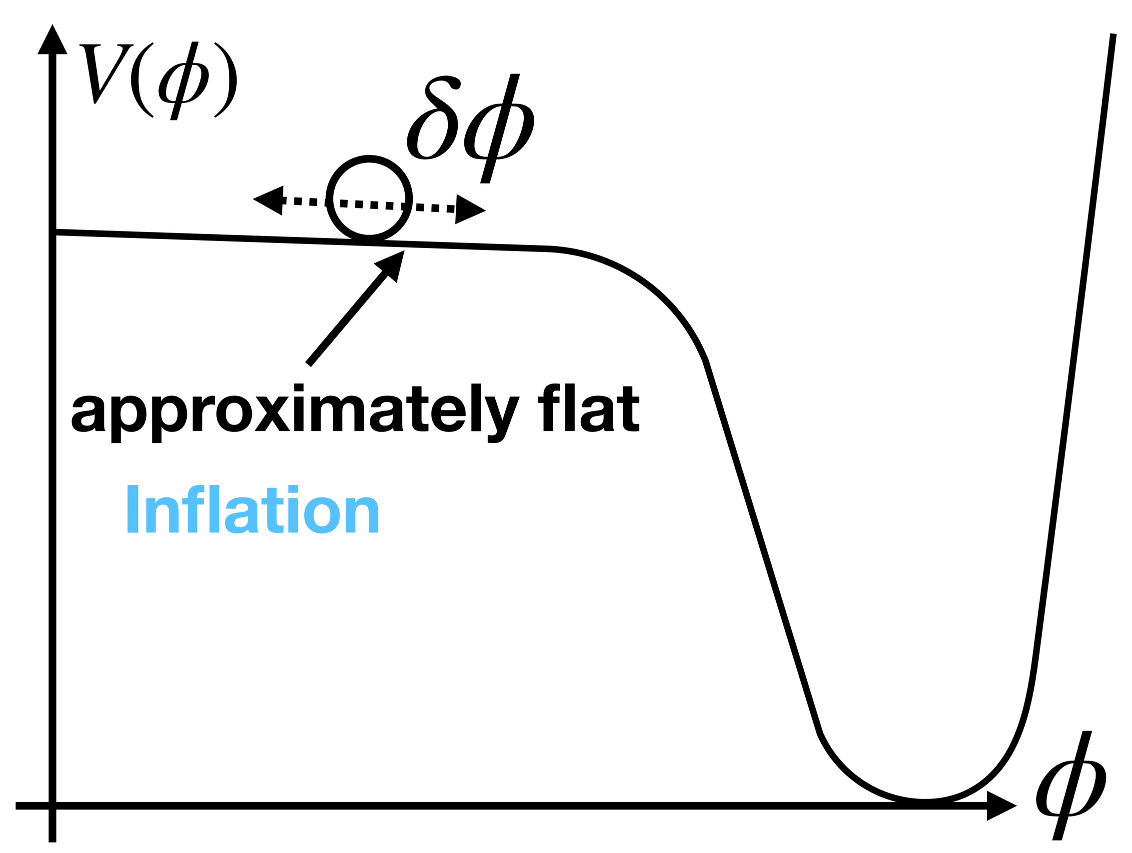
$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]. \tag{1}$$

Inflaton

is the scalar field ϕ which has an approximately flat potential.

$$\begin{cases} (\partial_\mu\phi)^2 \ll V(\phi) \\ \epsilon \quad \equiv \frac{M_{\rm P}^2}{2}\left(\frac{V_\phi}{V}\right)^2 \ll 1 \,, \\ \eta \quad \equiv M_{\rm P}^2\frac{V_{\phi\phi}}{V} \ll 1 \end{cases} \qquad \begin{array}{l} \epsilon, \eta : \text{ slow-roll parameters} \\ \phi \ \ : \text{ inflaton} \end{array}.$$

$\Leftrightarrow$



3.Effective Couplings and Amplitudes

The IR effective Lagrangian of inflaton ϕ

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial_\mu\phi)^2 + \frac{\alpha}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \frac{\beta}{\Lambda^6}(\nabla_\mu\partial_\nu\phi)^2(\partial_\rho\phi)^2 + \cdots \tag{2}$$

The four-point amplitudes of inflaton are given by $M(s,t) = \frac{4\alpha}{\Lambda^4}\left(s^2+st+t^2\right) - \frac{3\beta}{\Lambda^6}\left(s^2t+st^2\right)$ in terms of IR coefficients.

