Rectangular W-algebras of types so and sp and dual coset CFTs

Takahiro Uetoko (Ritsumeikan Univ.)

Based on: [arXiv:1906.05872]

w/ Thomas Creutzig (Alberta Univ.), Yasuaki Hikida (YITP, Kyoto Univ.)

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Introduction

- Strings and Higher spins

String theory → **Higher spin gravity**

First Regge trajectory → Vasiliev theory

Tensionless limit [Gross '88]

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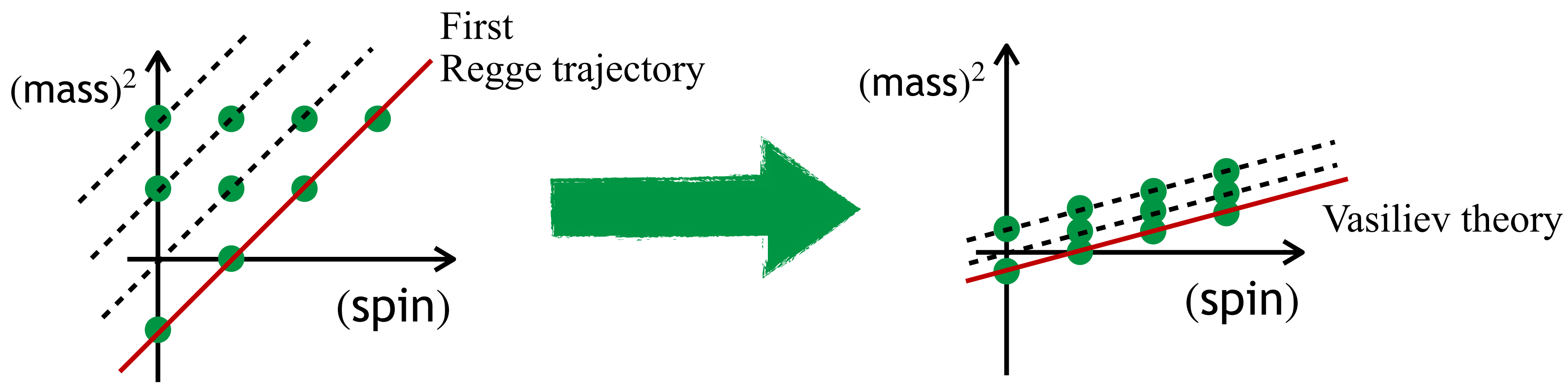
- String spectrum



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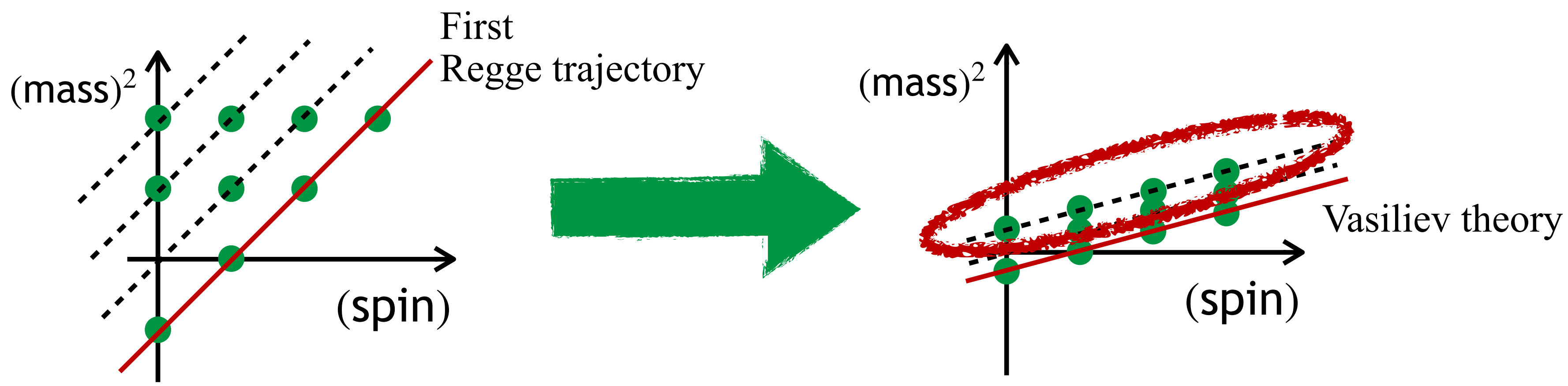
How to explain the higher Regge trajectories?

