Partial Deconfinement

Hiromasa Watanabe (Univ. of Tsukuba)

Collaborator: M. Hanada (Univ. of Southampton), G. Ishiki (Univ. of Tsukuba)

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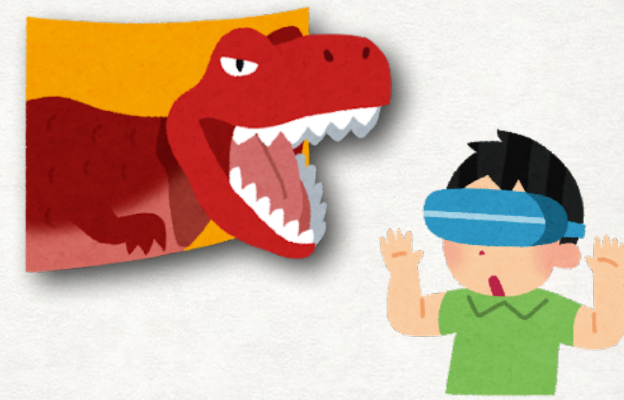
Motivation



Holographic principle or gauge/gravity correspondence

Quantum Gravity
Black Hole

"Equivalent" ⟷

A certain QFT
in lower dimension

e.g.) AdS/CFT correspondence
[Maldacena, 1997]

Gravity theory
in Anti de Sitter space

⟷

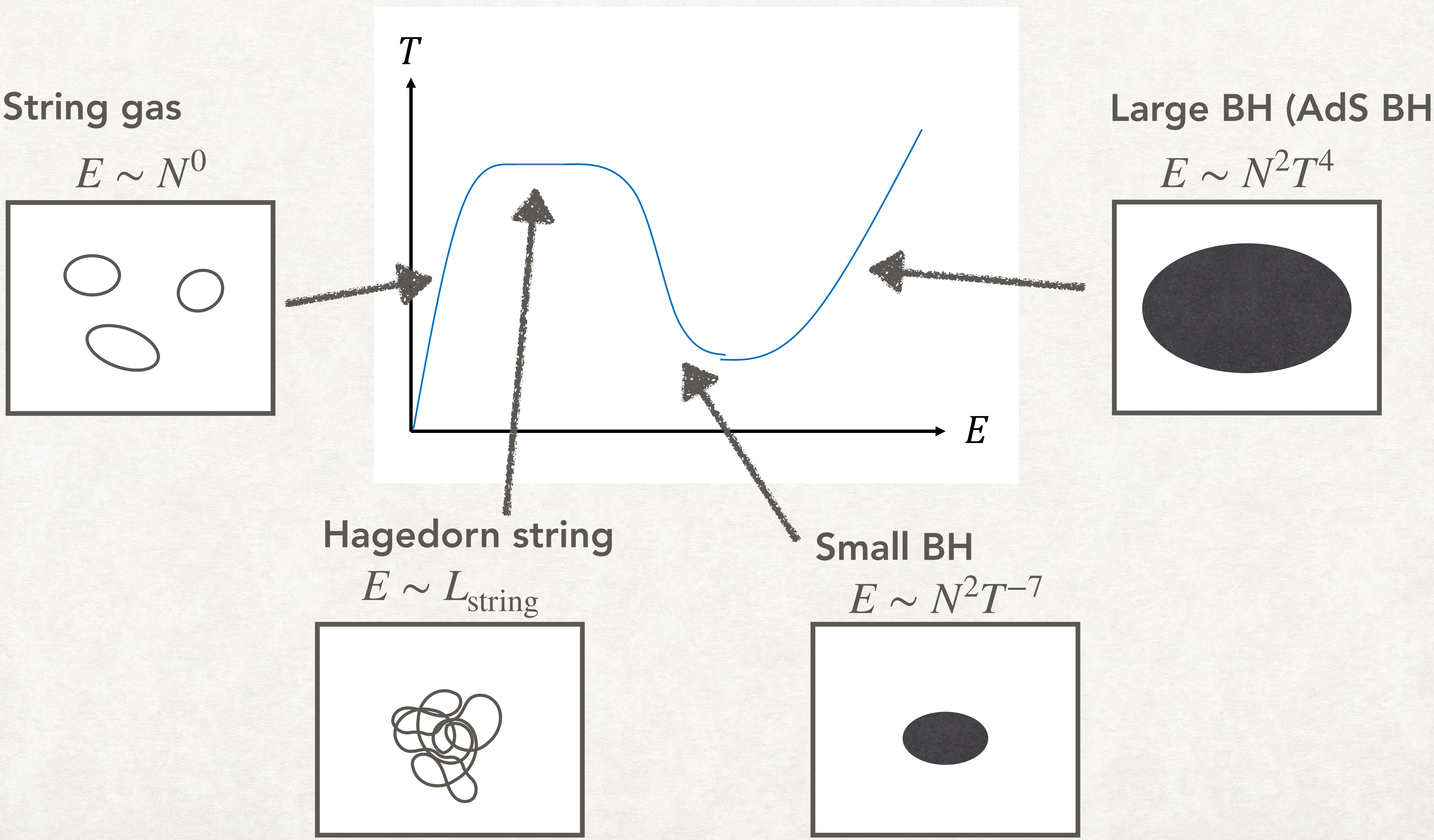
SU(N) super Yang-Mills theory
(SYM)

- Large N
- Adjoint representation

We want to study it
to learn about **quantum gravity**. →

Black hole in $AdS_5 \times S^5$ <=> 4d N=4 SU(N) SYM

Strongly coupled 4d SYM / dual string theory (E: fix)