For example, if the three-point amplitude of two inflaton and one massive scalar is given as

$$A_{\phi\phi\sigma}(k_1,k_2,k_3) = g, \tag{3}$$

the four-point amplitudes by integrating out the intermediate massive scalar at IR can be expressed as

$$M(s,t) = \frac{g^2}{m_\sigma^2} + \frac{2g^2}{m_\sigma^6}\left(s^2+st+t^2\right) - \frac{3g^2}{m_\sigma^8}\left(s^2t+st^2\right) + \cdots. \tag{4}$$

IR coefficients from UV analysis (cf. Suro Kim's talk)

In general, the coefficients of the four-point amplitudes with intermediate particles, which are labeled by n, can be expressed in the following way.

IR coefficients

The IR coefficients of s^2 and s^2t terms can be obtained as

$$\frac{4\alpha}{\Lambda^4} = 2\sum_n \frac{g_n^2}{(m_n^2)^3}, \qquad -\frac{3\beta}{\Lambda^6} = \sum_n \frac{g_n^2}{(m_n^2)^4}(2l_n^2+2l_n-3). \tag{5}$$

where the UV amplitudes should be bounded as (cf. Froissart-Martin bound)

$$|M(s,t)| < |s|^2.$$

This analysis implies

- The sign of four-derivative effective coupling is always positive. (cf. positivity bound by arXiv:hep-th/0602178)
- The sign of six-derivative effective coupling depend on the spin of intermediate particles.

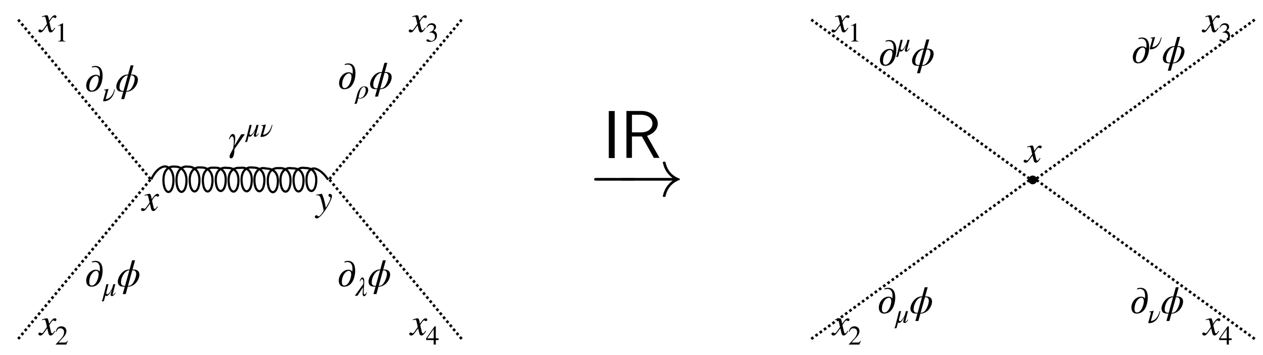
KK gravitons do not satisfy the preceding Froissart-Martin bound because the UV amplitude mediated by KK gravitons behaves as $\sim s^2$, but they are one of the main targets in the cosmological collider program.

What is the sign of effective coupling mediated by KK graviton??

4.Kaluza-Klein Graviton

The Lagrangian of KK graviton and inflaton reads

$$\mathcal{L} = \frac{1}{2}\phi\Box\phi - \frac{1}{4}\gamma_{\mu\nu}(\mathcal{E}_\gamma^{\mu\nu\rho\lambda})\gamma_{\rho\lambda} - \frac{m_\gamma^2}{8}\left(\gamma_{\mu\nu}^2 - \gamma^2\right) + \mathcal{L}_{\rm int}, \quad (6)$$



where $\mathcal{E}$ is kinetic operator of massive spin-2 particle (arXiv:1401.4173):

$$\mathcal{E}_{\mu\nu}^{\rho\lambda}\gamma_{\rho\lambda} = -\frac{1}{2}\left[\Box\gamma_{\mu\nu} - \partial_\mu\partial_\alpha\gamma_\nu^\alpha - \partial_\nu\partial_\alpha\gamma_\mu^\alpha + \partial_\mu\partial_\nu\gamma - \eta_{\mu\nu}\left(\Box\gamma - \partial_\alpha\partial_\beta\gamma^{\alpha\beta}\right)\right]. \quad (7)$$