- String spectrum



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All Regge trajectory

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Higher spin gravity

Vasiliev theory

with $M \times M$ matrix valued fields

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  - Dual model is 2d Grassmannian-like coset

$$\frac{su(N+M)_k}{su(N)_k \oplus u(1)_{kNM(N+M)}}$$

With $M = 1$, this reduce to the Gaberdiel-Gopakumar duality [Gaberdiel-Gopakumar '10]

  - Evidence: spectrum, asymptotic symmetry, …

[Creutzig-Hikida-Rønne '13, Creutzig-Hikida '18]

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Can we generalize this analysis to other models?

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Introduction

- Our question and summary

Can we generalize this analysis to other models?

An answer (my talk)

- **We consider 2 ways to truncate the DOF**
  - Restricted matrix extensions; $so(M)$, $sp(M)$
  - Even spin truncation of $hs[\lambda]$
- **We propose the dual coset model and examine the asymptotic symmetry**

Plan of talk

1. Introduction

2. HS gravity with $sl(M)$ gauge sector

3. Some generalization for extended HS gravity

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- Chern-Simons description of HS gravity
  - 3d gravity: $sl(2)$ CS theory [Witten '88]

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Include higher spin

  - 3d HS gravity: [Prokushkin-Vasiliev '98]

$$hs[\lambda]$$

$$hs[\lambda] = B[\lambda] \ominus \mathbf{1}$$

$$B[\lambda] = \frac{U(sl(2))}{\langle C_2 - \frac{1}{4}(\lambda^2 - 1)\mathbf{1} \rangle}$$

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$$hs[\lambda] \longrightarrow sl(n) \text{ CS theory with gravitational } sl(2)$$

$[\lambda = n \ (n = 2,3,\ldots)]$

Ex) principal embedding

$$sl(n) = sl(2) \oplus \left(\bigoplus_{s=3}^{n} g^{(s)}\right)$$

spin-(s-1) representation

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  - 3d HS gravity with $M \times M$ fields:

$$hs_M[\lambda] \simeq gl(M) \otimes B[\lambda] \ominus \mathbf{1}_M \otimes \mathbf{1}$$

$$\simeq sl(M) \otimes \mathbf{1} \oplus \mathbf{1}_M \otimes hs[\lambda] \oplus sl(M) \otimes hs[\lambda]$$

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$$\xrightarrow{[\lambda = n]} sl(M) \otimes \mathbf{1}_n \oplus \mathbf{1}_M \otimes sl(n) \oplus sl(M) \otimes sl(n) \simeq sl(Mn)$$

Include gravitational $sl(2)$

HS gravity with $sl(M)$ gauge sector

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We examine $sl(Mn)$ CS theory decomposed as

$$sl(Mn) \simeq sl(M) \otimes \mathbf{1}_n \oplus \mathbf{1}_M \otimes sl(n) \oplus sl(M) \otimes sl(n)$$

  - 3d HS gravity with $M \times M$ fields:

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$$\longrightarrow sl(M) \otimes \mathbf{1}_n \oplus \mathbf{1}_M \otimes sl(n) \oplus sl(M) \otimes sl(n) \simeq sl(Mn)$$

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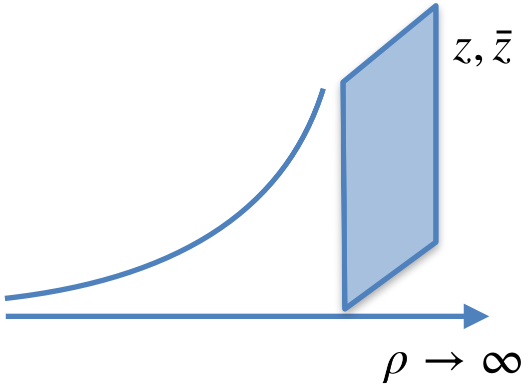
HS gravity with $sl(M)$ gauge sector