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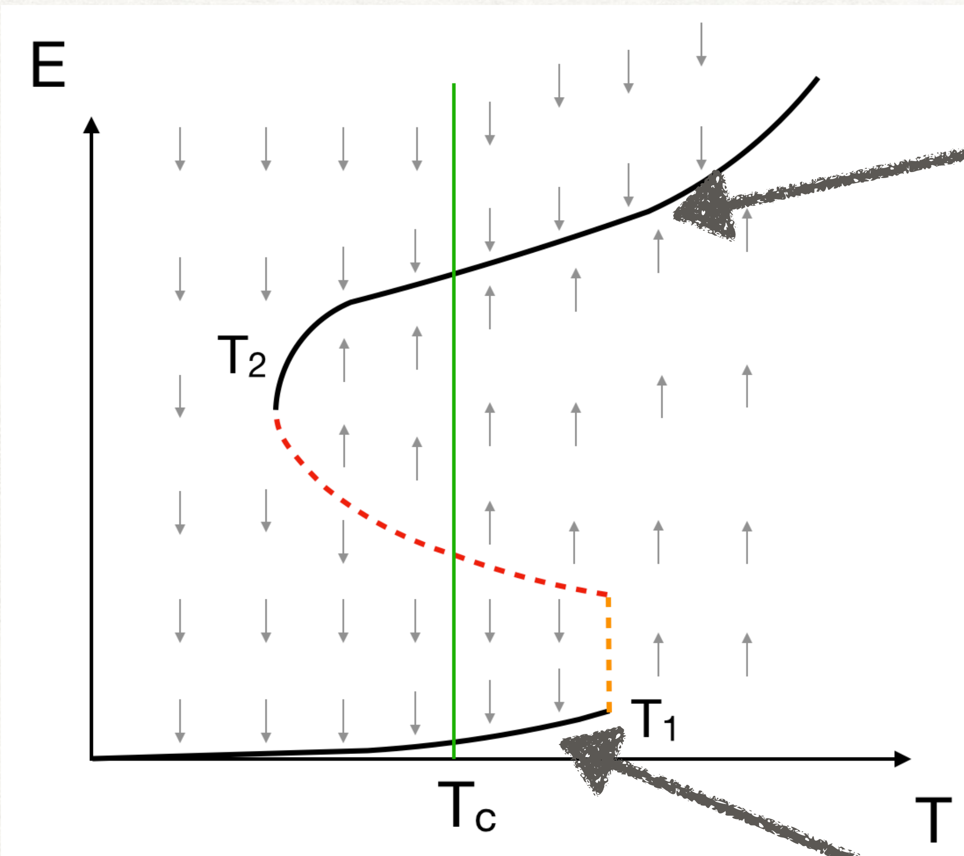
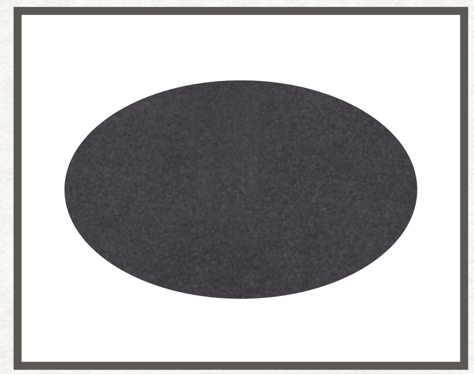


(T: fix)

How about
small BH or Hagedorn string?

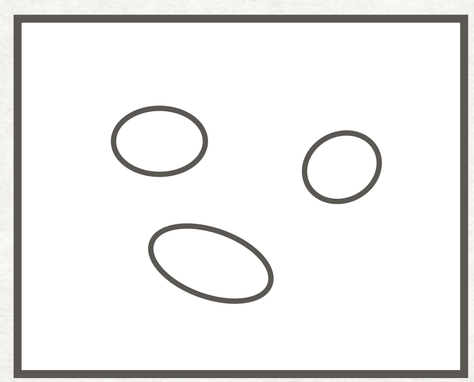
Large BH (AdS BH)

$E \sim N^2 T^4$



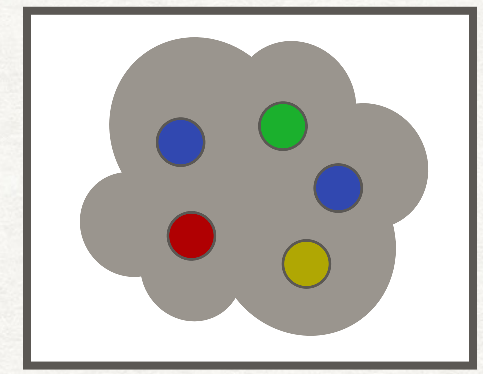
String gas

$E \sim N^0$



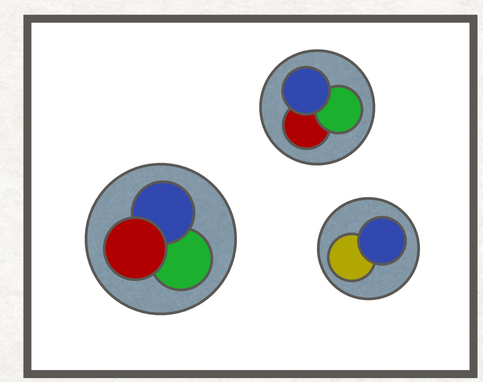
Gauge theory side;

Deconfined phase



phase transition

Confined phase



[Witten, (1998)]

D-branes with open strings & BH

Dp-brane : the objects that open strings can put their endpoints.

