Extended supersymmetric quantum mechanics from symmetries in higher dimensional Dirac action

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① Introduction and Conclusion

◆ **Quantum mechanical supersymmetry (QM SUSY) has a wide range of applicable topics**

exactly solvable quantum mechanics, AdS/CFT, Black hole, SYK model, **extra dimension**, ⋯ (**spinor**, gauge, gravity fields)

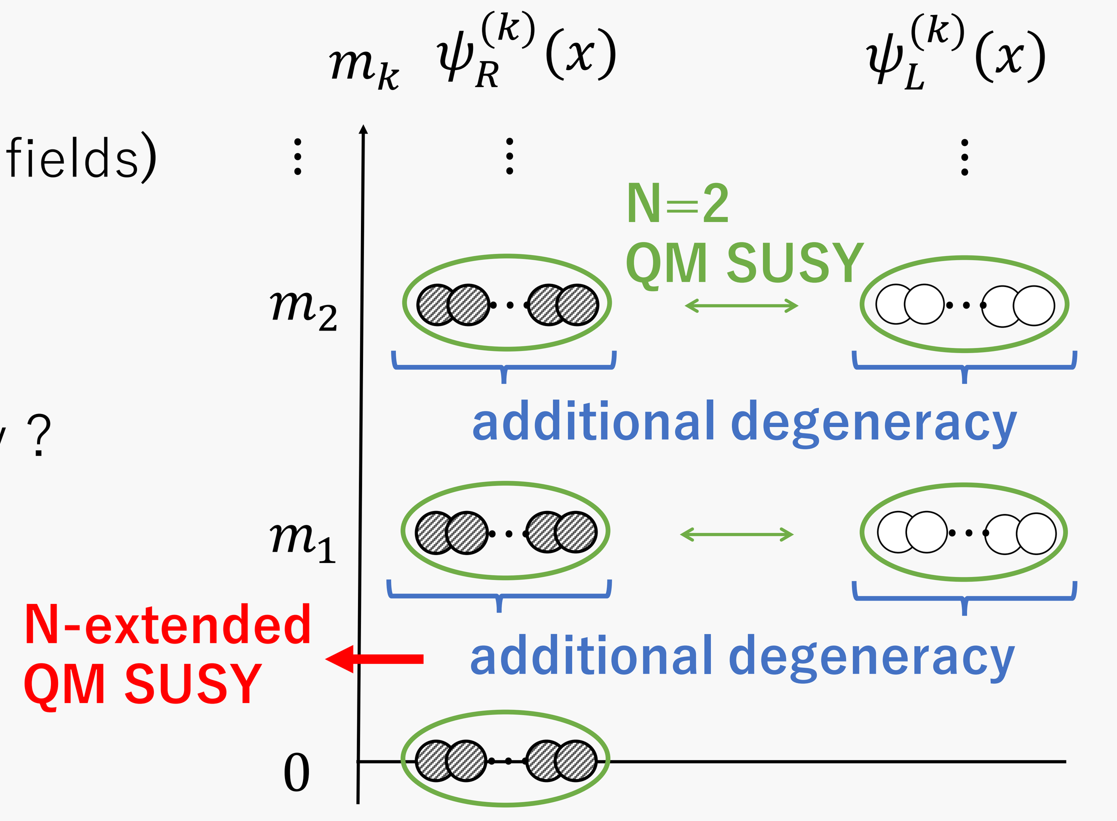
◆ **KK decomposition of higher dimensional Dirac field**

Degeneracy between 4D left and right – handed spinors ➡ N=2 QM SUSY

◆ **Problem :** There is additional degeneracy due to the degrees of freedom of higher dimensional spinor. What further structures are hidden in this degeneracy ?