- Gauge field (Solution of EOM)
  - 3d gravity: $A = e^{-\rho L_0} a(z) e^{\rho L_0} dz + L_0 d\rho$



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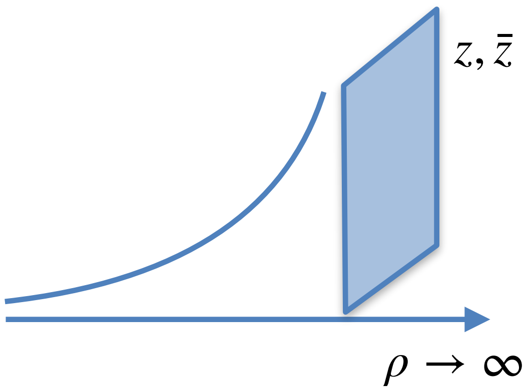

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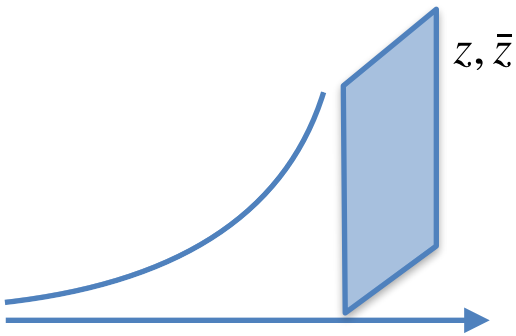

Matrix extension

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HS gravity with $sl(M)$ gauge sector

Asymptotically AdS
$$(A - A_{AdS})|_{\rho\to\infty} = \mathcal{O}((e^{\rho})^0)$$

- Asymptotically AdS condition
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  ⬇

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Virasoro generator

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$$a(z) = J_a(z)(t^a \otimes \mathbf{1}_n) + \mathbf{1}_M \otimes V_1^2 + \sum_{s=2}^{n} W^{(s)}(z)(\mathbf{1}_M \otimes V^s_{-s+1}) + \sum_{s=2}^{n} Q_a^{(s)}(z)(t^a \otimes V^s_{-s+1})$$

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W_N generators

generators of $sl(M)$

$$+ \sum_{s=2}^{n} W^{(s)}(z)(\mathbf{1}_M \otimes V_{-s+1}^{s}) + \sum_{s=2}^{n} Q_a^{(s)}(z)(t^a \otimes V_{-s+1}^{s})$$

HS gravity with $sl(M)$ gauge sector

- Asymptotically AdS condition

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$$(A - A_{AdS})|_{\rho\to\infty} = \mathcal{O}((e^{\rho})^0)$$

They are the generators of asymptotic symmetries

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HS gravity with $sl(M)$ gauge sector

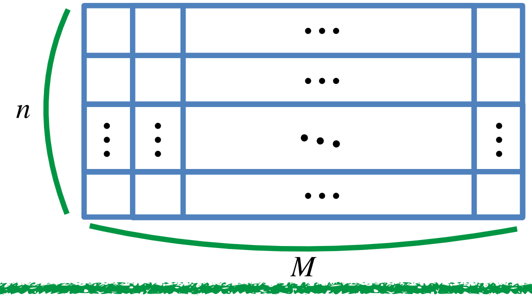
- Asymptotic symmetry

$$sl(Mn) \simeq sl(M) \otimes \mathbf{1}_n \oplus \mathbf{1}_M \otimes sl(n) \oplus sl(M) \otimes sl(n)$$

  - Hamiltonian reduction of $sl(Mn)$ with the $sl(2)$ embedding

→ We obtain the rectangular W-algebra

Principal embedding correspond to the partition



→ Young diagram of rectangular type

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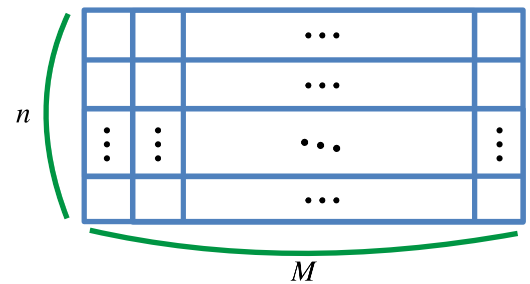

→ Young diagram of rectangular type

  - The central charge and level of $sl(M)$ are evaluated (e.g. [Kac-Wakimoto '03])

  - These are consistent with the dual coset model with finite N

[Creutzig-Hikida '18]

  - We need the proper t' Hooft parameter

$$\lambda = \frac{k}{k+N}$$

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Let's turn to the other matrix extension!