Classical vacua (:minima of potential)

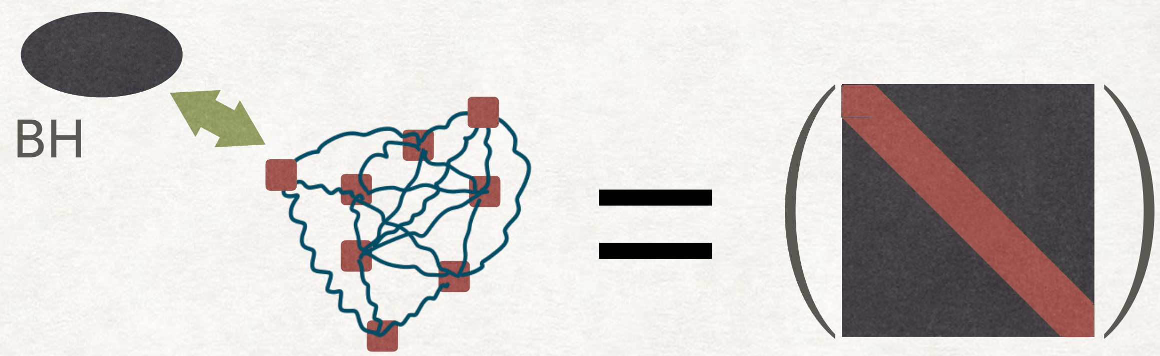
$$X_M = \mathrm{diag}\left(x_M^1, x_M^2, \cdots, x_M^N\right)$$

$(X_M)_{ii} = x_M^i$: Position of i th Dp-brane

$(X_M)_{ij}$'s fluctuation

: Open strings between i th and j th Dp-brane

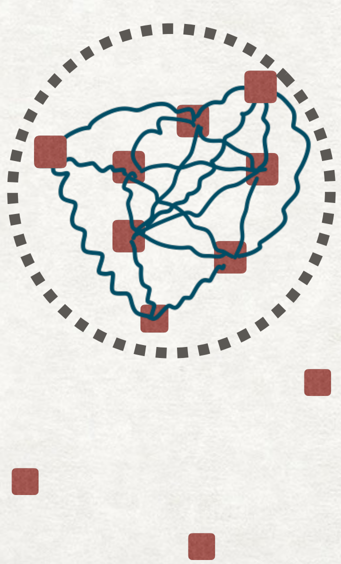
High temperature T region,

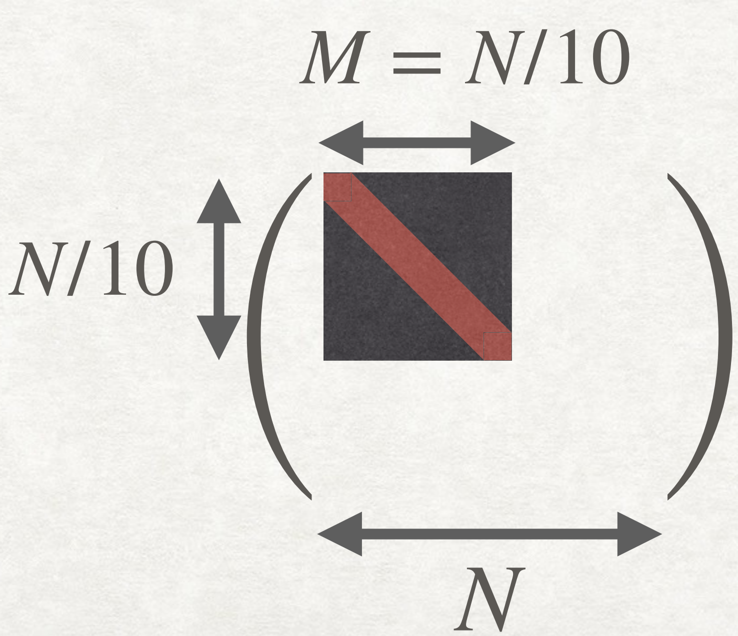


The bound state of D-branes & open strings

Configuration of scalar fields X_M

How about $E \sim M^2 = N^2/100$? [Hanada & Maltz, (2016)/Berkowitz, Hanada & Maltz, (2016)]





$M \times M$ subblock is formed

M^2 d.o.f. is deconfined

➡ "partial deconfinement"

Check of partial deconfinement

The order parameter of transition (review)

Polyakov loop : an order parameter of confine/deconfine transition

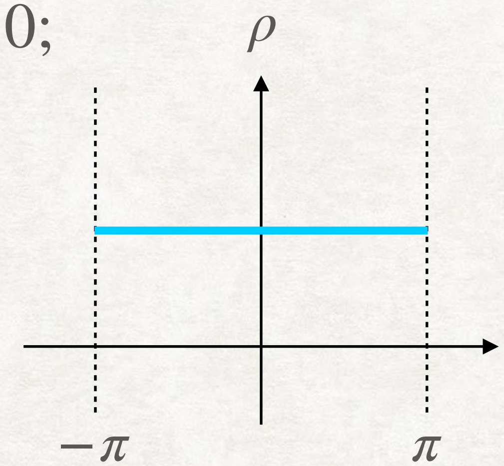
with $\mathrm{SU}(N)$ adjoint fields and large N.

$$P = \frac{1}{N}\mathrm{Tr}\mathscr{P}\exp\left[-\oint_{\text{temporal}} A_t\right] = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j} = \int d\theta\, \rho(\theta) e^{i\theta}$$

$$\rho(\theta) = \frac{1}{N}\sum_j \delta(\theta - \theta_j)$$: phase distribution