The coupling between 4D graviton $h_{\mu\nu}$ and scalar field is expressed as

$$S_{int} = \frac{\delta S_{int}}{\delta g^{\rho\sigma}}h^{\rho\sigma} = \int d^4x\sqrt{-g}\,T_{\rho\sigma}h^{\rho\sigma}, \quad T_{\rho\sigma} = \partial_\rho\phi\partial_\sigma\phi - \frac{1}{2}\eta_{\rho\sigma}\partial_\alpha\phi\partial^\alpha\phi \quad (8)$$

Similarly, the coupling between KK graviton and scalar field can be expressed as,

$$S_{int} = \int d^4x\sqrt{-g}\,\mathcal{C}T_{\rho\sigma}\gamma^{\rho\sigma} + \cdots, \qquad \mathcal{C} \ : \ \text{coupling constant}, \ \gamma^{\mu\nu} \ : \ \text{KK graviton}. \quad (9)$$

Four-point amplitude mediated by KK graviton at IR

$$M(s,t) = \frac{4\,\mathcal{C}^2}{3m_\gamma^2}\left(s^2 + st + t^2\right) + \frac{5\,\mathcal{C}^2}{2m_\gamma^4}\left(s^2t + st^2\right) + \cdots. \quad (10)$$

We find that the signs of effective couplings are the same as previous discussion even though their values are different from eq.(5).

5.Bispectrum

Bispectrum

From the effective Lagrangian eq.(2), we get bispectra as follows.

$$\langle\delta\phi_{\vec{k}_1}\delta\phi_{\vec{k}_2}\delta\phi_{\vec{k}_3}\rangle \equiv (2\pi)^7P_{\delta\phi}^2\frac{1}{k_1^2k_2^2k_3^2}\delta(\sum_i \mathbf{k}_i)\left[\alpha\mathcal{A}_{\rm 3pt}(\mathbf{k}_i) + \beta\mathcal{B}_{\rm 3pt}(\mathbf{k}_i)\right], \qquad P_{\delta\phi} = \frac{H^2}{(2\pi)^2} : \text{scalar power spectrum} \quad (11)$$

where $\mathcal{A}_{\rm 3pt}$ and $\mathcal{B}_{\rm 3pt}$ are contributions from the four-derivative and six-derivative operators, respectively.

Shape Function

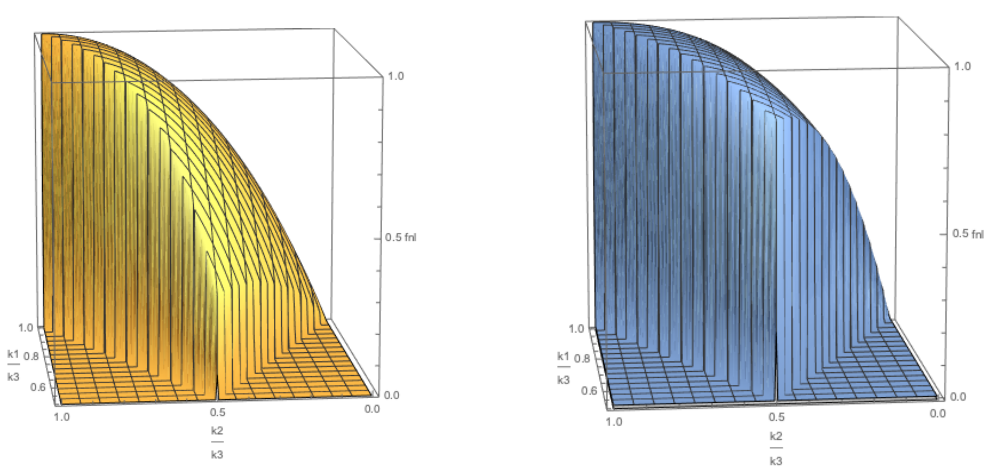


Fig 1: Bispectra of the 4th derivative term (left) and the 6th derivative term (right) respectively.