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[Creutzig-Hikida '18]

  - We need the proper t' Hooft parameter

$$\lambda = \frac{k}{k+N} \text{ or } \lambda' = -\frac{k}{k+N+M}$$

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Some generalization

- Restricted $M \times M$ matrix extension
  - 2 restrictions on the extra matrix DOF

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$\lambda = 2n + 1$ $\lambda = 2n$

$$so(2n+1) \qquad sp(2n)$$

Some generalization

- Restricted $M \times M$ matrix extension

We consider 4 CS gravities

$hs[\lambda]$ \ $M \times M$	$so(M)$	$sp(2m)$
$so(2n+1)$	$so(M(2n+1))$	$sp(2m(2n+1))$
$sp(2n)$	$sp(2Mn)$	$so(4mn)$

Ex) Decomposition of previous model

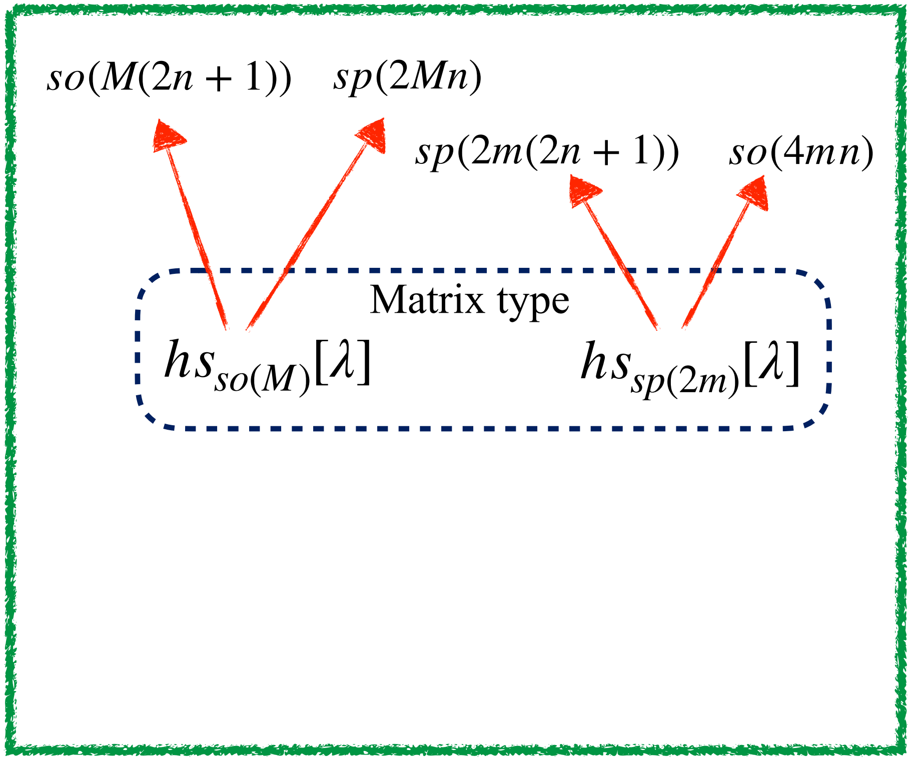
$$sl(Mn) \simeq sl(M) \otimes \mathbf{1}_n \oplus \mathbf{1}_M \otimes sl(n) \oplus sl(M) \otimes sl(n)$$

$so(2n+1)$ $sp(2n)$

Some generalization

- Asymptotic symmetry
  - We obtain 4 types of rectangular W-algebras by Hamiltonian reduction