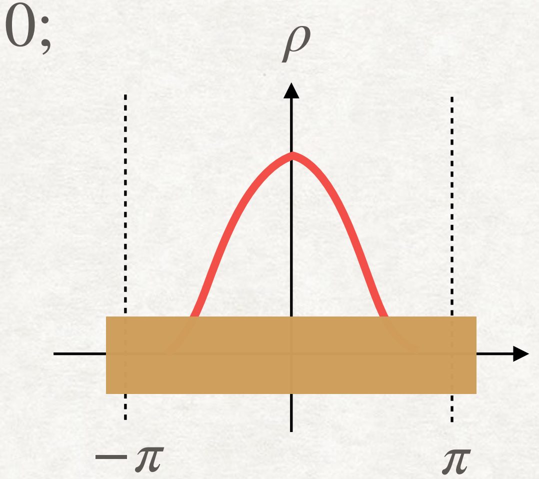
Can be regarded as continuous function in large N limit

$P = 0;$



Confined phase

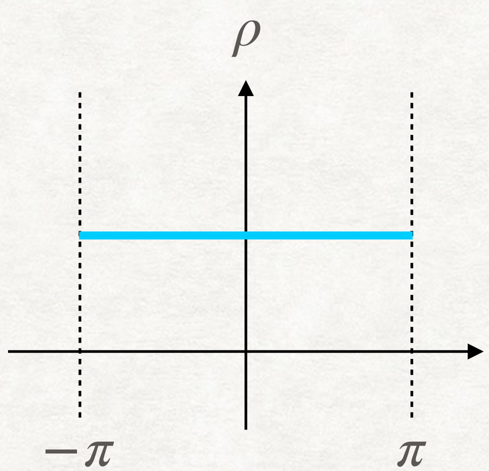
$P \neq 0;$



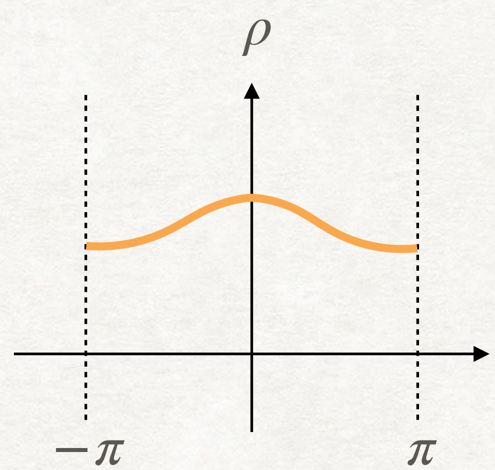
Deconfined phase

Partial deconfinement

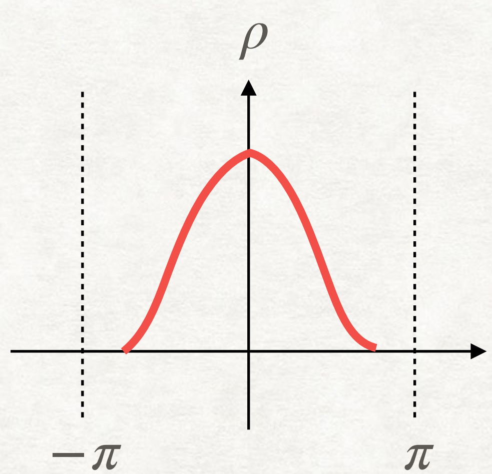
Polyakov loop : an order parameter of confine/deconfine transition



confined



"partially" deconfined



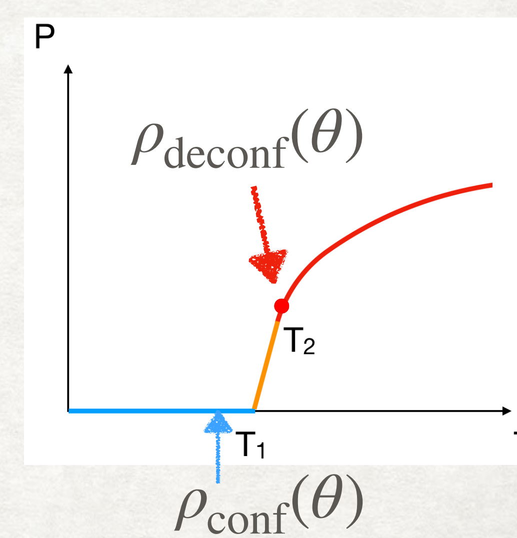
"fully" deconfined

Deconfinement transition

"Gross-Witten-Wadia" transition
[Gross & Witten, (1980)/ Wadia, (1980)]

Partial deconfinement $(M < N)$

$$\rho(\theta) = \frac{N-M}{N}\rho_{\rm conf}(\theta) + \frac{M}{N}\rho_{\rm deconf}(\theta) = \frac{N-M}{N}\cdot\frac{1}{2\pi} + \frac{M}{N}\rho_{\rm deconf}(\theta)$$



Partial deconfinement is "the mixture."

M θ_j s are in deconfined phase and N-M θ_j s are in confined phase

Examples of partial deconfinement

Examples of partial deconfinement

- 4d $\mathrm{U}(N)$ Yang-Mills theory with matters on S^3 (weak coupling);

[Aharony, Marsano, Minwalla, Papadodimas & Raamsdonk, (2003)/Schnitzer, (2004)]

At zero 't Hooft coupling;

$$Z(x) = \int [dU] \exp\left\{ \sum_{m=1}^{\infty} \frac{1}{m} \left(z_{\mathrm{B}}(x^m) + (-1)^{m+1} z_{\mathrm{F}}(x^m) \right) \mathrm{tr}(U^m) \mathrm{tr}((U^\dagger)^m) \right\} \qquad x \equiv e^{-\beta}, \quad z(x) = \sum_i x^{E_i}$$

$$\int [dU] \to \prod_i \int_{-\pi}^{\pi} [d\theta_i] \prod_{i<j} \sin^2\left(\frac{\theta_i - \theta_j}{2}\right), \quad \mathrm{tr}(U^n) \to \sum_j e^{in\theta_j}$$