◆ **Conclusion**

- **N-extended QM SUSY with central charges from symmetries in extra dimensions is hidden in the 4D mass spectrum**
- **KK mode functions correspond to BPS states**



② KK decomposition of Dirac field and N=2 QM SUSY

◆ $4+d$ dimensional Dirac action with a curved extra dimension Ω

$$S=\int_{M_4} d^4x \int_\Omega d^dy\sqrt{-g}\times\bar{\Psi}(x,y)\left[i\Gamma^\mu\partial_\mu+i\Gamma^y\left(\nabla_y+iA_y(y)\right)-W(y)\right]\Psi(x,y)$$

background

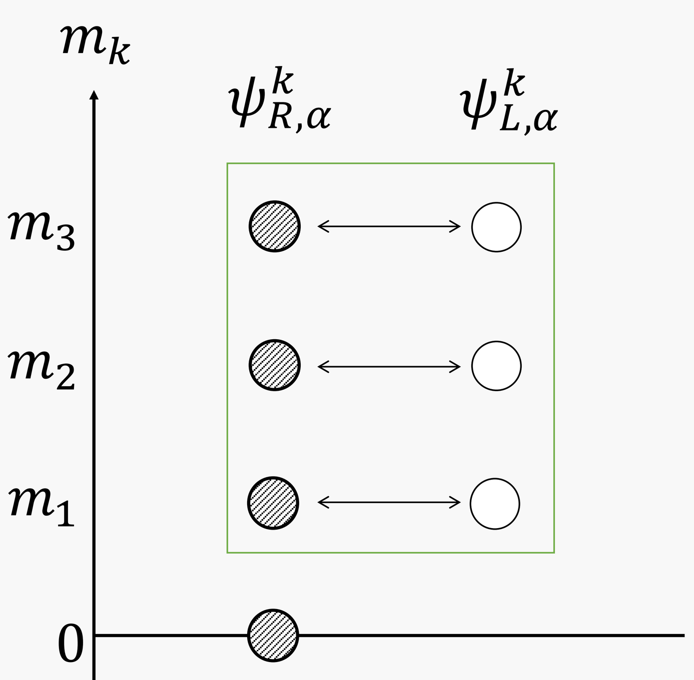
$\Psi(x,y)$ has $2^{[d/2]}\times 4$ components

$$\Psi(x,y)=\sum_{k,\alpha}\left[f_\alpha^{(k)}(y)\otimes\psi_{R,\alpha}^{(k)}(x)+g_\alpha^{(k)}(y)\otimes\psi_{L,\alpha}^{(k)}(x)\right]$$

$2^{[d/2]}$ components KK mode function — 4D chiral spinor

k : KK mode number
$\alpha=1,2,\cdots$: label of additional degeneracy

◆ 4D mass terms consist of the pair of left and right fields



◆ $f_\alpha^{(k)}(y)$ and $g_\alpha^{(k)}(y)$ should be related with each other

N=2 QM SUSY

$$Q\,f_\alpha^{(k)}(y)=m_k\,g_\alpha^{(k)}(y)$$
$$Q\,g_\alpha^{(k)}(y)=m_k\,f_\alpha^{(k)}(y)$$

supercharge $\sim i\Gamma^y(\nabla_y+iA_y(y))-W(y)$

"bosonic" and "fermionic" states in QM SUSY

N=2 SUSY algebra

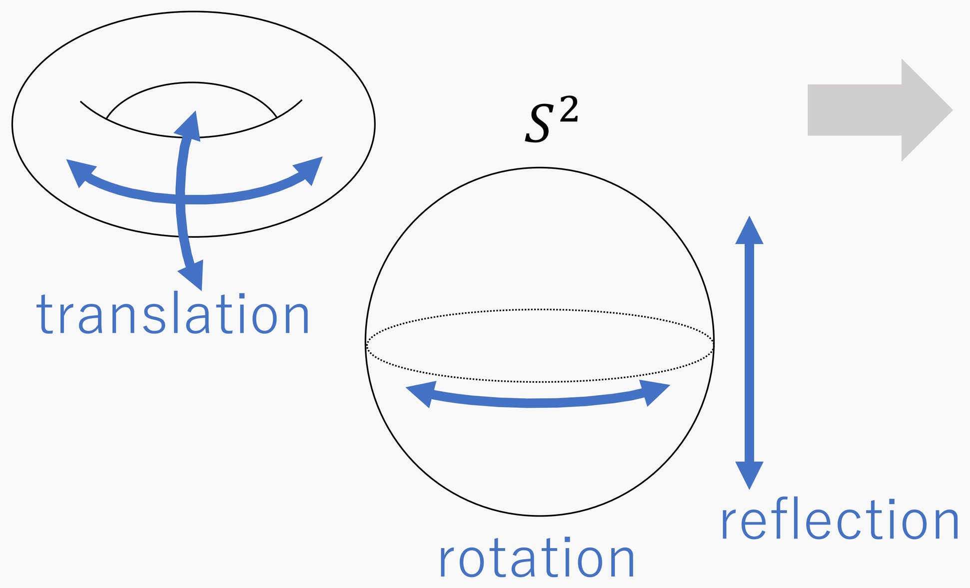
$$\{Q_i,Q_j\}=2H\delta_{ij}\ ,\quad [Q_i,H]=0\ ,\quad Q_1=Q\ ,\ Q_2=i(-1)^FQ$$