Their central charge and level are evaluated



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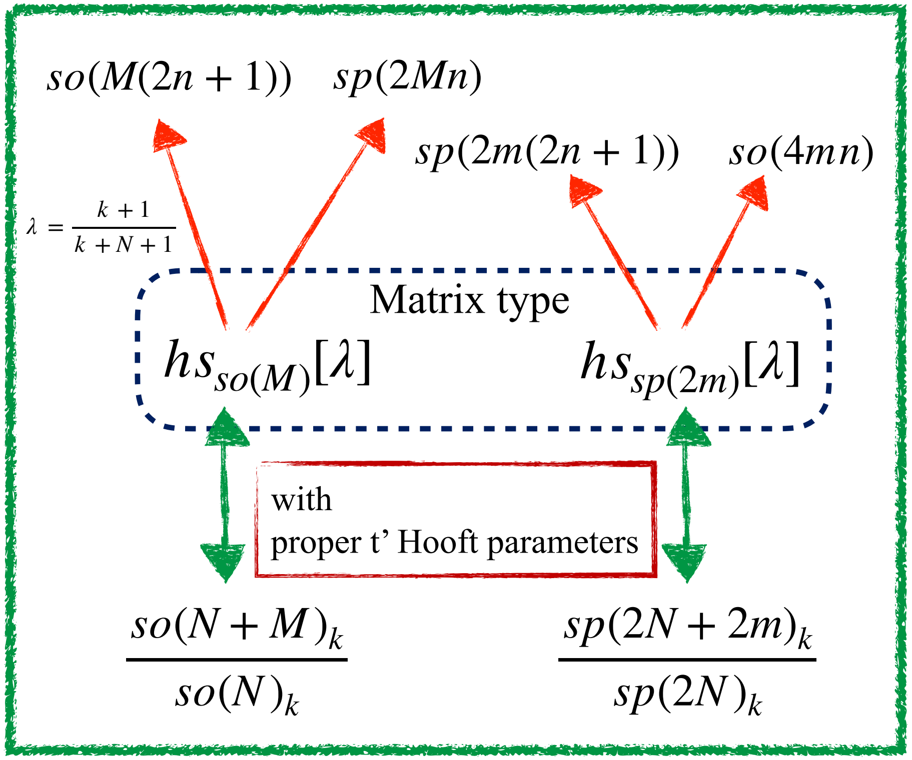

- Dual coset model
  - We propose 2 dual coset models

    Their central charge and level are consistent with above algebras with finite N

Some generalization

- Asymptotic symmetry
  - We obtain 4 types of rectangular W-algebras by Hamiltonian reduction

$so(M(2n+1))$ $sp(2Mn)$

$so(N)_k$ $sp(2N)_k$

Other checks of the duality

We focus on the OPEs

Their central charge and level are consistent with above algebras with finite N

Some generalization

- OPEs (for $\lambda = 2$)

  $sl(2)$ with $M \times M$ matrix

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$so(2M)$ or $sp(2m)$ currents

spin1: J^a

spin2: $T,\ Q^a$

EM tensor

Spin 2 charged currents

Some generalization

- OPEs (for $\lambda = 2$)

  $sl(2)$ with $M \times M$ matrix

  - We compute the OPEs among generators of each algebras by requiring associatively

    - spin1: J^a — $so(2M)$ or $sp(2m)$ currents
    - spin2: T, Q^a — T: EM tensor; Q^a: Spin 2 charged currents

  - We compute OPEs among generators of each coset algebras

Above OPEs are reproduced !!

Plan of talk

1. Introduction
2. HS gravity with $sl(M)$ gauge sector
3. Some generalization for extended HS gravity
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Summary

- Our question and summary

Can we generalize this analysis to other models?

An answer (my talk)

- **We consider 2 ways to truncate the DOF**
  - Restricted matrix extensions; $so(M)$, $sp(M)$
  - Even spin truncation of $hs[\lambda]$
- **We propose the dual coset model and examine the asymptotic symmetry**

That's all for my presentation

Back up slides

Some generalization

- $\mathcal{N} = 1$ super symmetric models
  - We analyze 4 types algebras and propose 2 coset models