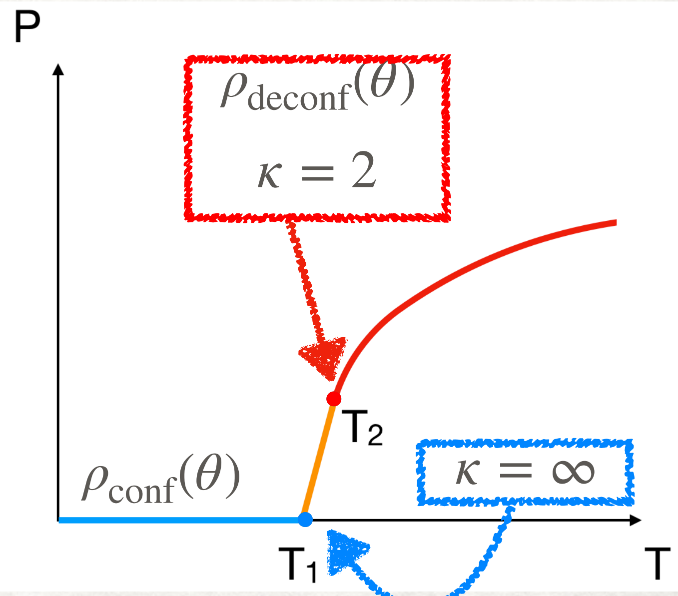
$$Z(x) = \int [d\theta_i] \ \exp\left(- \sum_{i \neq j} V(\theta_i - \theta_j) \right) \qquad V(\theta) = \ln(2) + \sum_{n=1}^{\infty} \frac{1}{n} \left(1 - z_{\mathrm{B}}(x^n) - (-1)^{n+1} z_{\mathrm{F}}(x^n) \right) \cos(n\theta)$$

At small nonzero 't Hooft coupling;

$$Z(\beta) = \int [dU] \ \exp\left[- \left(|\mathrm{tr}(U)|^2 (m_1^2 - 1) + b |\mathrm{tr}(U)|^4 / N^2 \right) \right]$$

When $b > 0$ $\quad (\kappa^{-1} = u_1(1 - m_1^2 - 2bu_1^2), \ u_1 = \mathrm{tr}(U)/N)$

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} & (T \leq T_1) \\ \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right) & (T_1 < T < T_2) \\ \frac{2}{\pi\kappa}\cos\frac{\theta}{2}\sqrt{\frac{\kappa}{2} - \sin^2\frac{\theta}{2}} & \left(T \geq T_2, |\theta| < 2\arcsin\sqrt{\kappa/2}\right) \end{cases}$$



At ● ; "GWW transition"

Examples of partial deconfinement

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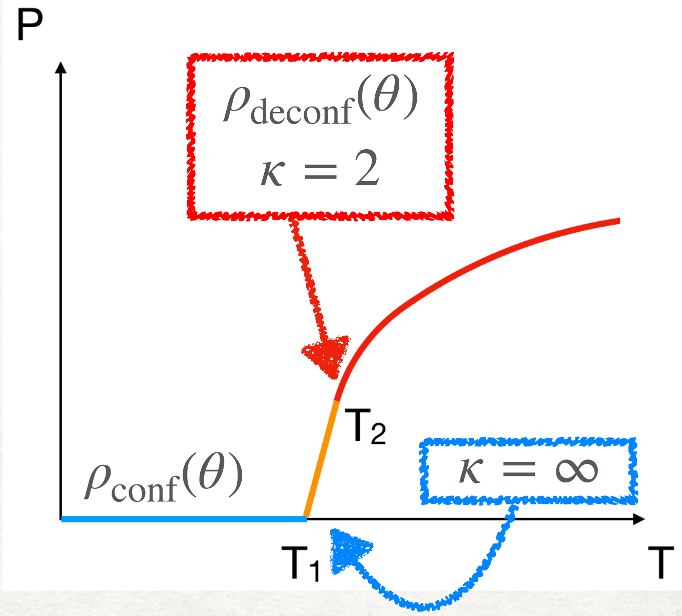
$$\frac{M}{N} = \frac{2}{\kappa}$$



$$\rho(\theta) = \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right) = \left(1 - \frac{2}{\kappa}\right)\cdot\frac{1}{2\pi} + \frac{2}{\kappa}\cdot\frac{1}{2\pi}(1 + \cos\theta)$$

At ● ; "GWW transition"

Partial deconfinement $(M < N)$

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- 4d $\mathcal{N}=4$ SYM on S^3 (weak coupling) ; [Sundborg, (2000), Aharony *et al*, (2003)]
- Free vector model, etc…