$(-1)^F$: counterpart of 4D chirality (+1 for $f_\alpha^{(k)}(y)$ and -1 for $g_\alpha^{(k)}(y)$)

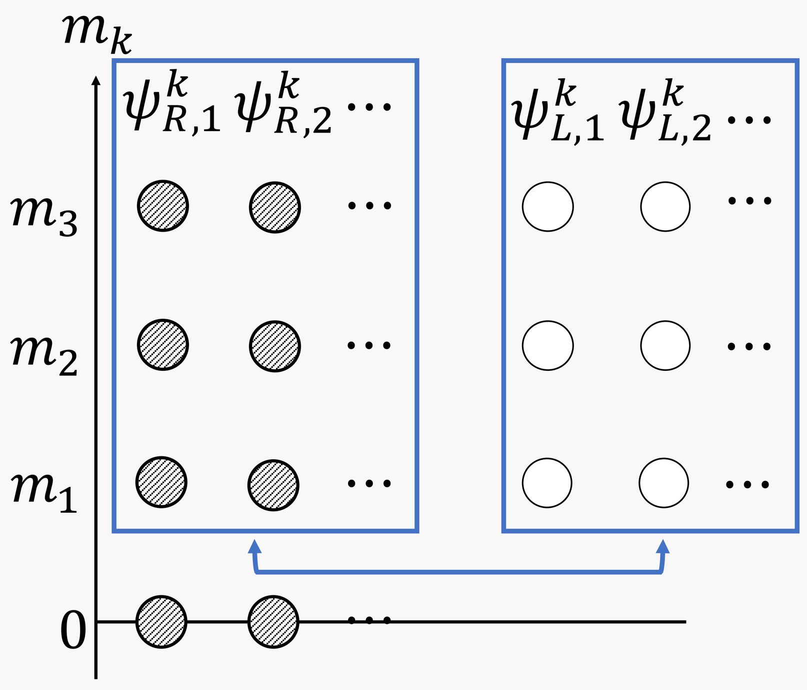
③ Extended QM SUSY from symmetries

◆ Symmetries of extra dimensions

example: torus



◆ Additional degeneracy of 4D spinors



N-extended QM SUSY

$$f_\alpha^{(k)}(y)\ \overset{Q_i^{(A)}\ (i=1,2,\cdots,N_A)}{\longleftrightarrow}\ g_\beta^{(k)}(y)$$

- $Q_i^{(a)}\sim(-1)^F\times\hat{a}_i\times Q$
- $Q_i^{(b)}\sim(-1)^F\times\hat{b}_i\times Q$
⋮
- $Q_i^{(\alpha)}\sim\hat{\alpha}_i\times Q$
- $Q_i^{(\beta)}\sim\hat{\beta}_i\times Q$
⋮

$\hat{a}_i,\cdots,\hat{\alpha}_i,\cdots$ are operators related to symmetries and satisfy

anticommute

H — commute — $(\hat{a}_1,\hat{a}_2,\cdots,\hat{a}_{N_a})$ $(\hat{b}_1,\hat{b}_2,\cdots,\hat{b}_{N_b})$ ⋯

$(\hat{\alpha}_1,\hat{\alpha}_2,\cdots,\hat{\alpha}_{N_\alpha})$ $(\hat{\beta}_1,\hat{\beta}_2,\cdots,\hat{\beta}_{N_\beta})$ ⋯

④ SUSY algebra with central charges

◆ **N-extended SUSY algebra with central charges**

$$\left\{Q_i^{(A)},Q_j^{(B)}\right\}=2H\delta_{ij}\delta^{AB}+2Z_{ij}^{(A)}\delta^{AB}$$
$$\left[Z_{ij}^{(A)},Q_k^{(B)}\right]=\left[Z_i^{(A)},H\right]=\left[Z_i^{(A)},Z_j^{(B)}\right]=\left[Q_i^{(A)},H\right]=0$$
$$(i,j,k=1,2,\cdots,\quad A,B=a,b,\cdots,\alpha,\beta,\cdots)$$