We can distinguish them due to the ratio between the equilateral configuration and the folded one.

$$\frac{\mathcal{A}_{3\,\rm pt}(0.5,0.5,1)}{\mathcal{A}_{3\,\rm pt}(1,1,1)} \sim 0.32, \quad \frac{\mathcal{B}'_{3\,\rm pt}(0.5,0.5,1)}{\mathcal{B}'_{3\,\rm pt}(1,1,1)} \sim 0.84 \quad (12)$$

$$\mathcal{B}'_{3\,\rm pt}(k_1,k_2,k_3) \equiv \mathcal{B}_{3\,\rm pt}(k_1,k_2,k_3) + \frac{H^2}{\Lambda^2}\mathcal{A}_{3\,\rm pt}(k_1,k_2,k_3) \quad (13)$$

6.Trispectrum

Trispectrum

From the effective Lagrangian eq.(2), we get trispectra as follows.

$$\langle\delta\phi_{\vec{k}_1}\delta\phi_{\vec{k}_2}\delta\phi_{\vec{k}_3}\delta\phi_{\vec{k}_4}\rangle \equiv (2\pi)^3\delta(\sum_i \mathbf{k}_i)\left[\alpha\mathcal{A}_{\rm 4pt}(\mathbf{k}_i) + \beta\mathcal{B}_{\rm 4pt}(\mathbf{k}_i)\right]. \quad (14)$$

In equilateral limit,

$$\mathcal{A}_{\rm 4pt}(\mathbf{k}_i) = \frac{H^8}{256\Lambda^4k^9}\left(\cos^2\theta(103\cos^2\gamma + 309) - 206\cos\theta\sin^2\gamma + 103\cos^2\gamma + 173\right) \quad (15)$$

$$\mathcal{B}_{\rm 4pt}(\mathbf{k}_i) = \frac{H^{10}}{4096\Lambda^6k^9}\left(-1311\cos^3\theta\sin^2\gamma - \cos^2\theta(1111\cos^2\gamma + 8577)\right.$$
$$\left. +6155\cos\theta\sin^2\gamma - (3733\cos^2\gamma + 3155)\right). \quad (16)$$

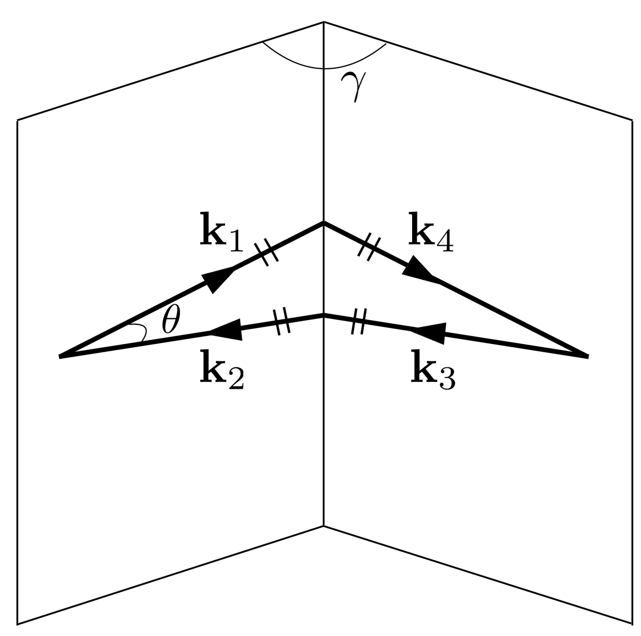


Fig 2: Equilateral configurations of four momenta

The coefficient $\cos^3\theta$ can be used to probe the sixth derivative operator without swamped by the fourth derivative one.

7.Conclusion

- The signs of higher order derivatives coupling depend on the spin of intermediating particles.
- Fourth order derivative term is universally positive $\alpha > 0$ independent of spin of intermediating particles.
- Sixth order derivative term depends on the spin of intermediating particles. ($\beta > 0$ for scalars, $\beta < 0$ for spin $2, 4, \ldots$)
- This conclusion applies to KK graviton even though it violates the Froissart-Martin bound.
- Non-Gaussianities generated by these effective couplings can be used to probe the spin of heavy particles.