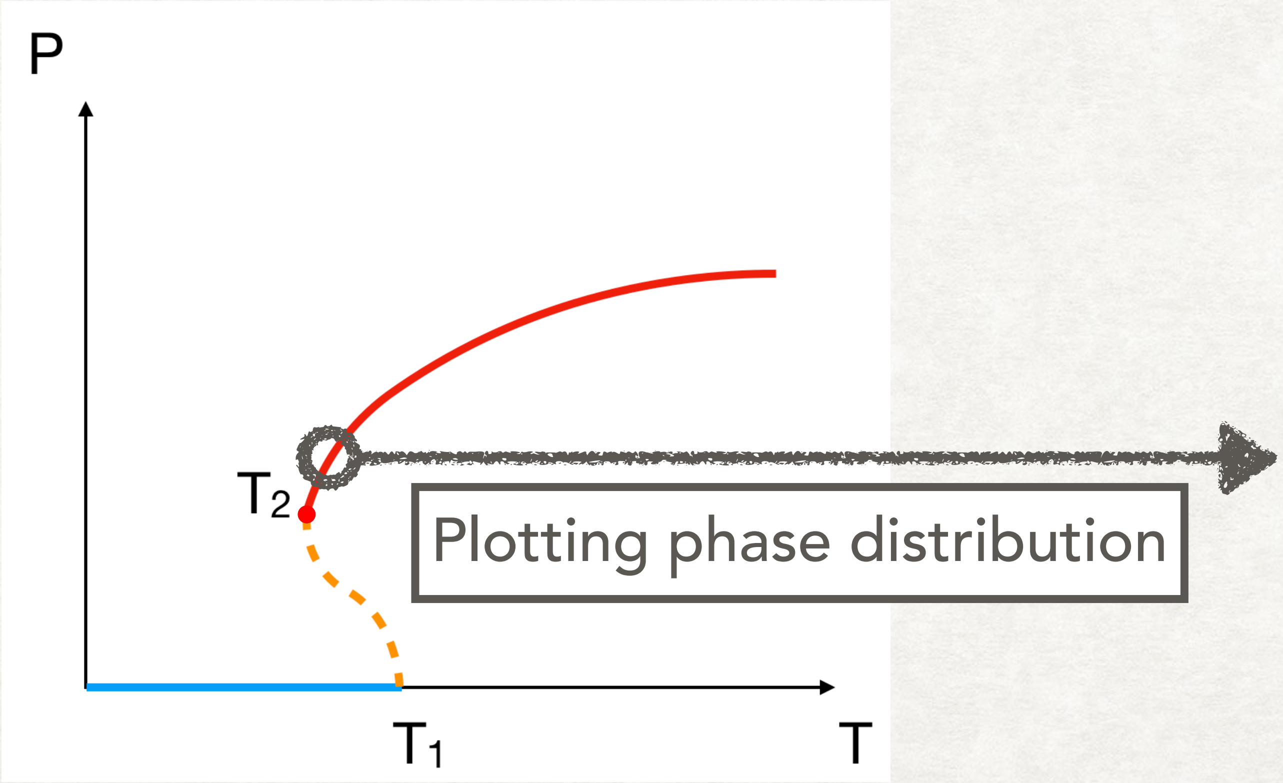
Examples of partial deconfinement

- The bosonic part of **plane wave matrix model** (PWMM or BMN matrix model)
  = the mass deform. of (0+1)d SYM / Matrix quantum mechanics.

$$L = N\,\mathrm{Tr}\left(\frac{1}{2}\sum_{I=1}^{9}\left(D_t X_I\right)^2 + \frac{1}{4}\sum_{I,J=1}^{9}\left[X_I, X_J\right]^2 - \frac{\mu^2}{2}\sum_{i=1}^{3} X_i^2 - \frac{\mu^2}{8}\sum_{a=4}^{9} X_a^2 - i\sum_{i,j,k=1}^{3}\mu\epsilon^{ijk}X_iX_jX_k\right)$$

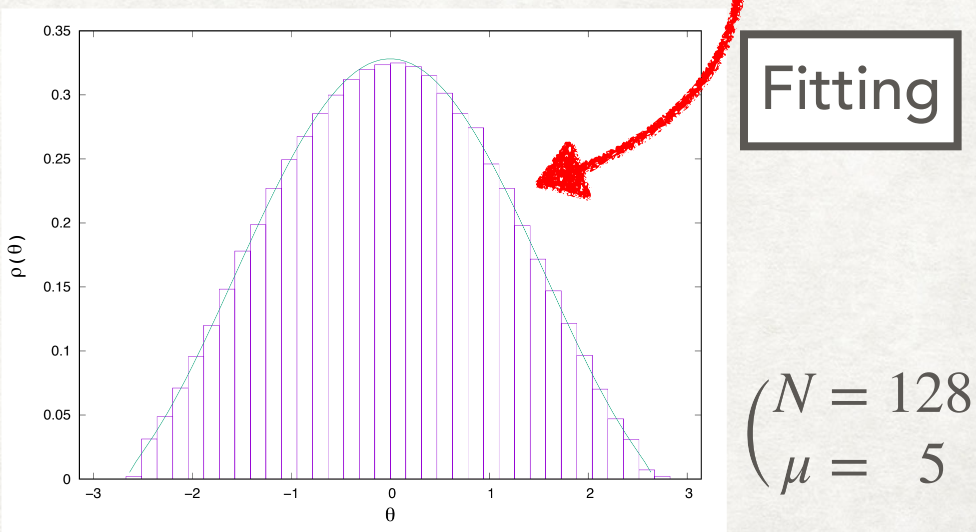
Check by Monte Carlo simulation;

- Hysteresis $(T_2 \leq T_1)$
- Phase distribution



Assumed GWW transition;

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} \\ \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right) \\ \frac{2}{\pi\kappa}\cos\frac{\theta}{2}\sqrt{\frac{\kappa}{2} - \sin^2\frac{\theta}{2}} \end{cases}$$



$$\begin{pmatrix} N = 128 \\ \mu = 5 \end{pmatrix}$$

Summary & Discussion

- We proposed the partial deconfinement which implies a part of color d.o.f. is deconfined in theory.
    - It's relating to small BH in dual gravity via holography
- We demonstrated the existence of partial deconfinement in several $SU(N)$ gauge theories.
    - This should happen in which does not have the center symmetry.
    - How about finite N case such as real world QCD?
    - Can we apply it to the unstable black hole?