Central charges : $Z_{ij}^{(a)}\sim(\hat{a}_i\,\hat{a}_j-\delta_{ij})\times H,\ \cdots,\ Z_{ij}^{(\alpha)}\sim(\hat{\alpha}_i\,\hat{\alpha}_j-\delta_{ij})\times H,\ \cdots$

◆ **BPS states**

diagonalize the eigenvalues of $Z_{ij}^{(A)}$ and redefine $Q_i^{(A)}\to Q_i'^{(A)}$

$$\left\{Q_i'^{(A)},Q_j'^{(B)}\right\}=2(m_k+z_i^{(A)})\delta_{ij}\delta^{AB}\quad\text{for eigenstates}$$

$\boldsymbol{Q_i'^{(A)}=0}$ **for mode functions with** $\boldsymbol{z_i^{(A)}=-m_k}$ (BPS states)

In this SUSY $\left[Q_i^{(A)},Q_j^{(A)}\right]=0$ ➡ $Q_i'^{(A)}Q_j'^{(A)}=0\ (i\neq j)$

⬇ $Q_1'^{(A)},\cdots,Q_{i_{N_A}}'^{(A)}=0$ except for at most one supercharge

mode functions become BPS states if $N_A>1$

⑤ Example

◆ Extra dimension : 2d torus with periodic BC

reflection symmetry

R_i : reflection operator $y_i\to -y_i$
P : reflection operator $\vec{y}\to-\vec{y}$

- $Q_\pm'^{(R_i)}\sim\frac{1\pm R_i}{2}\times\sigma_i\times Q\quad(i=1,2)$
- $Q_\pm'^{(P_3)}\sim\frac{1\pm P}{2}\times\sigma_3\times Q$
- $Q_\pm'^{(P)}\sim\frac{1\pm P}{2}\times Q$

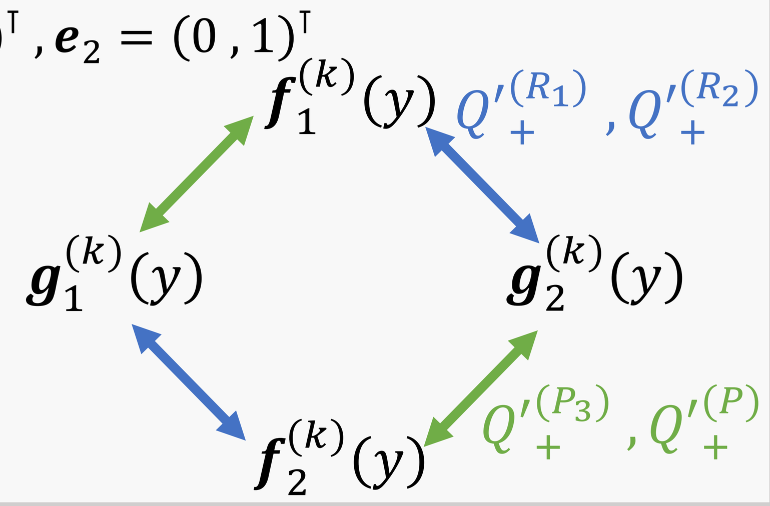
◆ example of mode function $e_1=(1,0)^\top, e_2=(0,1)^\top$

$$f_\alpha^{(k)}(y)\propto\cos\left(\frac{2\pi k_1}{L_1}y_1\right)\cos\left(\frac{2\pi k_2}{L_2}y_2\right)e_\alpha$$

⬇ $z_\pm^{(A)}=\pm(reflection)\times H$

$Q_+'^{(R_1)}$ $Q_+'^{(R_2)}$ $Q_+'^{(P_3)}$ $Q_+'^{(P)}$

~~$Q_-'^{(R_1)}$ $Q_-'^{(R_2)}$ $Q_-'^{(P_3)}$~~ $Q_-'^{(P)}$



⑥ Discussion

◆ Boundary condition, shape of extra dimension, non-trivial background

➡ Breaking of N-extended QM SUSY

◆ Further hidden structures in higher dimensional model ?

◆ Are there new insights obtained from this structure?

◆ Applications to new exactly solvable quantum-mechanical models ?