Backup Slides

Examples of partial deconfinement

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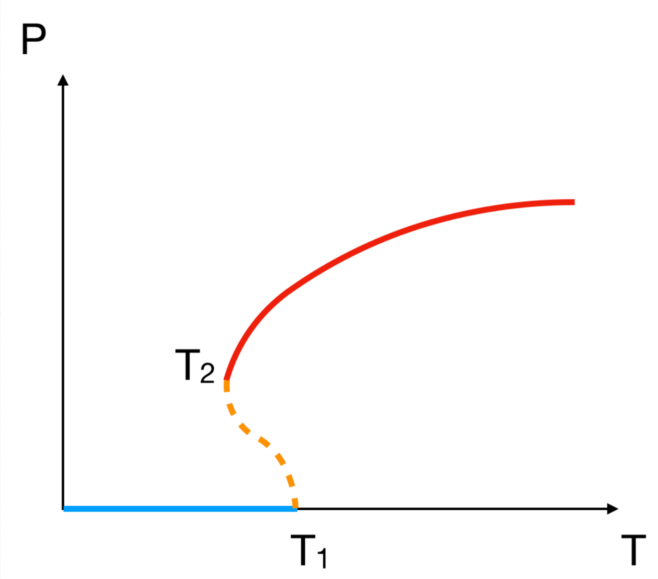

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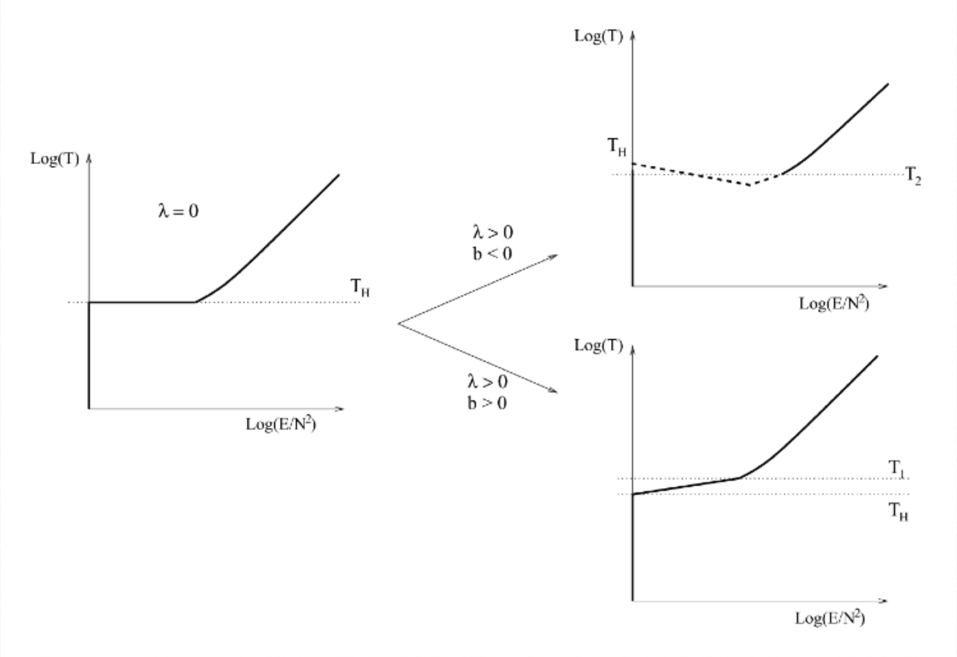
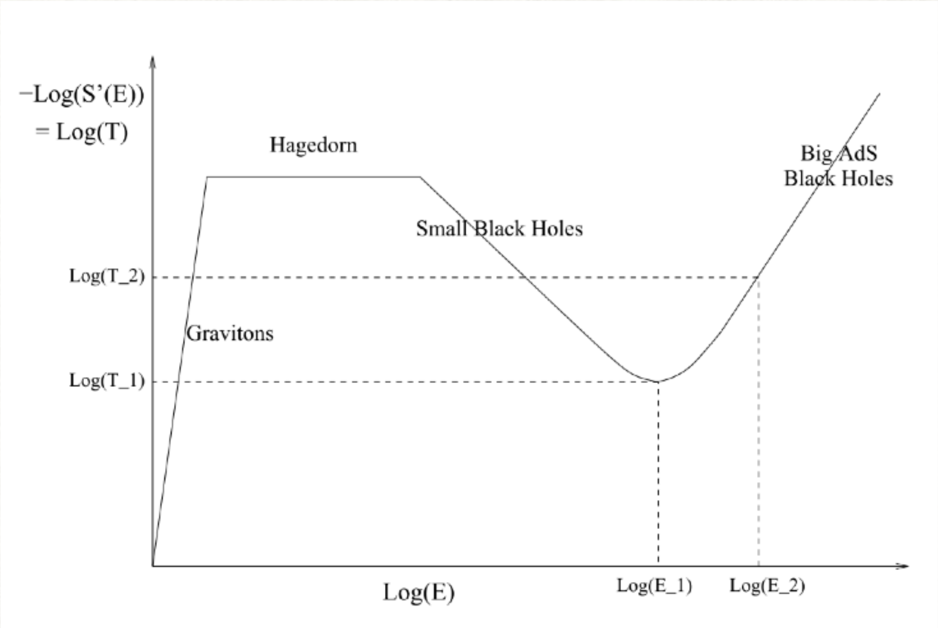
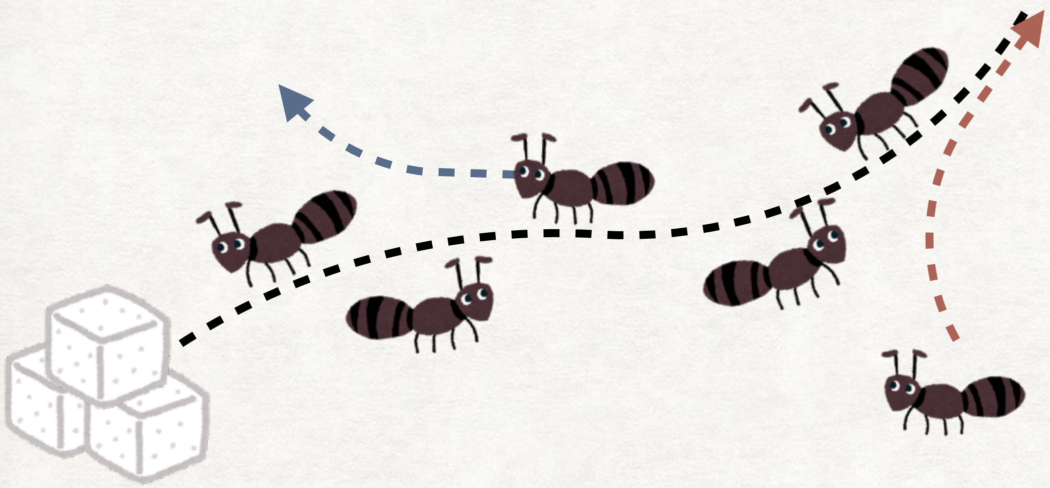


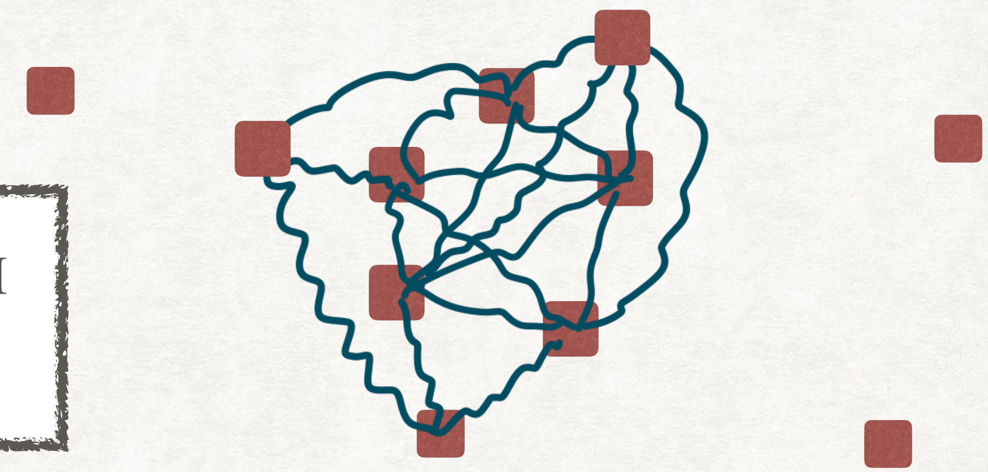


From [Aharony, Marsano, Minwalla, Papadodimas & Raamsdonk, (2003)]

Ant model and partial deconfinement



$$N_{\text{trail}} \leftrightarrow N_{\text{BH}}$$
$$p \leftrightarrow T$$



Ant trail : ants bound by pheromone

p : pheromone from each ant

BH : D-branes bound by open strings

T : index for excitation of open string

$$\frac{dN_{\text{trail}}}{dt} = (\alpha + pN_{\text{trail}})(N - N_{\text{trail}}) - \frac{sN_{\text{trail}}}{s + N_{\text{trail}}}$$

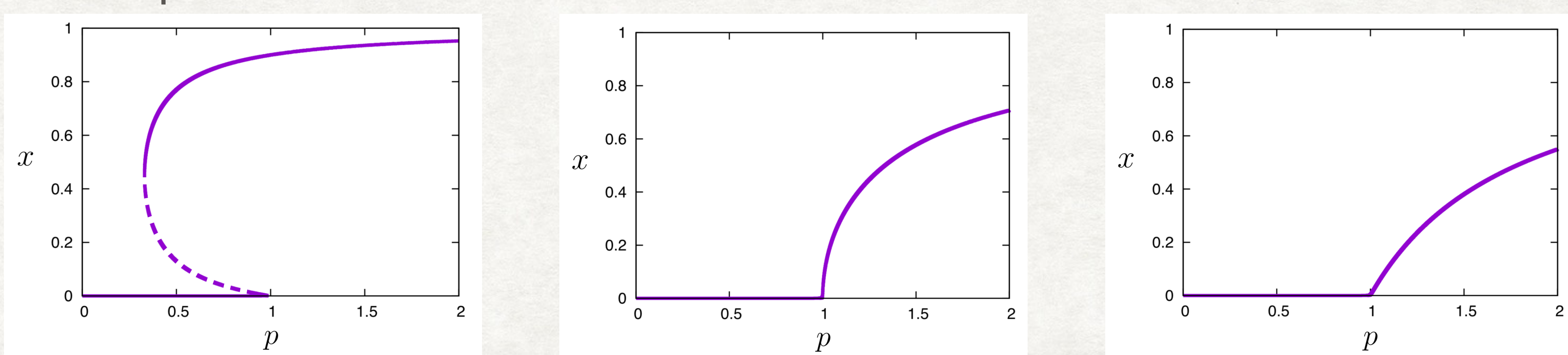
Inflow effect

Outflow effect

[Beekman, Sumpter & Ratnieks, (2001)]

saddle point

$$\frac{dN_{\text{trail}}}{dt} = 0, \quad x \equiv \frac{N_{\text{trail}}}{N}$$



small s ⟷ large s